Arbitrary Lagrangian-Eulerian approach in Reduced Order Modelling of a Flow with a Moving Boundary

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Abstract

Flow-induced deflections of aircraft structures result in oscillations that might turn into such a dangerous phenomena like flutter or buffeting.

In this paper the design of an aeroelastic system consisting of Reduced Order Model of the flow with moving boundary is presented. The model is based on Galerkin projection of governing equation onto space spanned by modes obtained from high-fidelity computations. The motion of the boundary and mesh is defined in Arbitrary Lagrangian-Eulerian approach and results in additional convective term in Galerkin system. The developed system is demonstrated on the example of a flow around oscillating wing.

1. Introduction

Interactions between fluid and moving (deforming) boundaries are ones of the most important issues of fluid dynamics. They occur in wind turbines [19], civil engineering (e.g. the influence of a wind on bridges and buildings [24, 50]) and aerospace industry.

In the last case, Fluid-Structure Interaction (FSI) plays an important role in the design process of an aircraft. The examples include dangerous phenomena like flutter and buffeting of the wings and fuselage [14, 52, 41], vibrations in turbine engines [47] and helicopter blades [25], as well as applications in the design of bio-inspired air vehicles [9, 6]. Furthermore, recent research on the growth of the lift force and drag reduction by active electromorphing [35] and aeroelastic boundary actuators [28, 23] require analysis of Fluid-Structure Interactions.

The design of new aircraft requires an analysis of a huge number of variants. One has to check different aircraft configurations, mass cases, gusts an maneuvers, giving (even with engineering experience for current configurations and technologies) hundreds of thousands of simulations [40].

One time step of RANS calculation on the viscous grid using 32-core cluster may take up to 400 s, giving several weeks per simulation. On the other hand in the case of feedback flow control design the model should be small enough to accurately predict the flow response and ensure that the actuators will work in the correct phase. Long time required to find the flow solution prevents the development of a real-time flow control. This means that further growth in the aerospace industry, leading to more economical and environment-friendly solutions, is possible only through the significant reduction of computation time and memory requirements. Reduced order Galerkin models [18, 34] meet these requirements, approximating the governing equations (e.g. Navier-Stokes) with a system of a few ordinary differential equations.

The present paper is organized as follows. The high-dimensional algorithm of Fluid-Structure Interactions is described in section 2. Then, the details on governing equations in Arbitrary Lagrangian-Eulerian (ALE) approach, describing a flow with a moving boundaries (section 3) and the flow model reduction techniques based on Galerkin projection (section 4) are given. In particular, Galerkin Method (section 4.1), projection of convective term in ALE approach (section 4.2) and Proper Orthogonal Decomposition used in mode expansion (section 4.3) are described. Some remarks on the improvement of model's accuracy are given in section 5. Finally, the Reduced Order Models of a flow around an oscillating airfoil are presented in section 6. The results are summarized in section 7.

2. Fluid-Structure Interaction algorithm

Computational Aeroelasticity [23] is a branch of mechanics which examines the interactions between a stream of fluid and a deformable body using the methods of Computational Fluid Dynamics (CFD) and Computational Structural Mechanics (CSM) [41].

The high-dimensional approach used in this work relies on the use of independent solvers for solid and fluid mechanics, exchanging information on the coupling interface. As a result, different discretizations (Finite Volume and Finite Element Methods) and element types (tetrahedra, plates,beams) are used, and the meshes used on both sides might vary in the number of nodes and elements on the coupling, “wet” surface (interface). Non-conforming grids are the reason of using of additional interpolation tools.

The computational FSI algorithm used in this work is shown in fig. 1.

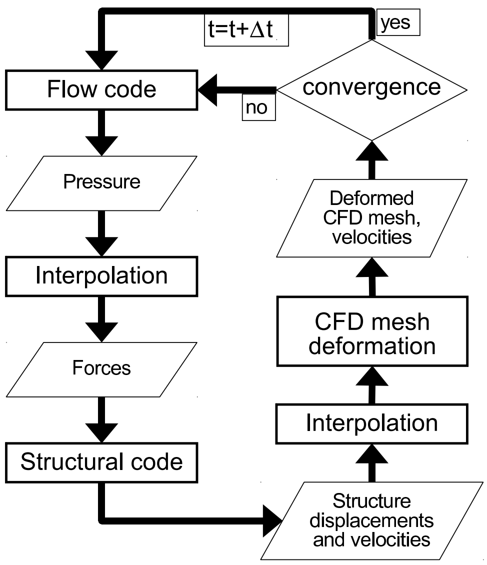

Figure 1: General Fluid-Structure Interaction algorithm

The velocity and pressure field is calculated by CFD code. In the case of 2D laminar flow, in-house DNS solver is used, while 3D Euler/RANS simulations with large numbers of degrees of freedom are performed on parallel and efficient DLR TAU-code [44].

