A model-based solution for fault diagnosis of thruster faults: Application to the rendezvous phase of the Mars Sample Return mission

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Abstract

This paper addresses the design of model-based fault diagnosis schemes to detect and isolate faults occurring in the orbiter thrusters of the Mars Sample Return (MSR) mission. The proposed fault diagnosis method is based on a $H(0)$ filter with robust poles assignment to detect quickly any kind of thruster faults and a cross-correlation test to isolate them. Simulation results from the MSR "high-fidelity" nonlinear simulator provided by Thales Alenia Space demonstrate that the proposed method is able to diagnose thruster faults with a detection and isolation delay less than $1.1s$.

1. Motivation

Future sciences space missions require critical autonomous proximity operations, e.g. rendezvous and docking/capture for the Mars Sample Return (MSR) mission. Mission safety is usually guarranteed through various modes of satellite operations, with ground intervention, except in these specific critical phases, for which the on-board robustness and on-board fault tolerance / recovery prevails in the dynamics trajectory conditions.

Satellite health (incl. outages) monitoring is classically performed through a hierarchical implementation of the fault diagnosis and fault tolerance in which several levels of faults containements are defined from local component/equipment up to global system, i.e. through various equipments (sensors like IMUs, thrusters, etc..) redundancy paths. Common Fault Detection Isolation and Recovery (FDIR) implementation uses four hierarchical levels with graduated detection/isolation/reaction to faults, see for instance [1, 2] where fault detection and isolation are performed by cross checks, consistency checks, voting mechanisms ...etc. Fixed thresholds (once validated with all the known delays and uncertainties) are used for rapid recognition of out-of-tolerance conditions but their setting tuned to avoid false alarms and to insure acceptable sensitivity to abnormal deviations. Unfortunately, such classical FDIR hierarchical implementation approach does not solve, sufficiently quickly, abnormal dynamics deviation or transient behavior in faulty situations, e.g. for rendezvous safety corridor during critical proximity operations, thus possibly leading to mission loss. Therefore, advanced model-basd FDI and fault tolerant control techniques are specifically developped to safely conjugate on-board (and on-line) the necessary robustness/stability of the satellite control and the necessary trajectory dynamics and vehicle operations.

The objective of this research is to develop an advanced model-based fault detection and isolation scheme, able to diagnose thrusters faults of the MSR orbiter, on-board/on-line and in time within the critical dynamics and operations constraints of the last terminal translation (last 20m) of the MSR rendezvous/capture phase. As mission scenario undertaken, the chaser stays in the rendezvous/capture corridor, such that it is possible to anticipate the necessary recovery actions to successfully meet the capture phase. Three main fault profiles are considered: locked closed thruster failure, cyclic forces/torques around the desired force/torque profile with small magnitude and monopropellant leakage. The innovation that we pursue with this study, is concerning the fault coverage capability, and more particularly, the ability of the fault diagnosis scheme to detect and isolate small faults which have no significative impact on the spacecraft dynamics and/or the GNC. For instance, a thruster locked closed is more difficult to diagnose because the thruster is not necessary used at the date of the failure, and because the thrusters, when they are used, achieve small pulses whose effect averaged over the control cycle is small. Such faults are highly non-detectable using the standard industrial on-board FDIR techniques and/or ground analysis. Moreover the uncertainty on the center of mass due to propellant motions in the tanks makes the detection and isolation more challenging.

Numerous fault diagnosis methods are applicable to this problem. In fact, most of the model-based diagnostic techniques reported in the literature have the potential to be applied. In recent years, some effective techniques of the fault detection and diagnosis for satellite attitude control systems based on inertial wheels have been developed, see for instance the books [3, 4, 5, 6] and the references given therein. The problem of thruster's faults is less considered in the literature. Among the contributions, one can refer to [7] where an iterative learning observer (ILO) is designed to achieve estimation of time-varying thruster faults. The method proposed in [8, 9] is based on the so-called unknown input observer technique and is applied to the Mars Express mission. Selected performance criteria are also used, together with Monte Carlo robustness tuning and performance evaluation, to provide fault diagnosis solutions. [10] addressed the problem of thrusters faults in the Microscope satellite and [11] considered the problem of faults affecting the micro-Newton colloidal thrust system of the LISA Pathfinder experiment. Both proposed FDI schemes are base on H_∞/H_- filters to generate residuals robust against spatial disturbances (i.e. third-body disturbances, J_2 disturbances, atmospheric drag and solar radiation pressure), measurement noises and sensor misalignment phenomena, whilst guaranteeing fault sensitivity performances. Additionally, a Kalman-based projected observer scheme is considered in [11]. [12] discusses several fault diagnostic observers using sliding mode and learning approaches.

