OPTIMAL DESIGN OF COMPOSITE LATTICE STRUCTURES FOR AEROSPACE APPLICATION

G. Totaro
Italian Aerospace Research Center – Advanced Materials and Technologies, Capua, Italy

Z. Gürdal
Delft University of Technology, Aerospace Structures Department, Delft, The Netherlands

The paper deals with numerical optimization of composite lattice structures without skin, consisting of regular, dense and symmetric system of intersecting hoop and helical ribs manufactured through winding technologies. This concept, if properly exploited, can lead to highly efficient design for structures having cylindrical or conical shape and experiencing high compressive or bending loads (i.e., interstages and load adapters for spacecraft launchers, fuselage barrels for aerial vehicles).

Design of axially compressed cylindrical lattice shells is deeply described in Ref. [1], whereas design constraints for cylindrical and conical lattice structures without skin are analytically formulated in Refs. [2], [3] with due regard to the corresponding main failure mechanisms of the structures. These formulations allow us to set up a minimization problem for the mass of the shell based on four design variables, i.e. the helical angle, the shell thickness and the widths over distances of hoop and helical ribs. The minimum mass solution and the corresponding optimal design parameters have been analytically derived for the general Anisogrid concept by means of the minimization of the safety factors corresponding to considered failure mechanisms [3]. In case of cylindrical shell, Anisogrid concept can also include the more specific Isogrid configuration just forcing hoop and helical ribs to have the same width and the cells to be equilateral triangles [4].

Nevertheless, since the minimum mass solution is based on buckling and strength constraints, axial or bending stiffness of the shell can not be addressed. In order to overcome this possible limitation of analytical design, an optimization routine has been implemented. The routine adopts the same formulated buckling and strength constraints used for analytical design, but is based on five design variables which characterize the structure, i.e., half the total number of helical ribs, n_h, the number of hoop ribs, n_c, the shell thickness, H, and the widths of hoop and helical ribs, b_c and b_h, respectively. The routine allows us to explore all the suboptimal configurations that are located in the vicinity of the minimum mass solution and to show which of these configurations can be used to satisfy also the stiffness and the manufacturing constraints [5].

Thus, obtained solutions can be substantially different from those following from the strength design, but in any case, the mass increase can be effectively minimized. Moreover, the routine can prospectively add the possibility to control directly the stiffness of the shell by imposing the stiffness requirement as additional constraint.

In order to demonstrate numerical optimization of composite lattice structures, a preliminary design is finally carried-out for a launch vehicle interstage structure.

Numerical versus analytical optimization

Analytical optimization allows us to address the minimum mass solution under strength and buckling constraints. The obtained results demonstrate an infinity of optimal isomass solutions as functions of applied loads and basic material properties (compressive strength and elastic modulus of ribs). All these solutions show the same helical angle and shell thickness. The last two design variables, i.e., the widths over distances of hoop and helical ribs are dimensionless. Thus, only one parameter needs to be preassigned to specify all of them. This parameter can be used to take into account some practical conditions (e.g., the manufacturing constraints) and to provide the adequate density of the grid cells. If, for example, the width of hoop ribs is preassigned, the spacing of hoop and helical ribs is consequently found, so that the number of hoop and helical ribs is approximately known as well. This means that for the unique optimal angle many optimal configurations exist.

Structural configurations can be also geometrically described in terms of discrete variables depending on the given shape of the shell. In this case, the helical angle is no more regarded as a design variable but turns out to be linked to the other variables, namely, the number of hoop ribs, n_c, and half the total number of helical ribs, n_h.

To simplify the geometrical description, we presume that the lattice structure end cross sections coincide with the first and the last hoop ribs. Equations for the constraints and for the objective function are the same that those used for the analytical minimization.

Optimization routine has been initially verified by analytical results corresponding to minimum mass solution for both the general Anisogrid concept and for the particular Isogrid case.

