Dynamic Method Used for Determining the Aerodynamic Forces Effecting the Models Tested in Short Duration Wind Tunnel

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Introduction

Time-dependent forces and moments acting on the model tested in a short-duration wind tunnel by the means of a strain-gauge balance can be determined by the following methods of processing: 1) averaging technique; 2) analytical method; 3) statistical method; 4) simplified statistical method [1]. The averaging technique requires a rather long time of wind-tunnel operation (so that oscillations caused by starting shock loads decay) and a comparatively long period with constant flow parameters. Still, even if these conditions are satisfied, there remains some uncontrollable systematic error. Three other methods implementing the principle of energy compensation are based on the following assumptions: the model is a rigid body; the oscillating system consists of the model and a certain part of the strain-gauge balance to be determined; the products of velocities of revolution around the axes of the system are rather small and can be neglected as compared to angular accelerations; the accelerations are measured at certain point by accelerometers.

The main difficulty in the analytical method is associated with determining part of the system involved into motion and subsequent determining mass, center of mass, and moments of inertia. The accuracy of reconstruction of the acting aerodynamic loads depends also on the coordinates where the accelerometers are located, which is a significant restriction of the method.

The statistical method is based on the fact that relations between the forces of inertia and accelerations measured by accelerometers can be linearized under an additional assumption that aerodynamic coefficients are constant over a chosen time interval determining the test time of the wind tunnel. It turns out that the number of equations (6) is smaller than the number of unknowns (42). Nevertheless, as the system is valid at an arbitrary time during the test, the system is solved by the least squares technique on a certain interval.

The simplified statistical method implies a linear dependence between acceleration and the corresponding "force", which reduces the number of unknowns. Three accelerometers are used (one in each direction), and the measurement axis coincides with the coordinate direction. The unknown proportionality coefficients depend on the model and on frequency with which they are used. These coefficients have to be determined in each test for the model mounted in the test section. The system is excited by a shock pulse, which leads to oscillations around the zero line without the action of aerodynamic forces. The measurements of sting accelerations and the balance measurements of forces and moments allow us to determine the coefficients. The averaging procedure is used to determine the aerodynamic coefficients.

The problem of choosing the interval to be used in statistical methods is not discussed in the original [1]. Nevertheless, the solution significantly depends on the place and length of the interval. The assumption about constant aerodynamic coefficients can also be invalid on the interval length needed for solving the problem. The required correlation between the oscillation periods of the system and the length of the steady-flow time window can impose additional restrictions on the model weight.

1. Dynamic method

In the present work, we propose a novel technique largely devoid of the above-mentioned drawbacks and taking into account the dynamics of model motion and inconstancy of flow parameters. The technique described below is a continuation of [2]. It is assumed that the signals registered during the response time of the strain-gauge balance are described by ordinary differential equations with constant coefficients, i.e., the system "model + sting + strain-gauge balance + attachment unit" is a linear dynamic object. The grounds for this assumption are as follows:

– the loads are such that the relation between strains and stresses is described by Hooke's law;
– the dependence of structural strains on the distribution functions of loads acting on the model is weak, which is confirmed, for instance, by the results of testing a scramjet and its elements in a blowdown wind tunnel with flow Mach numbers $M_\infty = 2$, 4, and 6 with simultaneous measurements of forces and moments by mechanical and strain-gauge balances [3, 4];
– the gauge size is small with respect to the characteristic size of the model;
– the changes in temperature of the model and gauges during the test are insignificant

In this case, obtaining the experimental responses of the system to unit loads, we determine the system of integral equations for reconstruction of the acting loads on the basis of responses of the strain-gauge balance registered in time.