The pressures are interpolated onto the structure using modules based of finite-element meshes, as well as bucket [2] and oct-tree [29] neighbour search algorithms. Under applied aerodynamic load CSM (elastic) solver calculates the deformations of the structure. CSM solvers used in this task include in-house CSM system [30] and open-source Calculix solver [10].

The nodal displacements on the boundary of the structure are interpolated onto CFD mesh. Then the displacements and velocities in the interior of CFD mesh are calculated using deformation tool based on spring analogy [15].

The flow in modified (deformed) domain results in modified velocity and pressure fields, and another values of structure’s node displacements. The loop presented above runs until the convergence in a given time step is reached. Then, time step is increased using coupling procedures described in [36].

The most time-consuming part of such a coupled analysis is a high-fidelity flow solver. To accelerate FSI analyses, e.g. in the aircraft certification procedure and real-time flow control applications, full-dimensional CFD solver might be replaced by Reduced Order Model of a flow with moving boundaries.

3. Governing equation

The viscous, incompressible flow is described by Navier-Stokes equations, that might be written in the form:

$$\dot{\mathbf{u}} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \nabla \mathbf{p} - \frac{1}{Re} \triangle \mathbf{u} = 0 \quad (1)$$

The velocities are defined in respect to the fixed computational mesh. This Eulerian approach, widely used in fluid dynamics, means that the flow particles move through the mesh elements. This description allows large distortions in the fluid motion (separations, vortices, reverse flow, etc.), but it requires precise definition of domain's boundary.

A technique that overcomes the shortcomings of Eulerian algorithms and allows the simulation of a flow with a moving boundaries is Arbitrary Lagrangian-Eulerian (ALE) approach [27, 11, 51, 42, 12]. It combines the best features of Eulerian and Lagrangian approaches, by letting the nodes of fluid mesh move independently of the fluid particles.

In ALE formulation, the velocity of the boundary and the fluid mesh $\mathbf{u}_{grid}$ is included in the modified convective term:

$$\dot{\mathbf{u}} + \nabla \cdot ((\mathbf{u} - \mathbf{u}_{grid}) \otimes \mathbf{u}) + \nabla \mathbf{p} - \frac{1}{Re} \triangle \mathbf{u} = 0, \tag{2}$$

where $\mathbf{c} = \mathbf{u} - \mathbf{u}_{grid}$ is a relative velocity between the material and the mesh and is called convective velocity [12]. Mesh acceleration plays no role in the ALE formulation.

The movement of CFD mesh nodes is independent of the fluid particle motion. In particular, it might be associated with the movement of the structural grid boundary nodes (in Lagrangian approach equal to the material velocity), ensuring that both CFD and structural meshes will not overlap or disconnect.

In the case of viscous fluid model, the velocity (of fluid particles) on the boundary of the domain is equal to the velocity of the structure (grid) ($\mathbf{u} = \mathbf{u}_{grid}$). In the case of inviscid flows, only the normal components of the velocity are coupled ($\mathbf{n} \cdot \mathbf{u} = \mathbf{n} \cdot \mathbf{u}_{grid}$).

4. Model reduction

4.1 Galerkin Method

In this paper, Galerkin method [34, 48] is used to develop Reduced Order Model that preserves the main flow dynamics. This approach consists in approximation of the velocity field by base solution $\mathbf{u}_0$ (steady or time-averaged flow) and a weighted sum of modes $\mathbf{u}_j$:

$$\mathbf{u}^{[N]} = \mathbf{u}_0 + \sum_{j=1}^{N} a_j \mathbf{u}_j = \sum_{j=0}^{N} a_j \mathbf{u}_j, \qquad a_0 \equiv 1, \tag{3}$$

that results in the separation of space ($\mathbf{u}_j$) and time (mode amplitudes a_j) variables.