In this paper, the proposed FDI scheme consists of a $H(0)$ filter with pole assignment which is in charge of residual generation for fault detection. This detection scheme allows to detect quickly any kind of thruster faults. The isolation task is solved using a cross-correlation test between the residual signal and the thrusters. For reduced computational burdens, the isolation test is based on a sliding time window.
Note that a great advantage of the proposed method is that the use of hyper-parameters used to specify the requirements in terms of robustness and fault sensitivity performance allows the proposed technique to be re-used for other space missions like ExoMars, Proba3, Mars Express ...etc... Furthermore, the existence of formal proofs in terms of fault sensitivity performance (thanks to the $H(0)$ index) allows to pinpoint critical faulty situations. This may lead to a useful tool that can be used to analyze the robustness properties of the GNC against faulty situations prior identified by this tool. Thus, specific MonteCarlo tests can be done before a complete campaign.

Notations. The Euclidean norm is always used for vectors and is written without a subscript; for example $\|x\|$. Similarly in the matrix case, the induced vector norm is used: $\|A\| = \overline{\sigma}(A)$ where $\overline{\sigma}(A)$ denotes the maximum singular value of A. Signals, for example $w(t)$ or w, are assumed to be of bounded energy, and their norm is denoted by $\|w\|_2$, i.e. $\|w\|_2 = \left(\int_{-\infty}^{\infty} ||w(t)||^2 dt\right)^{1/2} < \infty$. Linear models, for example, $P(s)$ or simply P, are assumed to be in $\mathbb{RH}_\infty$, real rational functions with $||P||_\infty = \sup_\omega \overline{\sigma}(P(j\omega)) < \infty$. In accordance with the induced norm, $||P||_- = \inf_{\omega\in\Omega} \underline{\sigma}(P(j\omega))$ is used to denote the smallest gain of a transfer matrix P. Here, $\underline{\sigma}(P(j\omega))$ denotes the minimum non-zero singular value of matrix $P(j\omega)$ and $\Omega = [\omega_1 ; \omega_2]$ the evaluated frequency range in which $\underline{\sigma}(P(j\omega)) \neq 0$. As a direct extension, the $H(0)$ gain of a MIMO filter is defined according to $||P||_0 = \lim_{\omega\to 0} \underline{\sigma}(P(j\omega)) \neq 0$ which is known as the zero frequency gain (dc-gain). Linear Fractional Representations (LFRs) are extensively used in the paper. For appropriately dimensioned matrices N and $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$, the lower LFR is defined according to $F_l(M,N) = M_{11} + M_{12}N(I - M_{22}N)^{-1}M_{21}$ and the upper LFR according to $F_u(M,N) = M_{22} + M_{21}N(I - M_{11}N)^{-1}M_{12}$, under the assumption that the involved matrix inverses exist.

2. Material backgrounds

Consider a dynamical system subject to q_f faults $f_i(t), i = 1...q_f$. The robust fault detection problem concerns the detection of $f_i(t) \neq 0$ while guaranteeing some robustness performance level to disturbances and model perturbations.

To formulate this problem, consider the uncertain model (1) in the LFR form i.e., all uncertain parameters and model perturbations have been "pulled out" so that the system's model appears as a nominal model P subject to an artificial feedback Δ (see figure 1 for easy reference):

$$\begin{pmatrix} y(s) \\ u(s) \end{pmatrix} = F_u\left(P(s), \Delta\right) \begin{pmatrix} d(s) \\ f(s) \end{pmatrix} \tag{1}$$

In this formulation, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $y \in \mathbb{R}^m$ denote the state vector associated to the transfer function P, the input and the output vectors, respectively. $d \in \mathbb{R}^{q_d}$ is a vector of all disturbance inputs. P denotes a LTI model that includes a control law model, and Δ is a block diagonal operator that behaves to the structure $\underline{\Delta}$ defined according to:

$$\underline{\Delta} = \{\text{block diag}(\delta_1^r I_{k_1}, ..., \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, ..., \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1^C, ..., \Delta_{m_C}^C), \delta_i^r \in \mathbb{R}, \delta_i^c \in \mathbb{C}, \Delta_i^C \in \mathbb{C}\} \tag{2}$$

Here $\delta_i^r I_{k_i}, i = 1, ..., m_r$, $\delta_j^c I_{k_{m_r+j}}, j = 1, ..., m_c$ and $\Delta_l^C, l = 1, ..., m_C$ are known respectively as the "repeated real scalar" blocks, the "repeated complex scalar" blocks and the "full complex" blocks. It is assumed that all model perturbations are represented by Δ so that $||\Delta||_\infty \leq 1$. This can be assumed without loss of generality since the model P can always be scaled.
Now, let us consider the following general form of a residual vector:

$$r(s) = F(s)\begin{pmatrix} y(s) \\ u(s) \end{pmatrix}, \quad r \in \mathbb{R}^{q_r} \tag{3}$$