2. Initial equations

Let the balance response $y_j(t)$ in the measured channels satisfy a system of ordinary differential equations with constant coefficients of the form

$$\sum_{j=1}^{n} \varphi_{ij}\left(\frac{d}{dt}\right) y_j(t) = f_i(t),\ i = 1,\ldots,n,\ n \le 6, \quad (2.1)$$

where $\varphi_{ij}(d/dt)$ are polynomials of d/dt and $f_i(t)$ are the acting external loads such that $f_i(t) = 0$ *for* $t \le 0$. If at $t = 0$ the values of the sought functions and their corresponding derivatives are equal to zero, the solution of system (2.1) is the normal response to the external load, whereas there is no solution of the homogeneous system in the general solution. This case corresponds to test conditions in short-duration wind tunnels, and the solution of system (2.1) has the following form (here, all quantities except for time have zero dimension, the time scale is arbitrary, and the time is normalized to the discretization step h):

$$y_j(t) = -\sum_{k=1}^{n} \int_0^t \frac{\partial U_{jk}(t-\tau)}{\partial \tau} f_k(\tau)\, d\tau, \quad (2.2)$$

Here $U_{jk}(t)$ is the normal response of the j - th component to a unit load on the k-th component.

Determination of normal responses

We introduce a matrix of generalized static loads $G=\{G_{ki}\}$ where k is the number of the component in the i-th variant of loading, a matrix of registered balance readings $Y(t)=\{y_{ji}(t)\}$ a matrix of coefficients of influence of static loads in terms of the k-th components on the readings of the j-th components $W=\{W_{jk}\}$, and a matrix of normal responses $U(t)=\{U_{jk}(t)\}$; $i,j,k=1,\dots,n$; where n is the number of components of the strain-gauge balance. It is convenient to determine the functions $U_{jk}(t)$ in an experiment by the unloading method: the model is preliminary loaded by a generalized force (force and/or moment of the force in some inertial coordinate system). After that, the load is removed within a short time δt, and the output signals $Y(t)$ are registered. Equation (2.2) with allowance for $f\equiv G=const$ yields the functions $U_{jk}(t)$:

$$U(t)=W-Y(t)G^{-1} \tag{2.3}$$

The number of loadings is normally greater than the number of strain-gauge components used, which allows choosing loading combinations with the conditionality number of the matrix $\mu(G^{-1})\sim 1$. Since the system is stable, then $\lim_{t\to\infty} Y(t)=0$, and its follows from (2.3) that $\lim_{t\to\infty} U(t)=W$. This circumstance is used to monitor the accuracy of dynamic calibrations. In reality, the measurements are performed discretely at the times; h is the time step determined by the equipment used.

3. Determination of forces and moments in the class of piecewise-constant functions

The problem of solving the integral equation (3.2) in the general case is ill-posed [5]. The method described above ensures problem regularization. The solution is constructed in the class of piecewise-constant functions on the set of segments $[t_i,t_{i+1}]$, which together form the total interval of solution determination $[0,t_{end}]$.

We write the initial equation (2.2) in the vector-matrix form:

$$Y(t)=-\int_0^t \frac{\partial U(t-\tau)}{\partial\tau} f(\tau)d\tau \tag{3.1}$$

Let the solutions $\{f_l\}$ be determined on the segments $[t_l,t_{l+1}], l=\overline{0,i-1}$. Then, the residue function for the i-th interval is written as the difference between the measured and calculated values of the strain-gauge balance signals:

$$\begin{aligned} z(t)&=w(t)-U(t-t_i)f_i, t\in[t_i,t_{l+1}]\\ w(t)&=Y(t)+\sum_{l=0}^{i-1}\left[U(t-t_{l+1})-U(t-t_l)\right]f_l \end{aligned} \tag{3.2}$$

