SHIMMY MODEL OF AIRCRAFT LANDING GEAR AND QUADRATIC FORMS

V.I. Goncharenko
Aeronautical Scientific/Technical Complex named after O.K.Antonov (ASTC “ANTONOV”), Kyiv, Ukraine

A.Yu. Obolensky
Institute of Mechanics named after S.P.Timoshenko, Kyiv, Ukraine

Introduction

The wheel is one of the most important inventions of our civilization. The pneumatic tyre plays the considerable and determining part in vehicle dynamics. The interaction between the landing gear of aircraft and tyre dynamic behaviour can be cause of shimmy vibrations.

The landing gear is an important and complex system. The landing gear should be free from excessive vibrations and dynamical instabilities, in particular a shimmy vibrations. Shimmy is the self-excited oscillatory motion of a wheel about (an almost) vertical steering axis. Such type of unstable motion about vertical steering axis is usually designated as the wheel shimmy oscillation.

Shimmy is a violent and possibly dangerous vibration. Shimmy does not only occur on aircraft but has also been encountered on the steerable wheels of cars, trucks, and motorcycles and on the caster wheelchairs too. The vehicle forward motion kinetic energy is transferred to self-excitation energy through the road to tyre side force and aligning moment [1].

Shimmy is an oscillatory combined lateral-yaw motion of the landing gear. There is a number of different tyre models developed for application to the shimmy problem. There are the tyre theories of Boris von Schlippe, Mstislav V. Keldysh, William J. Moreland, Hans B. Pacejka and others. The question which model is most accurate in predicting shimmy has been a source of disputes [2].

Shimmy can occur on both nose and main landing gears. Shimmy is complex phenomenon and is influenced by many design parameters. Most publications usually deal with detailed models of the landing gear and are generally focused on solving shimmy problems for this particular configuration [2].

This paper will be concentrated only on landing gear model or wheel suspension model. Inasmuch this paper focuses an attention on model of landing gear no attention will be given to a steering system. The steering system (or shimmy damper) is an important factor in landing gear shimmy vibrations. Presence of such system or damper can be always taken into account.

The real design of the aircraft landing gear is a complex 3-D design. It is necessary to be able to represent the landing gear as the generalized model suitable in most cases.

Generalized model of landing gear

It is usually possible to neglect mass of a wheel suspension. Therefore the suspension of wheels can be presented as an elastic element. Always it is possible to take into account inertia of wheel suspension known methods.

For simulation of shimmy phenomenon the simplified shimmy model with one degree of freedom is usually used. The model of a wheel suspension with two degrees of freedom is frequently applied to analysis of a real design. In the general case, it is necessary to take into account five degrees of freedom of a wheel suspension for the valid description of the shimmy phenomenon [3].

For the symmetrical landing gear it is enough to take into account three degrees of freedom of a wheel suspension: Z, Ψ, and Θ. The degrees of freedom reads: Z is the lateral coordinate, Ψ the roll angle and Θ the yaw (torsion) angle. In this case a flexible matrix on the basis of Hooke's law characterizes the elastic properties of landing gear design. The Hooke law connects corresponding deformations (δ_Z, φ_X, and φ_Y) of structure with applied force P_Z and moments M_X and M_Y [4]:

$$\begin{pmatrix} \delta_Z \\ \varphi_X \\ \varphi_Y \end{pmatrix} \equiv \begin{pmatrix} Z - Z_{rot} \\ \Psi - \Psi_{rot} \\ \Theta - \Theta_{rot} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} P_Z \\ M_X \\ M_Y \end{pmatrix}.$$

Here the elasticity matrix $A = (a_{ij})$ is symmetric ($A = A^T$) and positive definite ($A >> 0$). This property of elasticity matrix defines a physical realizability of the landing gear design. The index *"rot"* means corresponding displacement due to turn at presence of a steering system or shimmy damper.

In the flexible matrix $A = (a_{ij})$ is presented six independent coefficients. Accordingly, for the description of a landing gear model it is required six independent parameters. Hence there are six parameters instead of two parameters, four parameters or five parameters. Any model of a landing gear with smaller than six the number of parameters is rough approximation the real landing gear, for example, [5]. In the latter case it is necessary to prove in addition an opportunity of its application for mathematical analysis. As usually such substantiation is not easier than solving of a full problem.

Main principle of the solution of applied problems is a full definability of all used parameters. Starting from this, the solution of a full problem with six the number of independent parameters a_{ij} is practically valid. It is possible to explain it so. The matrix $A = (a_{ij})$ has concrete physical sense. Coefficients a_{ij} of the flexible matrix can be determined by calculation during of landing gear design or are measured during natural experiment of the landing gear structure. For this reason the approach described above for a long time has supporters, in particular [6].

Canonical form of landing gear model

Any model should be enough simple and exact. It means that the solution of a mathematical problem should give a result with accuracy comprehensible to practice and the mathematical model used at it should be is comprehensible simple at the same time.