The residual generation design problem we are interested in can be formulated as follows:

Problem 1 *Let the LFR model $Fu(P(s), \Delta)$ be robustly stable and the fault f_i be observable from the output y (these are prior conditions for the fault detection problem to be well posed). Consider the residual vector r defined by equation (3). Our aim is to derive the state space matrices A_F, B_F, C_F, D_F of the LTI filter F that solve the following optimization problem:*

$$\begin{array}{rll} \max_{A_F, B_F, C_F, D_F} & \varphi & \\ s.t. & ||T_{f \to r}||_0 > \varphi & , \forall \Delta : ||\Delta||_\infty \leq 1 \\ & \lambda_i(A_F) \in \mathscr{R} \subseteq \mathscr{D}, \forall i & \end{array} \tag{4}$$

In (4), $T_{f \to r}$ denotes the transfer between f and r and $\mathscr{D}$ denotes the left half complex plane. λ_i refers to the ith eigenvalue of the matric A_F and φ denotes the fault sensitivity performance index for the residual vector (3). The problem dimensions are $A_F \in \mathbb{R}^{n_F \times n_F}$, $B_F \in \mathbb{R}^{n_F \times (m+p)}$, $C_F \in \mathbb{R}^{q_r \times n_F}$, $D_F \in \mathbb{R}^{q_r \times (m+p)}$. □

The constraint $\lambda_i(A_F) \in \mathscr{R} \subseteq \mathscr{D}$, $\forall i$ refers to a robust pole assignment constraint and the performance index φ guarantees a maximum faults amplification $H(0)$ gain, see the notation section. In other words, the problem is formulated so that the robustness requirements against d are specified through $\mathscr{R}$ while specifying a high fault sensitivity level of the residual vector r through the maximization of φ. Note that, in practice, $\mathscr{R}$ is a parameter to be selected by the designer since finding an optimal region for $\mathscr{R}$ that guarantees high nuisances rejection, is highly related to the system under consideration.

The problem is now to establish a computational procedure for the $H(0)$ and robust pole assignment specifications. Thus, d is ignored from now and this boils down to a new setup as illustrated on figure 1 derived from (1) and (3) using some linear algebra manipulations, so that:

$$r(s) = F_u\left(F_l(\overline{P}(s), F(s)), \Delta\right) f(s) \tag{5}$$

2.1 The SDP formulation of the $H(0)$ specification

To achieve high fault detection performance, it is proposed in [13, 10, 14] to introduce a shaping filter W_f that allows to specify the fault sensitivity objectives. The solution of problem 1 is then handled using the following lemma, which is an application of lemma 2 in [13] to problem 1 taking into account the definition of the $H(0)$ gain. The proof is omitted here since it can be found in [13].

Lemma 1 *Let W_f be defined so that $||W_f||_0 \neq 0$. Introduce W_F, a right invertible transfer matrix so that $||W_f||_0 = \frac{\varphi}{\alpha}||W_F||_0$ and $||W_F||_0 > \alpha$, where $\alpha = 1 + \varphi$. Define the signal $\tilde{r}$ such that $\tilde{r}(s) = r(s) - W_F(s) f(s) : \tilde{r} \in \mathbb{R}^{q_r}$. Then a sufficient condition for the $H(0)$ specification in (4) to hold, is*

$$||T_{f \to \tilde{r}}||_\infty < 1, \quad \forall \Delta : ||\Delta||_\infty \leq 1 \tag{6}$$

where $T_{f \to \tilde{r}}$ denotes the closed-loop transfer between $\tilde{r}$ and f.

Using the above lemma, the filter design problem can be re-casted in a fictitious H_∞-framework: Including φ, α and W_F into the model $\overline{P}$, one can derive from (5) a new model $\tilde{P}$ so that (see figure 1 for easy reference)

$$\tilde{r}(s) = F_u\left(F_l\left(\tilde{P}(s), F(s)\right), \Delta\right) f(s) \tag{7}$$

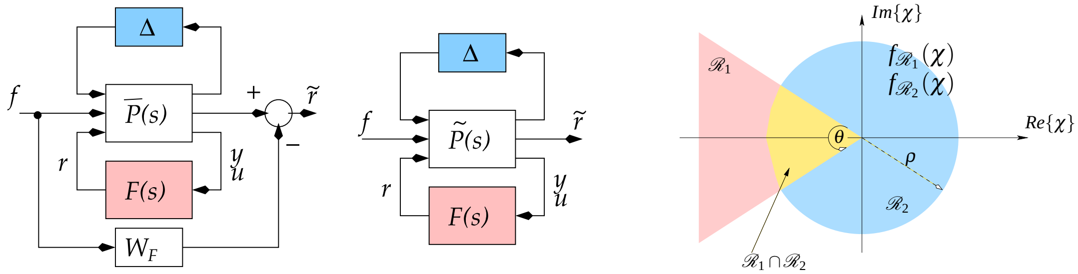

Figure 1: The fault detector design problem: $H(0)$ (left) and poles assignment (right) specifications