The (orthogonal) projection of the residual of approximated Navier-Stokes equation onto the space spanned by the modes (4) results in a system of ordinary differential equations (5).

$$\left(\mathbf{u}_i, R^{[N]}\right)_\Omega = \int_\Omega \mathbf{u}_i R^{[N]} d\Omega = 0 \tag{4}$$

$$\dot{a}_i = \frac{1}{Re} \sum_{j=0}^{N} a_j l_{ij} + \sum_{j=0}^{N} \sum_{k=0}^{N} a_j a_k q_{ijk} \tag{5}$$

where:

$$l_{ij} = \left(\mathbf{u}_i, \triangle \mathbf{u}_j\right)_\Omega \quad \text{and} \quad q_{ijk} = -\left(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_j \otimes \mathbf{u}_k)\right)_\Omega \tag{6}$$

In Hilbert space, the inner product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as:

$$\left(\mathbf{a}, \mathbf{b}\right)_\Omega = \int_\Omega \mathbf{a}\ \mathbf{b}\ d\Omega \tag{7}$$

4.2 Projection of convective term in ALE description

It is assumed, that velocity and displacements of the fluid mesh $\mathbf{u}_{grid}$ might be decomposed similarly to the velocity of fluid particles (3):

$$\mathbf{u}_{grid} = \sum_{j=1}^{N_G} a_j^G \mathbf{u}_j^G, \tag{8}$$

where modal mesh deformations $\mathbf{u}_j^G$, are time-invariant.

The projection of convective term of Navier-Stokes equation in ALE description leads to additional term in Galerkin system:

$$\begin{aligned} -\Big(\mathbf{u}_i, \nabla \cdot ((\mathbf{u} - \mathbf{u}_{grid}) \otimes \mathbf{u})\Big)_\Omega &= -\Big(\mathbf{u}_i, \nabla \cdot (\mathbf{u} \otimes \mathbf{u})\Big)_\Omega + \Big(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_{grid} \otimes \mathbf{u})\Big)_\Omega = \\ &= \sum_{j=0}^{N} \sum_{k=0}^{N} q_{ijk} a_j a_k - \sum_{j=1}^{N_G} \sum_{k=0}^{N} q_{ijk}^G a_j^G a_k \end{aligned} \tag{9}$$

where:

$$q_{ijk}^G = -\Big(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_j^G \otimes \mathbf{u}_k)\Big)_\Omega \tag{10}$$

As the mesh deformation modes are time-invariant, q_{ijk}^G term is only affected by the integration over elements of deforming mesh. The changing element shapes might be taken into account using continuous mode interpolation [31, 48].

4.3 Mode basis for model reduction

The mode bases used in the Galerkin approximation might be classified in terms of mathematical, physical and empirical approaches, as discussed in [34]. In the empirical approach, mode basis “is determined a posteriori using experimental or numerical data previously obtained for a given flow configuration“ [3]. The possible bases include centroidal Voronoi tessellations (CVT) [5], Lagrange, Hermite and Taylor bases [3], as well as Proper Orthogonal Decomposition [20, 46].

Although investigations in the area of empirical modes resulted in modifications of POD method like Sequential POD [22] and Double POD [45] and, recently, Dynamic Mode Decomposition [43, 16], POD is still one of the most popular modelling approaches in fluid dynamics, successfully used for flow control and aeroelastic analyses [34, 13, 26]. POD modes are optimal in energy representation by construction, so they possibly better describe the Navier-Stokes attractor (limit-cycle oscillations of periodic flow), than the same number of modes obtained in any different manner [32].

In this method, the M flow vectors (snapshots) of size N (number of Degrees of Freedom) are centered using time-averaged solution $\mathbf{u}_0$

$$\acute{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{u}_0, \qquad i = 1..M, \tag{11}$$

Resulting M fluctuation vectors $\acute{\mathbf{v}}_i$ form a matrix $\acute{V}$. POD modes used in model reduction are the eigenvectors $\mathbf{u}_i$ of standard eigenproblem $C\mathbf{u}_i = \lambda_i \, I \, \mathbf{u}_i$ of the autocorrelation matrix C of size $N \times N$:

$$C = \frac{1}{M} \acute{V} \acute{V}^T, \tag{12}$$

related to eigenvalues λ_i of largest magnitude.