Though the flexible matrix $A = (a_{ij})$ also characterizes unequivocally elastic properties of a design but at the same time the matrix does not give evident representation about a landing gear kind. Besides matrix representation does not allow to vary the landing gear parameters purposefully. Accordingly, the matrix representation does not allow recommending changes of the landing gear parameters in habitual terms: the landing gear (effective) length, the mechanical trail or caster length and the cant angle of landing gear; the landing gear torsional stiffness, the roll stiffness and the lateral stiffness of landing gear, see also figure 1.

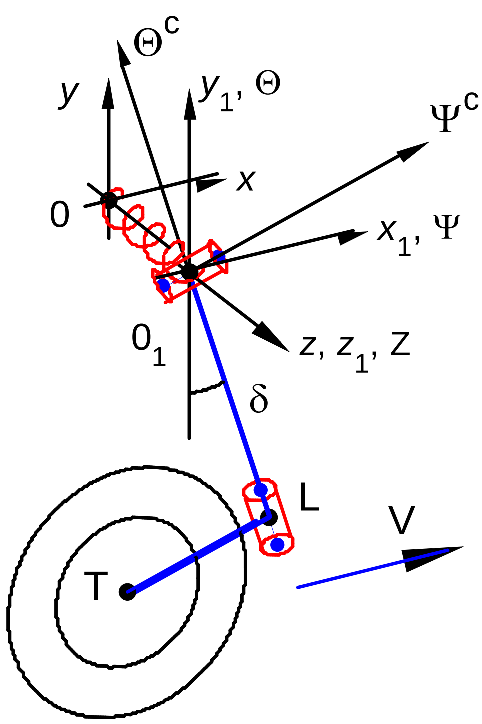

Fig. 1

Therefore it is expedient to compare to the flexible matrix $A=(a_{ij})$ some hypothetical design such as a landing gear to physically evident parameters. The first author has offered such description with the help of the six parametrical model of a landing gear at the end of the seventieth years that is marked in the thesis of the first author.

Any symmetric and positive definite matrix A can be represented in the form $A=R^T L^T D\, L\, R$ [7], where matrix

$$D = diag\left(\frac{1}{C_Z}\ \frac{1}{C_\Psi}\ \frac{1}{C_\Theta}\right), \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ -\ell & 1 & 0 \\ t & 0 & 1 \end{pmatrix},$$

and $R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{pmatrix}$.

If we have the flexible matrix $A=(a_{ij})$, then we know a number of parameters δ, ℓ, t, C_Z, C_Ψ, and C_Θ. These parameters have a physical sense: δ is the cant angle of landing gear, ℓ is the landing gear (effective) length, t is the mechanical trail or caster length of landing gear, C_Z is the lateral stiffness of landing gear, C_Ψ is the roll stiffness of landing gear, and C_Θ is the landing gear torsional stiffness. The canonical model of an aircraft landing gear is represented in figure 1. In this figure $LO_1=\ell$ and $TL=t$.

If there are some elastic units, then the matrix $A=\sum R_i^T L_i^T D_i\ L_i\ R_i >> 0$. Therefore, the stiffness matrix of the landing gear structure is $C=C^T >> 0$ and $C \equiv A^{-1} = R^T L^{-1} D^{-1} \left(L^{-1}\right)^T R$, where

$$L^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \ell & 1 & 0 \\ -t & 0 & 1 \end{pmatrix}, \quad D^{-1} = diag\left(C_Z\ C_\Psi\ C_\Theta\right).$$

Example. Let us consider an elastic landing gear with a self-castering axis of wheels. In this case $i=0$ and

$$L_0 R_0 = \begin{pmatrix} 1 & 0 & 0 \\ -\ell_0 & \cos\delta_0 & \sin\delta_0 \\ t_0 & -\sin\delta_0 & \cos\delta_0 \end{pmatrix}.$$

We have $A_0 = (L_0 R_0)^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_\Theta^0} \end{pmatrix} (L_0 R_0)$, whence $A_0 = \frac{1}{C_\Theta^0} (e_0 L_0 R_0)^T (e_0 L_0 R_0)$, where $e_0 = (0\;0\;1)$. If $u_0 = e_0 L_0 R_0$, i. e., $u_0 = e_0 L_0 R_0 = (t_0 \; -\sin\delta_0 \; \cos\delta_0)$, then we write $A_0 = \frac{1}{C_\Theta^0} u_0^T u_0 \geq 0$. Finally, we obtain $A = A_1 + A_0 = (a_{ij}) + \frac{1}{C_\Theta^0} u_0^T u_0 >> 0$. Note that there is similar procedure in the paper [8].