Noting that $F_u\left(F_l\left(\tilde{P}(s), F(s)\right), \Delta\right)$ is nothing else than the transfer $T_{f\to\tilde{r}}$, it follows by virtue of lemma 1 and the small gain theorem, that a sufficient condition for the $H(0)$ specification to hold is

$$\exists F(s) : \left\|F_l\left(\tilde{P}(s), F(s)\right)\right\|_\infty < 1 \tag{8}$$

Let $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ be the state space matrices of $\tilde{P}$ and consider the following partition of $\tilde{B}, \tilde{C}$ and $\tilde{D}$:

$$\tilde{B} = \left(\tilde{B}_1\, \tilde{B}_2\right), \quad \tilde{C} = \begin{pmatrix} \tilde{C}_1 \\ \tilde{C}_2 \end{pmatrix}, \quad \tilde{D} = \begin{pmatrix} \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{D}_{21} & \tilde{D}_{22} \end{pmatrix}, \quad \tilde{A} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}, \quad \tilde{D}_{22} \in \mathbb{R}^{(m+p)\times q_r} \tag{9}$$

It could be verified that $\tilde{B}_2 = 0$ and $\tilde{D}_{22} = 0$, showing that the fault detection filter F operates in open-loop versus the system. Then, using some linear algebra manipulations, it can be verified that the closed-loop model $F_l(\tilde{P}(s), F(s))$ admits the state realization (A_c, B_c, C_c, D_c) which is deduced from $\tilde{P}$ and F as follows:

$$A_c = \begin{pmatrix} \tilde{A} & 0 \\ B_F\tilde{C}_2 & A_F \end{pmatrix}, B_c = \begin{pmatrix} \tilde{B}_1 \\ B_F\tilde{D}_{21} \end{pmatrix}, C_c = \begin{pmatrix} \tilde{C}_1 + \tilde{D}_{12}D_F\tilde{C}_2 & \tilde{D}_{12}C_F \end{pmatrix}, D_c = \tilde{D}_{11} + \tilde{D}_{12}D_F\tilde{D}_{21} \tag{10}$$

From [15], $F_l(\tilde{P}(s), F(s))$ is stable (and F is a robustly stable filter due to the triangular structure of A_c) and there exists a solution to (8) if and only if there exists $\gamma < 1$ and matrices $\mathbf{A} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}, \mathbf{B} \in \mathbb{R}^{\tilde{n}\times(m+p)}, \mathbf{C} \in \mathbb{R}^{q_r\times\tilde{n}}, \mathbf{D} \in \mathbb{R}^{q_r\times(m+p)}$, $\mathbf{X} = \mathbf{X}^T \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$ and $\mathbf{Y} = \mathbf{Y}^T \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$ that solves the following SDP (Semi Definite Programming) problem:

$$\begin{gathered} \min \gamma \text{ s.t.} \\ \begin{pmatrix} \tilde{A}\mathbf{X} + \mathbf{X}\tilde{A}^T & \mathbf{A}^T + \tilde{A} & \tilde{B}_1 & (\tilde{C}_1\mathbf{X} + \tilde{D}_{12}\mathbf{C})^T \\ \mathbf{A} + \tilde{A}^T & \tilde{A}^T\mathbf{Y} + \mathbf{Y}\tilde{A} + \mathbf{B}\tilde{C}_2 + (\mathbf{B}\tilde{C}_2)^T & \mathbf{Y}\tilde{B}_1 + \mathbf{B}\tilde{D}_{21} & (\tilde{C}_1 + \tilde{D}_{12}\mathbf{D}\tilde{C}_2)^T \\ \tilde{B}_1^T & (\mathbf{Y}\tilde{B}_1 + \mathbf{B}\tilde{D}_{21})^T & -\gamma I & (\tilde{D}_{11} + \tilde{D}_{12}\mathbf{D}\tilde{D}_{21})^T \\ \tilde{C}_1\mathbf{X} + \tilde{D}_{12}\mathbf{C} & \tilde{C}_1 + \tilde{D}_{12}\mathbf{D}\tilde{C}_2 & \tilde{D}_{11} + \tilde{D}_{12}\mathbf{D}\tilde{D}_{21} & -\gamma I \end{pmatrix} < 0 \\ \begin{pmatrix} \mathbf{X} & I \\ I & \mathbf{Y} \end{pmatrix} > 0 \end{gathered} \tag{11}$$