While the number of snapshots M is substantially smaller than the number of degrees of freedom N, a modification of traditional POD, proposed by Sirovich [46], is used. In the Method of Snapshots the autocorrelation matrix $\acute{C}$ of size $M \times M$ is introduced:

$$C = \frac{1}{M} \acute{V}^T \acute{V} \tag{13}$$

The eigenvalues λ and $\acute{\lambda}$ of matrices C and $\acute{C}$ are the same, while the eigenvectors (modes) are connected:

$$\mathbf{u}_i = \frac{V \acute{\mathbf{u}}_i}{\|V \acute{\mathbf{u}}_i\|} \tag{14}$$

5. Model calibration

The mode basis calculated using Proper Orthogonal Decomposition of a given data set is truncated and a limited number of the most energetic modes, corresponding to the largest eigenvalues λ, are used in the construction of ROM's mode basis. The neglect of small scales results in filtering of high frequencies and vanishing of energy transfers between resolved and unresolved scales of fluid flow [8], that decrease the quality of the model.

The possible inconsistency of data set and the reduced-order formulation (neglect or inaccurate treatment of pressure and boundary terms, not verified incompressibility of the flow) [7], as well as structural instability of Galerkin Projection [34, 39, 21] are another sources of discrepancies between Reduced-Order Galerkin model and high-fidelity model (like Direct Numerical Simulation of Navier-Stokes equations or Large Eddy Simulation).

The deterioration of model's quality is particularly noticeable in distorted frequencies, phases and amplitudes of mode coefficients, under- or over-prediction of turbulent kinetic energy level and different dynamical responses.

To correct the behaviour and improve the accuracy of Reduced Order Galerkin Model, the coefficients of the Galerkin system of ODE is adjusted [8].

Such a calibration might be done by addition of artificial, “eddy” viscosities to recover the effects of truncated modes [1, 37, 38].

This artificial viscosity might be defined as a single, constant value ν_T (15), or N parameters $\nu_{T,i}$ related to each one of the modes (15).

$$l^+_{ij} = \frac{\nu_T}{\nu} l_{ij} \tag{15}$$

$$l^+_{ij} = \frac{\nu_{T,i}}{\nu} l_{ij}, \qquad i = 1 \cdots N \tag{16}$$

Instead of calibrating “eddy” viscosities, all linear coefficients l^+_{ij} [17] or both linear and quadratic coefficients q^+_{ijk} [8] of Galerkin System might be modified in order to improve the results of calibration.

The resulting system of equations might be written as follows:

$$\dot{a}_i = \underbrace{\nu \sum_{j=0}^{N} \left(l_{ij} + l^+_{ij} \right) a_j + \sum_{j=0}^{N} \sum_{k=0}^{N} \left(q_{ijk} + q^+_{ijk} \right) a_j a_k}_{f_i(\mathbf{a})} \tag{17}$$

The model presented above is a subject of the optimization procedure, where objective function, related to the prediction error of the model, is minimized.

The choices of mean square error of the mode coefficients (18) or their time-derivatives (19), referred as Floquet and Poincaré calibration, respectively [33, 4], are prevalent:

$$\chi_0 := \sum_{i=1}^{N} \int_0^T \left(a_i^{ROM}(t) - a_i^{DNS}(t) \right)^2 dt = \mathrm{Min} \tag{18}$$

$$\chi_1 := \sum_{i=1}^{N} \int_0^T \left(\dot{a}_i^{ROM}(t) - f_i(\mathbf{a}^{DNS}(t)) \right)^2 dt = \mathrm{Min} \tag{19}$$

where a_i^{ROM} and $\dot{a}_i^{ROM}$ represent mode coefficients and their time derivatives for Galerkin Model, a_i^{DNS} - the coefficients calculated from Proper Orthogonal Decomposition of reference simulation data, and $f_i(\mathbf{a}^{DNS}(t))$ - the values resulting from the substitution of a_i^{DNS} to the function on the right-hand side of equation 17.

Another error definitions might based on the turbulent kinetic energy:

$$\chi_2 := \int_0^T \left(\sum_{i=1}^{N} \left(a_i^{ROM}(t) \right)^2 - \sum_{i=1}^{N} \left(a_i^{DNS}(t) \right)^2 \right)^2 dt = \mathrm{Min} \tag{20}$$

or modal energy flow balance:

$$\chi_3 := \sum_{i=1}^{N} (P_i + C_i + D_i + T_i + F_i)^2 = \mathrm{Min} \tag{21}$$

(where P_i, C_i, D_i, T_i and F_i represent modal production, convection, dissipation, transfer and pressure power, respectively), leading to E-flow calibration, proposed by Noack [33].