Cylindrical shell

In case of the cylindrical lattice structures (Fig.1), the helical angle φ is expressed in terms of the rest design variables with the aid of the following equations:

$$n_h d_c = 2\pi R; \quad d_c \cos\varphi = 2a_c \sin\varphi \qquad (1)$$

$$n_h 2a_c \frac{\sin\varphi}{\cos\varphi} = 2\pi R\,; \quad L = (n_c - 1)a_c \qquad (2)$$

$$\frac{n_h 2L}{n_c - 1} tg\varphi = 2\pi R \qquad (3)$$

$$\varphi = tg^{-1}\left[\frac{\pi R}{L}\left(\frac{n_c - 1}{n_h}\right)\right] \qquad (4)$$

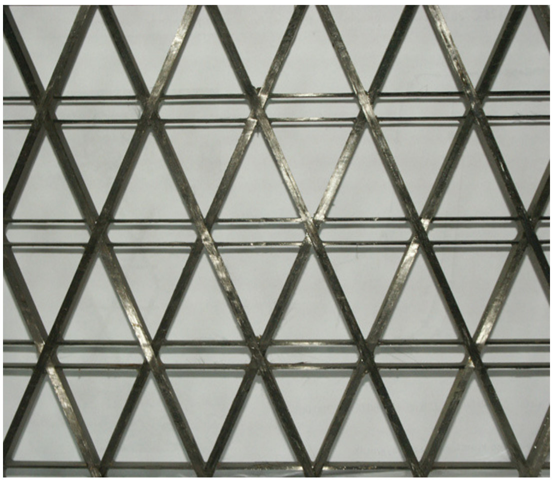

Fig. 1

in which R, L and d_c are the cylinder radius and height, and the hoop distance between two consecutive helical ribs.

As follows from the foregoing equations, the helical angle can take only discrete values as a function of two discrete variables of the structure. Thus, the optimization problem can be approached imposing the discrete variables and finding the optimal values for the continuous variables, i.e., the widths of hoop and helical ribs, b_c, and b_h, and the shell thickness, H. In this procedure, the number of hoop ribs is fixed once for all the structural configurations, whereas the number of helical ribs is varied within a certain range.

So each step of the design procedure involves only three continuous variables (b_c, b_h, H) and finds the suboptimal solution for a particular configuration (n_h, n_c). Plotting the structure mass as a function of the number of helical ribs, we can obtain the family of suboptimal configurations which are located in the vicinity of the minimum mass and thus simplify the choice of the proper design.

This approach is rather efficient because, as stated above, for a fixed number of hoop ribs, there exists one only optimal number of helical ribs and the structure mass can be minimized changing this number. Moreover, some additional constraints can be readily introduced in the design procedure. For example, the Isogrid configuration can be designed introducing the 30° helical angle into Eq.(4)

$$tg\frac{\pi}{6} = \frac{\pi R(n_c - 1)}{L n_h} \quad \Rightarrow \quad n_h = \frac{\sqrt{3} R(n_c - 1)}{L}$$

and presuming that the helical and the hoop ribs have the same width ($b_c = b_h$).

Conical shell

For conical lattice structures (Fig.2) in which the hoop ribs are regularly located between the points of intersection of symmetric helical ribs, we can arrive at the following equations for geodesic angles at the small end cross-section, φ_0 , and the large end cross-section, φ_f [6]

$$\varphi_0 = tg^{-1}\left(\frac{R_f \sin\gamma}{R_f \cos\gamma - R_0}\right) \tag{5}$$

$$\varphi_f = \varphi_0 - \gamma \tag{6}$$

$$\gamma = \pi \sin\beta \frac{(n_c - 1)}{n_h} \tag{7}$$

in which β is the angle between the meridian and the axis of the cone, R_0 and R_f, are the cone radii at the small and the large cross-sections, respectively.