Moreover, F is of full-order i.e. $n_F = \tilde{n}$. The fault detector state space matrices A_F, B_F, C_F and D_F are then deduced from $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{X}$ and $\mathbf{Y}$ according to the following procedure which is a direct application of the procedure proposed in [15] to our problem: i) find nonsingular matrices M, N to satisfy $MN^T = I - \mathbf{XY}$ (this can be done easily using the singular value decomposition technique), and, ii) define the fault detector by

$$D_F = \mathbf{D}, \quad C_F = (\mathbf{C} - \mathbf{D}\tilde{C}_2\mathbf{X})M^{-T}, \quad B_F = N^{-1}\mathbf{B}, \quad A_F = N^{-1}(\mathbf{A} - NB_F\tilde{C}_2\mathbf{X} - \mathbf{Y}\tilde{A}\mathbf{X}M^{-T}) \tag{12}$$

2.2 The LMI formulation of the robust poles assignment specification

Consider now the specification $\lambda_i(A_F) \in \mathscr{R} \subseteq \mathscr{D}, \forall i$. Assume that the region $\mathscr{R}$ is formed by the intersection of N elementary LMI regions $\mathscr{R}_i$, i.e.$\mathscr{R} = \mathscr{R}_1 \cap \ldots \cap \mathscr{R}_N$, see figure 1 for easy reference. Each LMI region $\mathscr{R}_i$ is characterized as follows:

$$\mathscr{R}_i = \{\chi \in \mathbb{C} : L_i + \chi Q_i + \chi^* Q_i^T < 0\} \tag{13}$$

where L_i and Q_i are real symmetric matrices. The matrix-valued function $f_{\mathscr{R}_i}(\chi) = L_i + \chi Q_i + \chi^* Q_i^T$ is called the characteristic function of the ith LMI region $\mathscr{R}_i$. Then, it is shown in [16] that a sufficient condition for all eigenvalues of A_c given by (10), lying in the region $\mathscr{R}$ for all $\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ is the existence, for each region $\mathscr{R}_i$, of a matrix P_i and $\beta < 1$ so that

$$\begin{pmatrix} \mathbb{Q}(A_c, P_i) & Q_{1i}^T \otimes (P_i B_c) & Q_{2i}^T \otimes C_c^T \\ Q_{1i} \otimes (B_c^T P_i) & -\beta I & I \otimes D_c^T \\ Q_{2i} \otimes C_c & I \otimes D_c & -\beta I \end{pmatrix} < 0, \quad P_i > 0, \quad i = 1 \dots N \tag{14}$$

where "$\otimes$" denotes the Kronecker product of matrices. The matrix $\mathscr{Q}_{\mathscr{R}_i}(A_c, P_i)$ is defined according to

$$\mathbb{Q}(A_c, P_i) = L_i \otimes P_i + Q_i \otimes (P_i A_c) + Q_i^T \otimes (A_c^T P_i) \tag{15}$$

$Q_{1i}^T Q_{2i} = Q_i$ is a factorization of Q_i so that Q_{1i} and Q_{2i} have full column rank.

Due to the triangular structure of A_c, it is obvious that the set of the eigenvalues of A_c are equal to the set of the eigenvalues of $\tilde{A}$ and A_F. Thus, a sufficient condition for all fault detection filter poles lying in the LMI region $\mathscr{R}$ for all $\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ (i.e for the robust pole assignment specification to hold) is the existence of a solution to the inequalities (14). Unfortunately, since each inequality constraint involves products of a matrix P_i, $i = 1, \dots, N$ and the fault filter variables A_F, B_F, C_F, D_F , the resulting optimization problem is nonlinear. To reduce the problem to a linear optimization problem, the linearizing change of variables given by (12) can be used.
Let $\tilde{B}_1, \tilde{C}_1, \tilde{D}_{11}, \tilde{D}_{12}, \tilde{D}_{21}$ be partitioned according to the dimension of Δ such that

$$\tilde{B}_1 = \begin{pmatrix} B_\Delta & B_f \end{pmatrix}, \tilde{C}_1 = \begin{pmatrix} C_\Delta \\ C_r \end{pmatrix}, \tilde{D}_{11} = \begin{pmatrix} D_{\Delta\Delta} & D_{\Delta f} \\ D_{r\Delta} & D_{rf} \end{pmatrix}, \tilde{D}_{12} = \begin{pmatrix} D_{1\Delta} \\ D_{1r} \end{pmatrix}, \tilde{D}_{21} = \begin{pmatrix} D_{2\Delta} & D_{2f} \end{pmatrix} \tag{16}$$