In this work the optimization procedure based on Genetic Algorithm [49] is used.

6. Results of computations

6.1 2D flow around NACA-0012 airfoil

Two test-cases have been chosen for Reduced Order Modelling. In the first case, two-dimensional, incompressible, viscous flow around oscillating NACA-0012 airfoil is analysed using in-house DNS solver. Reynolds number, related to the chord length, is $Re = 100$, and the angle of attack is $\alpha = 15°$. For these parameters the flow with fixed boundaries has one stable, steady solution. Prescribed sinusoidal transverse oscillations of the airfoil, with amplitude equal to the one fourth of the chord length and period equal 5 seconds, perturb the flow making it periodic with bounded amplitude of (limit cycle) oscillations (fig. 2).

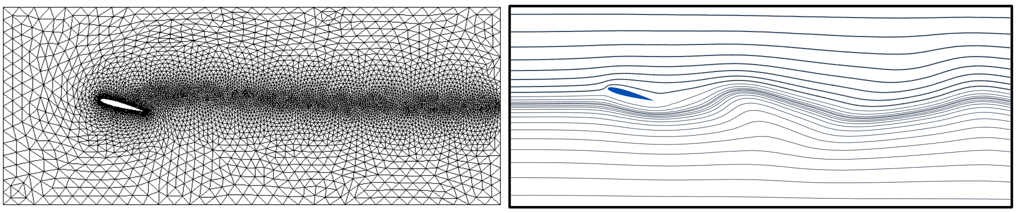

Figure 2: Finite element mesh for a flow around NACA-0012 airfoil (left) and a snapshot of limit-cycle oscillations

Only one form (transverse direction) of CFD mesh displacements $\mathbf{x}_{grid}$ and velocities $\mathbf{u}_{grid}$ makes the separation of space and time variables trivial:

$$\begin{aligned} \mathbf{x}_{grid} &= \mathbf{u}_1^G a_1^G = \mathbf{u}_1^G A sin(ft) \\ \mathbf{u}_{grid} &= \mathbf{u}_1^G \dot{a}_1^G = \mathbf{u}_1^G A f cos(ft), \end{aligned} \tag{22}$$

where f is the frequency and A is the amplitude of airfoil oscillation.

Proper Orthogonal Decomposition has been performed on snapshots from 7 periods of the flow described above. First four of resulting modes, depicted in fig. 3, carry almost 98% of information about kinetic energy of fluctuation.

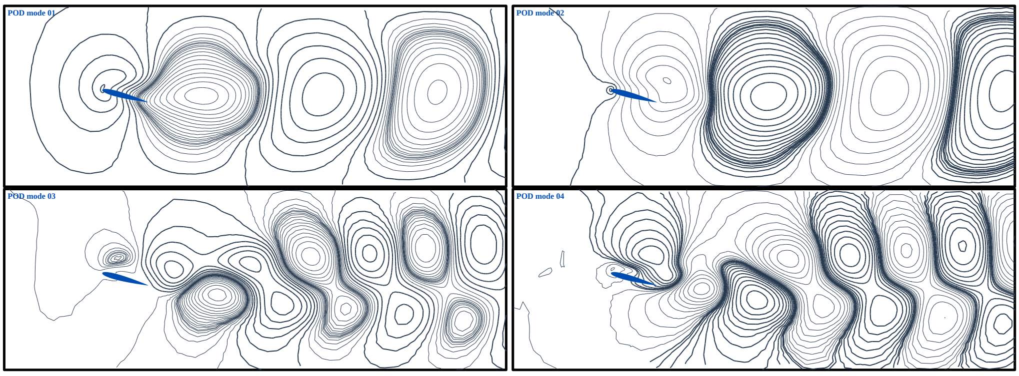

Figure 3: Streamlines of the most dominant POD modes for a flow around NACA-0012 airfoil

State equations have been projected onto the space spanned by first eight POD modes (covering 99.97% of kinetic energy of fluctuation). Two Galerkin models have been constructed.

In the first case (fig. 4, left), Galerkin model is formulated in Eulerian approach, neglecting motion of the boundary and mesh velocities. It can be seen, that initial oscillation is damped, as expected for subcritical values of Reynolds numbers.