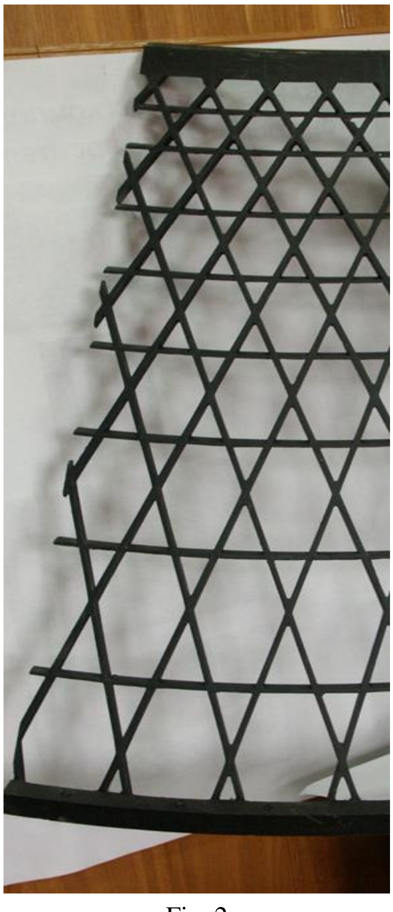

Fig. 2

Stiffness requirements

Analytical design does not allow for the stiffness requirements which can be imposed on the structure. In some cases, this restriction can become a serious drawback because the natural attempt to increase shell stiffness increasing the cross-sectional area of helical ribs from the area corresponding to the minimum mass solution can result in a rather inefficient design.

The proposed numerical optimization offers the possibility to provide the shell stiffness for each suboptimal configuration. Thus, the preliminary optimal design is simply identified with the suboptimal configuration which matches the stiffness requirement.

Application

Consider a preliminary design for a launch vehicle interstage structure which is mainly subjected to compressive and bending loads. Shell radius and height are respectively R = 1.5 m and L = 6.0 m. Bending load is reduced to an equivalent compressive force and added to the pure axial force, so that the total axial load for shell dimensioning is P = 2.5 MPa. Composite material is supposed to be the same for both hoop and helical ribs. Basic material properties of_ribs are E = 80 GPa (elastic modulus), $\sigma =$ 450 MPa (compressive strength) and ρ = 1580 Kg/m^3 (mass density).

Suppose to initially find minimum mass solution under strength and buckling constraints. Figure 3 represents results of numerical optimization in terms of mass with n_c = 16. The red spot highlights the best solution (n_h = 24, $M \cong$145 Kg, $\varphi \cong$ 26°) which is also given by analytical solution. Corresponding optimal design parameters for each configuration are shown in Fig.4. Figure 5 represents Eq.(4) for the current geometry of the shell. Moreover, axial compliance of found solutions is plotted (Fig.6). In particular, axial compliance in the minimum mass solution turns out to be about 11e-9 m/N.