It follows that all eigenvalues of A_F lye in the region $\mathscr{R}$ for all $\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ if there exist $\beta < 1, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ and $\mathbf{X}_i = \mathbf{X}_i^T \in \mathbb{R}^{\tilde{n} \times \tilde{n}}, \mathbf{Y}_i = \mathbf{Y}_i^T \in \mathbb{R}^{\tilde{n} \times \tilde{n}}$ $i = 1 \dots N$ that solve the following SDP problem:

$$\begin{array}{ll} \min \beta \; s.t. & \begin{pmatrix} L_i \otimes \Psi(\mathbf{X}_i, \mathbf{Y}_i) + Q_i \otimes \Phi_A + Q_i^T \otimes \Phi_A^T & Q_{1i}^T \otimes \Phi_B & Q_{2i}^T \otimes \Phi_C^T \\ Q_{1i} \otimes \Phi_B^T & -\beta I & I \otimes \Phi_D^T \\ Q_{2i} \otimes \Phi_C & I \otimes \Phi_D & -\beta I \end{pmatrix} < 0 \\ \text{with} & \Psi(\mathbf{X}_i, \mathbf{Y}_i) = \begin{pmatrix} \mathbf{X}_i & I \\ I & \mathbf{Y}_i \end{pmatrix} > 0, \; \Phi_A = \begin{pmatrix} \tilde{A}\mathbf{X}_i & \tilde{A} \\ \mathbf{A} & \mathbf{Y}_i \tilde{A} + \mathbf{B}\tilde{C}_2 \end{pmatrix}, \; \Phi_B = \begin{pmatrix} B_\Delta \\ \mathbf{Y}_i B_\Delta + \mathbf{B} D_{2\Delta} \end{pmatrix} \\ & \Phi_C = \begin{pmatrix} C_\Delta \mathbf{X}_i + D_{1\Delta}\mathbf{C} & C_\Delta + D_{1\Delta}\mathbf{D}\tilde{C}_2 \end{pmatrix}, \; \Phi_D = D_{\Delta\Delta} + D_{1\Delta}\mathbf{D} D_{2\Delta} \end{array} , \; i = 1 \dots N \tag{17}$$

2.3 Computational issues

From the above developments, problem 1 can be solved by jointly solving the SDP problems (11) and (17). This boils down to a multiobjective optimization problem in the form

$$\min \; \varepsilon\gamma + (1+\varepsilon)\beta \quad s.t. \; (11) \text{ and } (17) \tag{18}$$

whereby the choice of ε is guided by the Pareto optimal points. However, in practice, β is better considered as a parameter to be fixed to $\beta = 1$. Thus, the resulting optimization problem looks for the best achievable $H(0)$ objective whereas the robust pole assignment constraint is enforced. Any $\gamma < 1$ indicates that the obtained solution is admissible for problem 1. However, $\gamma \approx 1^-$ is required in order to obtain a low conservative solution.
Furthermore, as it is now well known, all aforementioned inequalities must be solved by using a single Lyapunov matrix for feasibility reasons. This boils down to the additional constraints $\mathbf{X}_1 = \dots = \mathbf{X}_N = \mathbf{X}$ and $\mathbf{Y}_1 = \dots = \mathbf{Y}_N = \mathbf{Y}$. Fortunately, the extra conservatism introduced by this additional restriction is modest in most applications.

3. Application to the MSR mission

The robust fault detection scheme presented in the above section is now considered for the detection and isolation of faults occurring in the orbiter thrusters unit.

3.1 Modeling the orbiter dynamics during the rendez-vous phase

The motion of the orbiter is derived from the 2nd Newton law. Because the distance between the Mars ascent vehicle and the orbiter is smaller than the orbit, it is possible to derive the so called Hill-Clohessy-Wiltshire equations by means of a first order approximation. This boils down to a linear six order state space model whose inputs are a three-dimensional forces vector. Then, considering the adequate change of coordinates, the motion of the orbiter can be described according to the following dynamical equations:

$$\begin{cases} \dot{x} = & Ax + BR(\hat{Q}_{tgt}(t), \hat{Q}_{chs}(t))M(I - \Psi(t))u_{thr}(t) + B_w w(t) \\ y = & Cx + n \end{cases}, \Psi(t) = diag(\psi_i(t))\, i = 1, \ldots, 8 \tag{19}$$