It is obvious, that both mode basis and approximated governing equations have to be formulated in ALE approach to model the flow with moving boundary, as is the case of second Galerkin model (fig. 4, right).

Including the velocity of the mesh in convective term (9) and proper calibration terms (17), Galerkin model in ALE approach is characterized by the same frequency and almost the same amplitude as reference data from ALE-based DNS.

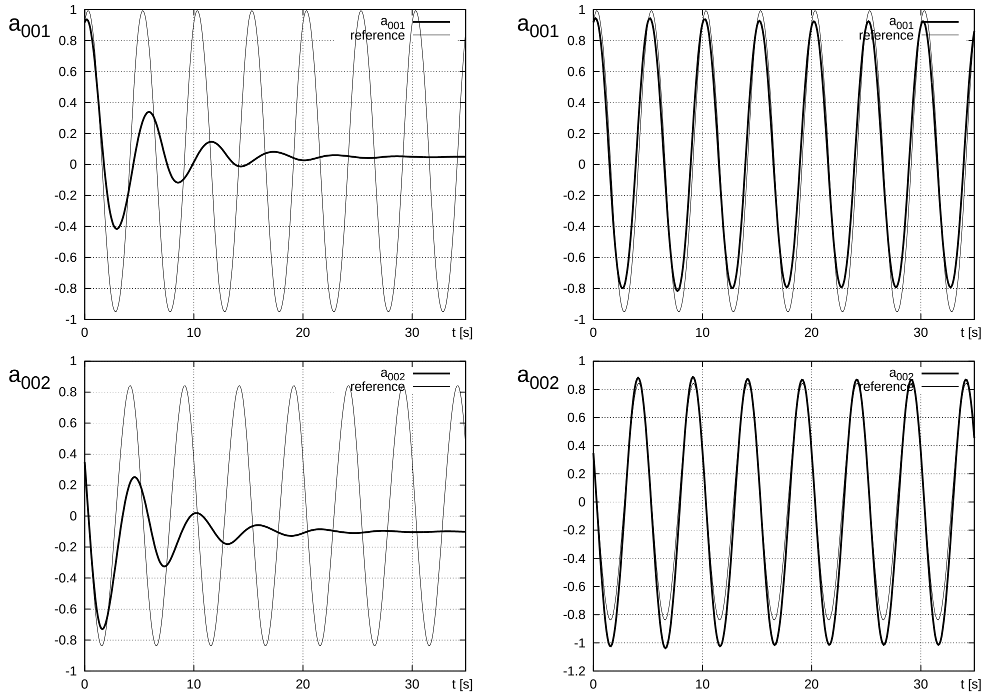

Figure 4: Amplitudes for first two modes resulting from POD decomposition of ALE DNS data (thin lines) and Galerkin models (thick lines). Eulerian ROM neglecting mesh velocities (left) and ALE-based ROM (right) are depicted

6.2 3D flow around AGARD 445.6 wing

The second configuration is AGARD 445.6 wing (fig. 5), analysed using DLR Tau Code solver.

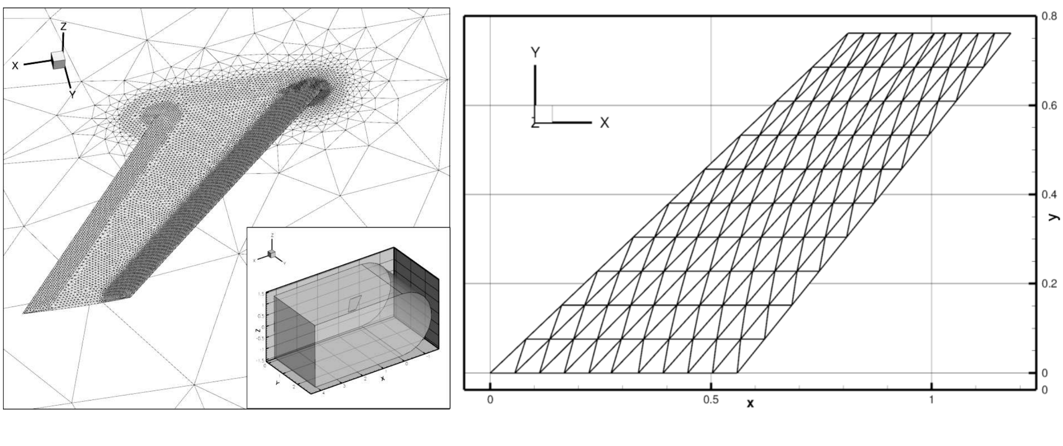

Figure 5: Computational domain and meshes for CFD (left) and CSM (right) analysis of AGARD 445.6 wing