Since $U(0)=dU(0)/dt=0$ the minimum length of the interval should be a finite quantity. We use the condition that the residue equals zero on the average over the interval. Assuming that $f_i=const$, we perform averaging in Eqs. (3.2) so that

$$<z_i>=\frac{1}{\Delta t_i}\int_{t_i}^{t_{i+1}} z(t)dt=0, <w_i>=\frac{1}{\Delta t_i}\int_{t_i}^{t_{i+1}} w(t)dt,$$

$$<U_i>=\frac{1}{\Delta t_i}\int_{t_i}^{t_{i+1}} U(t-t_i)dt$$

Then, the solution takes the form

$$f_i=<U_i>^{-1}\cdot<w_i> \tag{3.3}$$

The sought solution depends on the number of intervals N_{int} and on the distribution of their lengths (function $\varphi(\Delta t)$). To find this distribution, we introduce the functional of the root-mean-square residue

$$\Phi(N_{\text{int}},\varphi(\Delta t))=\frac{1}{t_{end}}\int_0^{tend} z^T(t)\cdot z(t)dt \tag{3.4}$$

where t_{end} is the time of the end of the process. The superscript $"T"$ indicates the operation of matrix transposition. Minimization of the functional (6.4) determines the required number of intervals, distribution of their lengths, and finally, the sought solution. It was experimentally found that the functional (3.4) has a set of local minimums. This circumstance should be taken into account in choosing the method of functional minimization. The algorithm used in the present paper does not ensure obtaining the absolute minimum of the functional in the general case, which requires a huge volume of computations. As compared to "local" algorithms, nevertheless, it offers substantially better results.

Algorithm of minimization of the functional Φ

1. The number of intervals N_{int} and a uniform initial distribution of their lengths are prescribed:
   $$t(i) = i \cdot \Delta t,\ \Delta t = t_{end} / N_{\text{int}},\ i = \overline{0, N_{\text{int}}}$$
2. For each point $t(i)$ a vicinity for its variation $T_i = \{t(i) : t(i) - M \cdot H \le t(i) \le t(i) + M \cdot H\}$ is defined, where H is the search step and $(2 \cdot M + 1)$ is the number of sampling points where the functional is calculated. In setting T_i, the following condition is satisfied:
   $$t(i-1) + L_{\min} \le t(i) - M \cdot H;$$
   $$t(i) + M \cdot H \le t(i+1) - L_{\min},$$
   $L_{\min}$ is a prescribed minimum allowable length of the interval. The values of H, M are also specified.
3. For T_i the lower limit of the functional is determined: $\Phi^* = \inf_{T_i} \Phi$. For $t(i)$ the value corresponding to Φ^* is used.
4. Items 2 and 3 are repeated for all internal points within $t(i)$.
5. The process is terminated if the admissible variations on T_i do not further reduce the functional.
6. Items 1–5 are fulfilled for several values of N_{int} among which the value corresponding to the minimum value of the functional with the corresponding distribution of integrals is chosen.

Comments

1) The above-described technique for reconstruction of time-dependent forces acting on the model does **not** impose restrictions on the model weight.
2) Since the technique takes into account the mutual influence of signals registered by all channels, requirements to designing and manufacturing the strain-gauge balance in terms of minimization of the mutual influence can be less severe.
3) Since deformation of strain-gauge sensors occurs under all loads, it is necessary to use a six-component balance even if it is necessary to acquire information on the number of components of the generalized force smaller than six.

4. Model problem

To illustrate operation of the algorithm, we consider a model linear dynamic object with three input actions and three output responses being registered. The following matrix of normal responses is set:

$$U(\tau) = W\left(1 - e^{-\tau}\left(\beta^{-1}\sin\beta\tau + \cos\beta\tau\right)\right),$$
$$\beta = 2\pi\omega_0 t_k,\ t = \tau t_k,\ t_k = 50[ms].$$

The matrices W and ω_0 [1/ms] are set, which determine the limiting values of the normal responses (analog of the static calibration matrix) and frequency of the mutual effects in the channels:

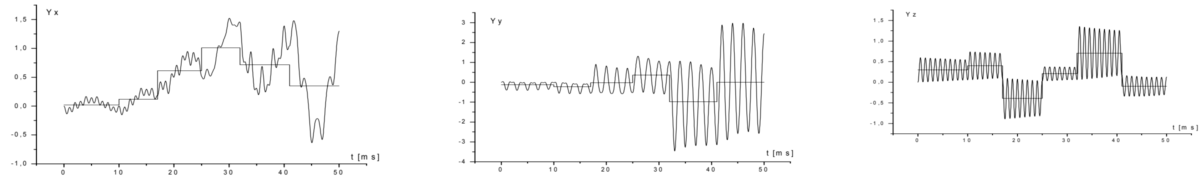
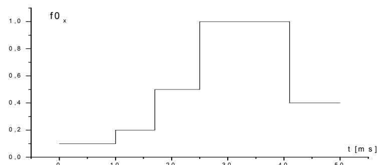


Fig.1. Responses of the model object to the actions f0

$$W = \begin{pmatrix} 1.0 & 0.1 & -0.2 \\ -0.2 & 1.0 & 0.3 \\ 10^{-2} & 10^{-3} & 1.0 \end{pmatrix},\ \omega_0 = \begin{pmatrix} 0.1 & 0.5 & 1.0 \\ 0.1 & 0.5 & 1.0 \\ 0.1 & 0.5 & 1.0 \end{pmatrix}$$

Six intervals are defined by partition points ti0, and the piecewise-constant forces f0 on these intervals are set (Fig.2). On the segment $[0, t_k]$, with a step $h = 0.04[ms]$, the responses of the object to the actions f0 are calculated as $Y(\tau_i) = U(\tau_i) \cdot f0(\tau_i)$ (their plots are shown below).

These values were further used as measurement results to reconstruct forces (Fig. 2).



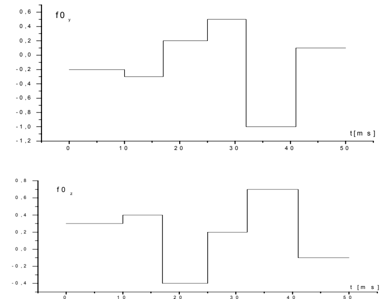


Fig.2. Reconstructed forces

The plots of responses (Fig. 1) show the calculated mean values over the intervals, determined by solving the problem. The accuracy of the solution for the functional and intervals is ~ 10^{-11}. Naturally, the accuracy of the solution significantly depends on noise, its intensity, frequency, distribution law, and properties of symmetry.