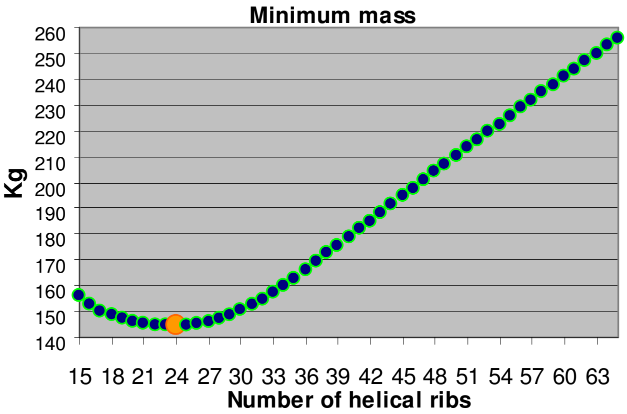

Fig. 3

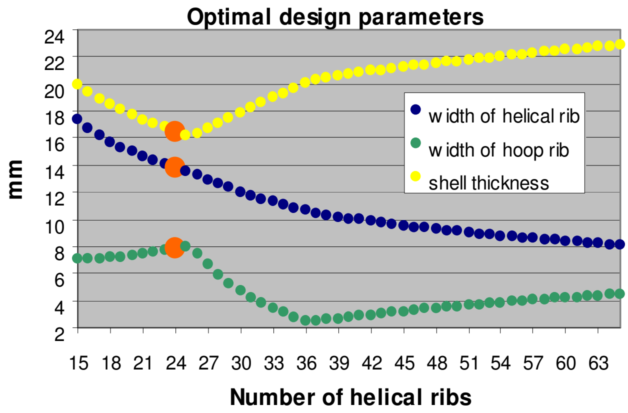

Fig. 4

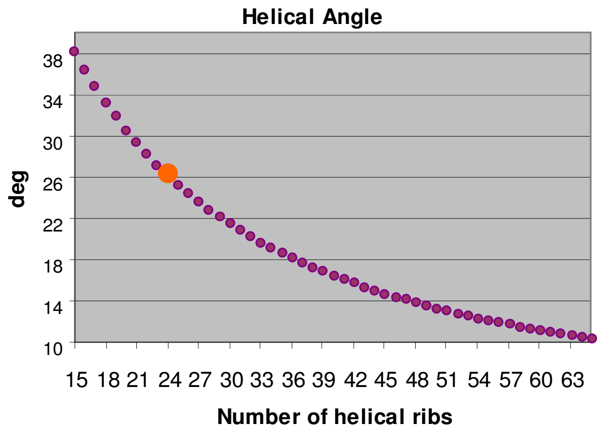

Fig. 5

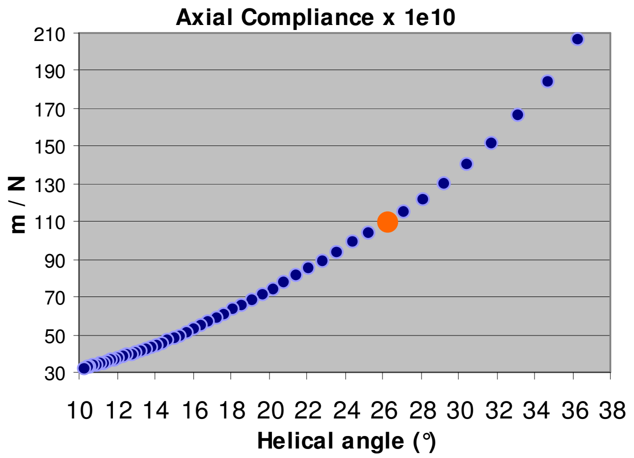

Fig. 6

Suppose we need to obtain an axial compliance requirement of the shell not greater than 6.0e-9 m/N. The first natural attempt is to start from the minimum mass solution and to increase cross-section of helical ribs. It can be verified this way that required compliance would be achieved for a mass of the shell greater than 210 Kg and corresponding mass increase with respect to analytical design would be more than 50%. Conversely, if we consider results of Fig. (6) the suboptimal solution which satisfies also compliance requirement exists. It is quite far from the minimum mass design in terms of angle and optimal parameters (n_h = 37, M ≅170 Kg, φ ≅ 17.5°) but mass increase would be only 17%. From a qualitative point of view it is reasonable to expect more axial stiffness from a configuration having orientation of helical ribs closer to the shell axis.

Conclusion

Numerical optimization of composite lattice structures overcomes some drawbacks and limits of analytical design by exploring suboptimal configurations that develop around the minimum mass solution (which is based on buckling and strength constraints). This possibility is especially true when also compliance requirement of the shell, manufacturing constraints and integration of substructures (namely, end rings) need to be considered and optimized for the specific application. In the present study the problem of axial compliance of a cylindrical structure has been approached showing the substantial weight saving that can be achieved by means of numerical optimization with respect to the configuration based on analytical design.

References

[1] *Optimal Design - Theory and Applications to Materials and Structures*, Edited by V.V. Vasiliev and Z. Gürdal, Technomic Publishing Co. Inc., Lancaster, Basel, 1999.

[2] V. Vasiliev, V. Barynin, A. Rasin, *Anisogrid Lattice Structures - Survey of Development and Application*, Composite Structures, 54 (2001) 361-370.

[3] A.F. Rasin and V.V. Vasiliev, *Development of Composite Anisogrid Spacecraft Attach Fitting,* Proc. of the 11th European Conf. on Composite Materials, May 31 - June 3, 2004, Rhodos, Greece, CD-ROM, 9p.

[4] G. Totaro, V.V. Vasiliev, F. De Nicola, A.F. Rasin, *Optimized Design of Isogrid and Anisogrid Lattice Structures*, 55th International Astronautical Congress, Vancouver, Canada, October 2004.

[5] F. De Nicola and G. Totaro, *Preliminary Design, Manufacturing and Testing of Grid Stiffened Structures*,C.I.R.A.,TR-04-358, Capua, Italy, 2004.

[6] V.V. Vasiliev, A.F. Rasin, G. Totaro, F. De Nicola, *Anisogrid Conical Adapters for Commercial Space Application*, 13th AIAA International Space Planes and Hypersonics Systems and Technologies Conference, Capua, Italy, 16-20 May 2005.