In this case, the Reynolds number is assumed high enough to neglect boundary layer effects and solve Euler equations. Mach number equals $M = 0.32$ and the angle of attack is $\alpha = 0.26°$. The deformations of the structure (modelled as a “plate model”) under aerodynamic load are calculated using in-house FEM solver. For a given configuration, flutter phenomena is observed (fig. 6).

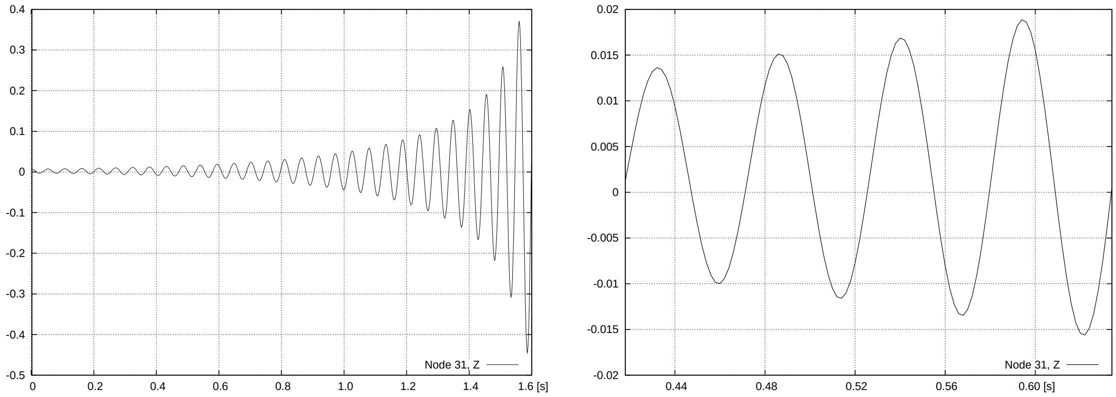

Figure 6: Z-component of the displacement of a node on the end of a trailing edge of the AGARD wing

The snapshots from 4 periods of oscillations (fig. 6, right) computed using high-fidelity aeroelastic system have been decomposed using POD. The most energetic of resulting modes, representing 99.9% of kinetic energy of fluctuation, are depicted in fig. 7.