In (19), $x \in \mathbb{R}^6$ that consists of the three-dimensional positions and velocities of the orbiter is the state vector and $y \in \mathbb{R}^3$ refers to the three-dimensional positions measured by means of a LIDAR unit, both given in the Mars ascent vehicle orbital frame. $u_{thr} \in \mathbb{R}^8$ is the controlled thrust signals given in the orbiter's frame. $\hat{Q}_{tgt} \in \mathbb{R}^4$ and $\hat{Q}_{chs} \in \mathbb{R}^4$ respectively refer to the attitude's quaternions of the Mars ascent and orbiter vehicles that are also provided by the navigation module. $R(\hat{Q}_{tgt}(t), \hat{Q}_{chs}(t))$ refers to a rotation matrix and w refers to the spatial disturbances, e.g. J_2 disturbances, atmosphere winds..etc. n denotes the measurement noise, considered here to be a white noise with very small variance due to the technology used for the design of the LIDAR. $M \in \mathbb{R}^{3\times 8}$ is the (static) allocation module and A, B, C are matrices of adequate dimension. Ψ models thruster faults, e.g. a locked-in-placed fault can be modeled by $\Psi_i(t) = 1 - \frac{c}{u_{ths}(t)}$ where c denotes a constant value (the particular values $c = \{0, 1\}$ allows to consider open/closed faults) whereas a fix value of Ψ_i models a loss of efficiency of the ith thruster. $\Psi(t) = 0 \forall t$ means that no fault occurs in the thrusters.
Then taking into account the controller actions (the attitude control loop is not considered here), considering $R(\hat{Q}_{tgt}(t), \hat{Q}_{chs}(t))$ $Mu_{thr}(t)$ as the input vector $u(t)$ and approximating the faults model $R(\hat{Q}_{tgt}(t), \hat{Q}_{chs}(t))M\Psi(t)u_{thr}(t)$ in terms of additive faults $f(t) \in \mathbb{R}^3$ acting on the state via a constant distribution matrix K_f (then $K_f = B$), it follows that the overall model of the orbiter's dynamics that takes into account both the rotational ($Q_{chs}(t)$) and linear translation ($x(t)$) orbiter motions can be written in the form (1) with $d = n$. Δ is also concerned by some unknown but bounded delays induced by the electronic devices and the uncertainties on the thruster rise times.

3.2 Design of the FDI scheme

3.2.1 Design of the fault detection filter

The robust fault detection scheme presented in section 2 is now considered. The problem dimensions are $q_f = 3, q_r = 3, m = 3, p = 3$. The shaping filter W_f involved in lemma 1 is chosen to be a low pass filter of first order with $H(0)$ gain the highest possible. With regards to the robust pole clustering constraint, it is required robust pole clustering in the LMI region defined as the intersection of the two following regions, i.e. $\mathscr{R} = \mathscr{R}_1 \cap \mathscr{R}_2$:
• $\mathscr{R}_1$: disk with center $(-q, 0)$ and radius ρ (to prevent fast dynamics). This region is defined according to:

$$\mathscr{R}_1 = \left\{ \chi \in \mathbb{C} : \begin{pmatrix} -\rho & q \\ q & -\rho \end{pmatrix} + \chi \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \chi^* \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} < 0 \right\}$$

where $q = 0.5$ and $\rho = 1$. By this choice, it is required all eigenvalues of A_F to be close to -0.5.
• $\mathscr{R}_2$: shifted conic sector with apex at ω and angle θ. $\mathscr{R}_2$ is characterized according to

$$\mathscr{R}_2 = \left\{ \chi \in \mathbb{C} : \begin{pmatrix} -2\omega cos(\theta) & 0 \\ 0 & -2\omega cos(\theta) \end{pmatrix} + \chi \begin{pmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{pmatrix} + \chi^* \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} < 0 \right\}$$

where the numerical values of ω and θ are fixed respectively to $\omega = 10$ and $\theta = 5^o$. This particular region is chosen to maintain a suitable damping ratio. Note that, as (11) enforces filter stability, it is inconsequential that the LMI region $\mathscr{R}$ intersects the right half-plane.

Following the discussion in section 2.3, the fault detection filter state-space matrices A_F, B_F, C_F and D_F are computed so that inequalities (11) and (17) are satisfied. As expected, the poles of the so computed filter are found to be close to ≈ -0.5. Figure 2 illustrates the principal gains $T_{u\to r}(j\omega)$ (the transfer between the inputs u and the residuals r) and $T_{y\to r}(j\omega)$ (the transfer between the measurements y and the residuals r) of the computed filter F. As it can be seen, $T_{u\to r}(j\omega)$ behaves like a low pass filter, whereas $T_{y\to r}(j\omega)$ behaves like a high pass filter. Furthermore, it can be noted that the gains of $T_{y\to r}(j\omega)$ is always lower than 1 showing that the measurement noise is not amplified on the residuals $r(t)$.

3.2.2 The isolation strategy

With regards to the fault isolation task and based on the method proposed in [10], the following normalized cross-correlation criterion between the residuals r and the associated controlled thrusters open rate u_{thr_i} is used here:

$$i(k) = argmin\, \frac{1}{N} \sum_{k=\tau-N}^{\tau} (r_j(k) - \overline{r})(u_{thr_i}(k) - \overline{u_{thr_i}}), \quad i = 1...8,\ j \in \{1,2,3\},\ t = k.T_s \tag{20}$$

In (20), $\overline{r}$, $\overline{u_{thr_i}}, i = 1...8$ and T_s denote the mean values of r and $u_{thr_i}, i = 1...8$ and the navigation module sampling period. For real-time reason, this criterion is computed on a N-length sliding-window. The resulting index $i(k)$ also refers to the identified faulty thruster. A key feature of this isolation strategy is that it is static and then, has low computational burdens.