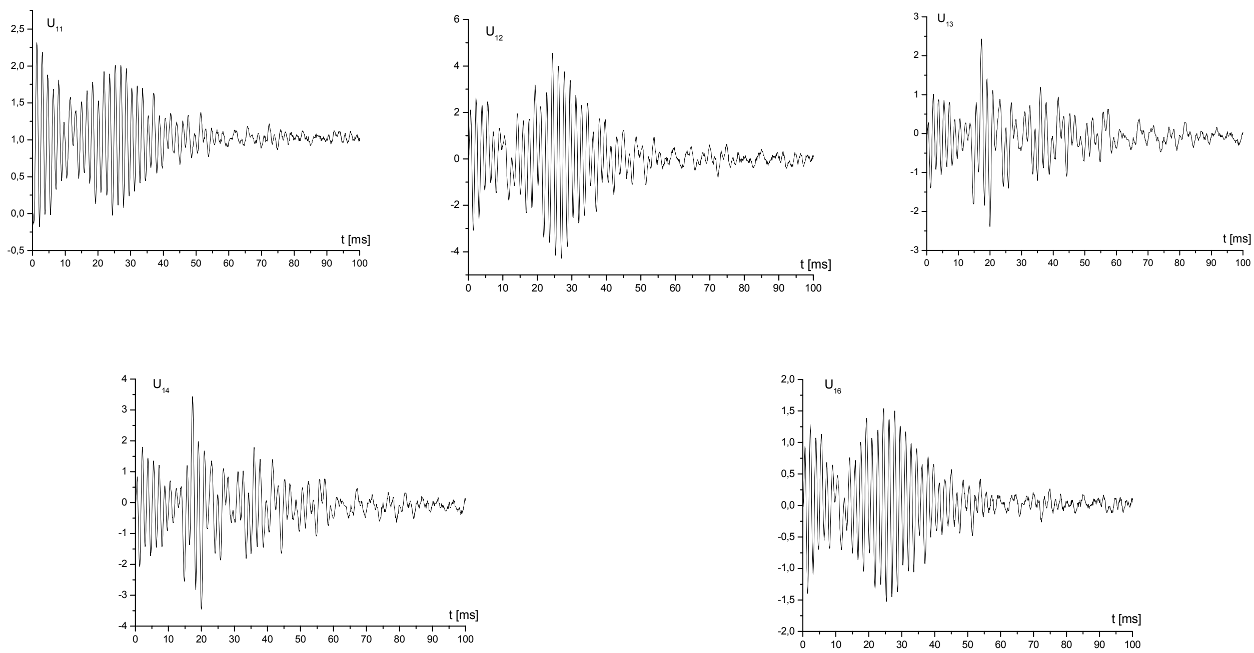


Fig.3. Normal responses of the **Fx**-component to unit loads in different channels

5. HB-2 reference model

Figure 3 shows the normal responses of the longitudinal component to unit loads, which were obtained in five channels for the HB-2 model. The model was mounted in the test section of the AT-303 wind tunnel of ITAM.

Figure 4 shows an example of the time evolution of the measured longitudinal force f_X and the corresponding drag coefficient C_X.

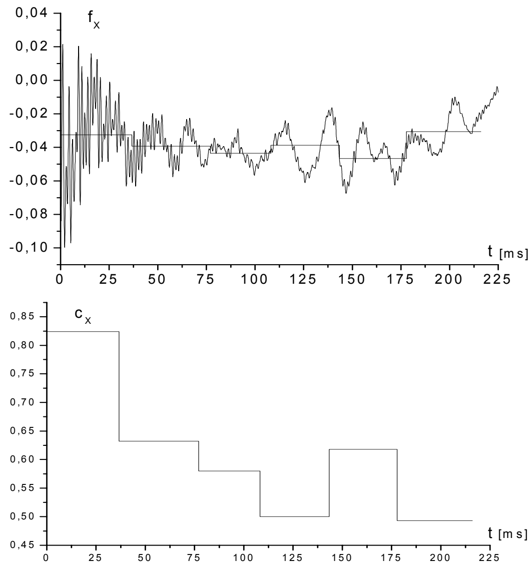


Fig.4. Time evolution of experimental values of the longitudinal force and the corresponding drag coefficient for M=12 and angle of attack 12°

Figure 5 shows the time-averaged drag coefficient with allowance for the correction for conicity by Newton's theory.

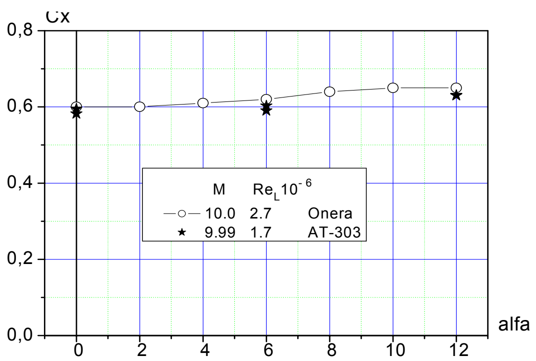


Fig. 5. Time –averaged drag coefficient

Conclusions

The results of solving the model problem and testing of aerodynamic characteristics of the HB-2 reference model on the basis of experimental data obtained in the AT-303 short-duration wind tunnel of ITAM testify to reliability of solutions obtained by the method described.

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