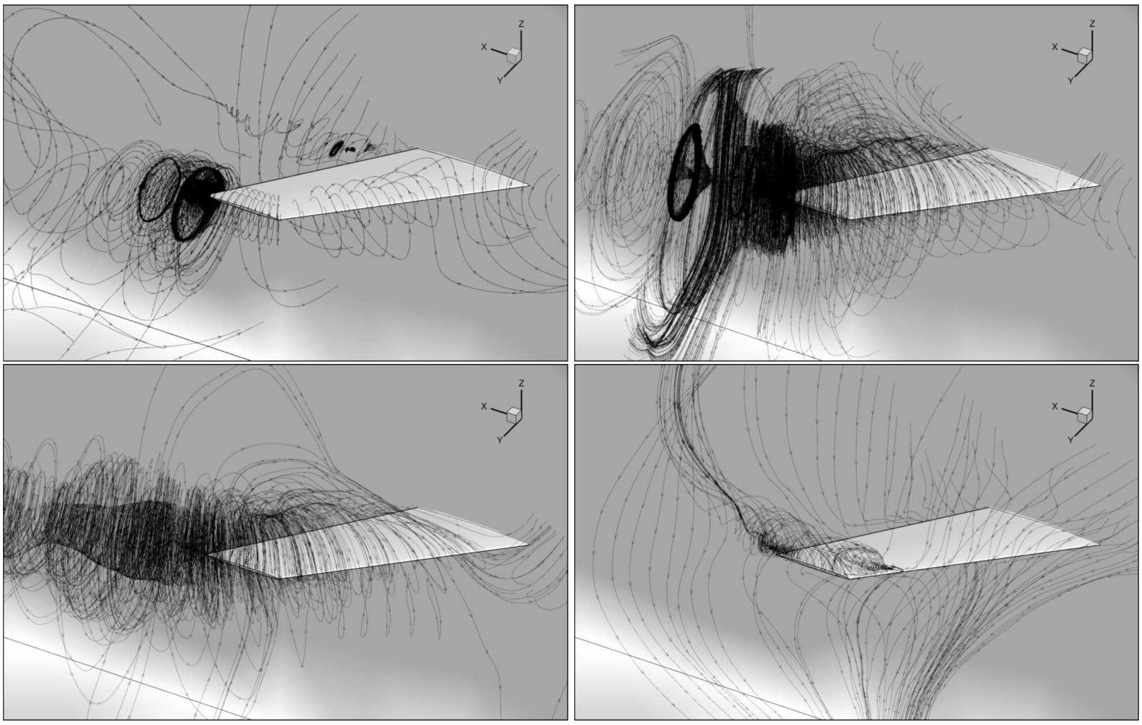

Figure 7: Streamlines of the most dominant POD modes for a flow around AGARD 445.6 wing

It can be seen that for Euler flow, as opposed to Direct Numerical Simulation of Navier-Stokes equations, the modes do not form pairs.

The projection of the flow snapshots onto the space spanned by POD modes allows to calculate the reference values of mode amplitudes. For the first two modes, they are depicted in fig. 8.

The resulting functions are characterized by the same frequency and growth rate as the graph of node displacements (fig. 6, right).

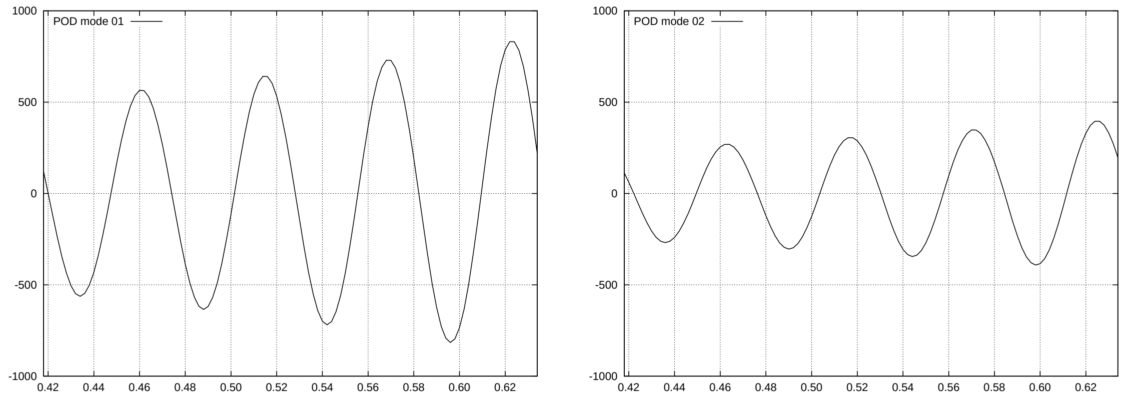

Figure 8: Amplitudes for first two modes resulting from POD decomposition of reference data

7. Summary

In this paper high-fidelity aeroelastic simulations and Reduced Order Models of a flow with a moving boundary are presented.

Arbitrary Lagrangian-Eulerian approach allows, by introducing convective velocity into the governing equations, the modelling of rapid boundary movement like airfoil and wing oscillation. Galerkin projection of governing equations (either Navier-Stokes or Euler) in ALE formulation results in additional quadratic term q^{G}_{ijk} in the Galerkin system, representing triadic interactions between two “flow” modes and one “mesh velocity” mode.

In this paper mode bases, resulting from Proper Orthogonal Decomposition of numerical simulation data, have been used. The mode basis construction Reduced Order models has been demonstrated on a 2D, viscous flow around NACA-0012 airfoil and 3D, inviscid flow around AGARD 445.6 wing. It has been shown, that the design of Reduced Order Models of the flows with moving boundary is possible using ALE approach.

Further investigations include the extension of the range of applicability of the model, by parametrization of mode bases using Continuous Mode Interpolation [31].

8. Acknowledgements

This work has been funded by the Polish Ministry of Science under research grant no. N N501 225437.

We thank Bernd R. Noack (CNRS, Université de Poitiers) and Gilead Tadmor (Northeastern University, Boston) for very stimulating discussions.

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