Such evident description of aircraft landing gear has the deep fundamentals and the roots connected to a canonical form of a symmetric matrix. Accordingly, such representation of aircraft landing gear should be named the canonical form of the landing gear too. The canonical form of the landing gear suggested by authors is characterized special by simplicity and differs by grace in comparison with known, for example, with the Stevens description [6] and the Kluiters description. The last description contains in the internal Fokker report. The given representation is in detail enough resulted in the thesis of Besselink [2].

Concluding remarks

For comparison of landing gears and for performance of mathematical analysis it is convenient to use the canonical representation of aircraft landing gear. The parameters (δ, ℓ, t, C_Z, C_Ψ, and C_Θ) of a canonical form uniquely determine the stiffness properties of the real landing gear design. And back, the stiffness properties of a landing gear design determine a set of parameters (δ, ℓ, t, C_Z, C_Ψ, and C_Θ). This correspondence is unequivocal to within rename of some parameters and definition of the landing gear cant angle δ to modulo $\pi/2$ and (or) within to change of δ for $\pi/2 - \delta$.

The canonical representation (δ, ℓ, t, C_Z, C_Ψ, and C_Θ) of landing gear models lets to make experimental model test correctly and to analyze shimmy of an aircraft landing gear, for example, ``Antonov 28", ``Antonov 32", ``Antonov 72", ``Antonov 74", ``Antonov 124", ``Antonov 225", ``Antonov 70", ``Antonov 38", ``Antonov 140", and ``Antonov 148".

Figure 2 shows results of research of the main gear shimmy phenomenon in some typical case.

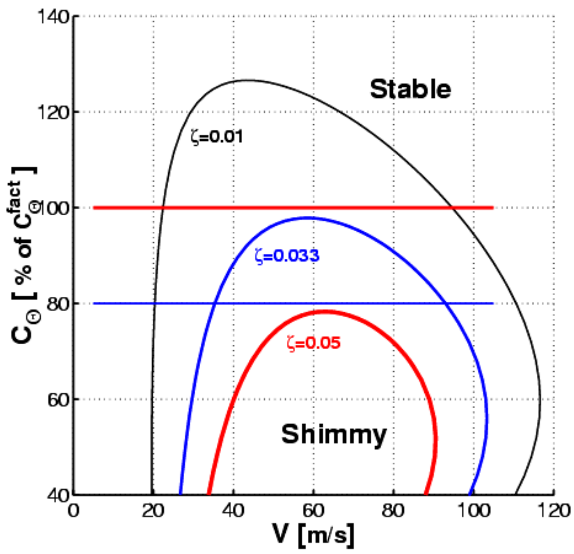

Fig. 2

The boundary of shimmy stability in the parameter space is represented on the plane: the forward velocity and the torsional stiffness [9]. The parameter V is a forward velocity of an aircraft model and the parameter C_Θ is a torsional stiffness of landing gear model on figure 2. The boundaries of shimmy on a plane (V, C_Θ) for three values of the damping factor $\zeta = 0.01$, 0.033, and 0.05 are represented at the figure.

In conclusion, shimmy of a landing gear remains a relevant problem today in spite of the long history. It is necessary should be to publish in the open literature all the results of research and the analyses of a shimmy vibration for a better understanding of the shimmy phenomenon and possibly developing of guidelines for a shimmy-free landing gear design. The contribution of authors to the decision of shimmy problem is the suggested canonical form of a landing gear model of an airplane.

References

[1] Pacejka H.B. *Tyre and Vehicle Dynamics.* Oxford: Butterworth-Heinemann, 2002.

[2] Besselink I.J.M. *Shimmy of Aircraft Main Landing Gears.* Delft: The Netherlands by DocVision, 2000.

[3] Goncharenko V.I. Shimmy of nose gear on an asymmetric suspension. *International Applied Mechanics*, Vol. 33, No. 2, pp 168 – 173, 1997.

[4] Goncharenko V.I. *The general statement of shimmy problem for a wheel with a 3 DOF suspension and the extension to the case of Coulomb friction in a rotary unit.* Submitted to VINITI, DO 4513. 1980.

[5] Keldysh M.V. Shimmy in the forward wheel of a 3-wheel or tricycle type landing gear. *Transactions of the TsAGI*, Vol. 564, 1945.

[6] Stevens J.E. Shimmy of a nose gear with dual co-rotating wheels. *Journal of the Aerospace Sciences,* Vol. 28, No. 8, pp 622 – 630, 1961.

[7] Obolensky A.Yu. and Obolensky I.A. *Lectures on Analytical Geometry*. Moscow, Izhevsk. Institute of Computer Science. 2004.

[8] Goncharenko V.I., Lobas L.G. and Nikitina N.V. Wobble in guide wheels. *Soviet Applied Mechanics*, Vol. 17, No. 8, pp 765 – 769, 1981.

[9] Goncharenko V.I. Fundamentals of shimmy fixed wheel prevention. *Proceedings, International Conference “AVIA – 2003”.* April 2003, Kyiv, Ukraine.