3.3 Simulation results

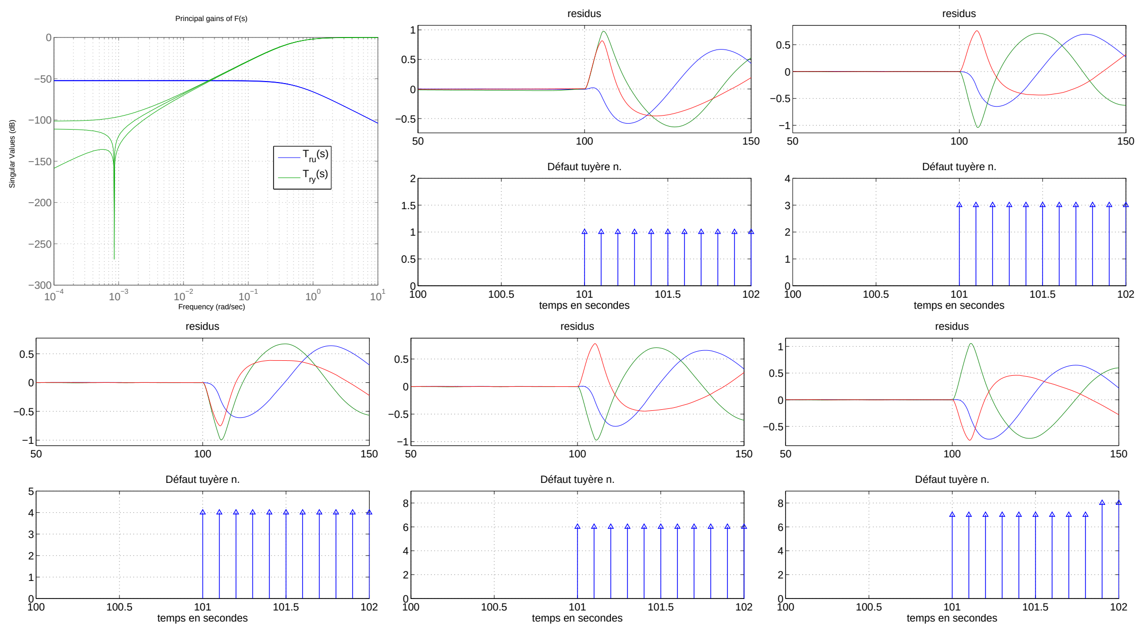

Figure 2: The principal gains of the filter F (top left) and behavior of $r(t)$ and $i(t)$ for some faulty situations.

The fault detection filter F is converted to discrete-time using a Tustin approximation and implemented within the nonlinear simulator of the MSR mission provided. The simulated faults correspond to a single thruster opening at 100% during the last 20m of the rendezvous. To make a final decision about the fault, a sequential Wald decision test applied to $||r(t)||_2$ is implemented within the simulator. The probabilities of non-detection and false alarms have been fixed to 0.1%. The isolation strategy is too implemented within the nonlinear simulator with $j = 1$, see (20). Figures 2 illustrate the behavior of the residual $r(t)$ and the isolation criteria $i(t)$, for some faulty situations, i.e. a fault occurs in the thruster n.1 (top middle), thruster n.3 (top right), thruster n.4 (bottom left), thruster n.6 (bottom middle) and thruster n.7 (bottom right). For each case, the fault occurs at $t = 100s$ and is maintained. The strategy works as follows: as soon as the fault is declared by the decision test, the cross-correlation criterion (20) is computed. As it can be seen on the figures, all thruster faults are successfully detected and isolated by the FDI unit with a detection and isolation delay less than $1.1s$. Note that such a strategy succeeds since both the rotational ($Q_{chs}(t)$) and linear translation ($x(t)$) orbiter motions have been considered. By this way, the effects that faults have on both the orbiter attitude and translation motion, are taken into account.

4. Concluding remarks

This paper addressed the design of robust model-based fault diagnosis schemes to detect and isolate faults occurring in the orbiter's thrusters unit of the Mars Sample Return mission. The presented study focused on the orbiter spacecraft during the rendezvous phase with the Mars ascent vehicle. The proposed fault diagnosis scheme consists of a $H(0)$ filter with robust poles assignment which is in charge of residual generation for fault detection. The isolation task is solved using a cross-correlation test between the residuals and the thrusters signals. For reduced computational burdens, the isolation test is based on a sliding time window. The key feature of the proposed method is the use of a judiciously chosen linear model for the design of the filter, i.e. the model consists of a 6-order model given in a judiciously chosen frame that takes into account both the rotational and linear translation spacecraft motions. This allows to propose a fault diagnosis solution with reduced computational burdens that is then thought to be a potential candidate for on-board implementation.

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