Guidance Gain Analysis via Oscillation Motion of Error Dynamics-Based Guidance Law for Stationary Target Observation

Mingu Kim⋆*, Youdan Kim*⋆*, and Chan Gook Park*⋆
⋆*School of Mechanical and Aerospace Engineering, Seoul National University*
599 Gwanak-ro, Gwank-gu, Seoul, 151-744, Korea

Abstract

This paper deals with the nonlinear guidance law based on error dynamics to perform the stationary target observation missions of multiple UAVs. In this paper, the guidance law utilizing the error dynamics is analyzed to provide the proper guideline for choosing the guidance gains in the consideration of the turning direction. Overdamped motion assumption as well as the relation between the turning direction and the rate of LOS(Line-Of-Sight) change is used to analyze the turning direction. Numerical simulations are performed to verify the performance of UAV guidance law and analyze the properties of the guidance law.

1. Introduction

Unmanned Aerial Vehicles(UAVs) are widely used to accomplish various military and civil missions. Guidance law is designed to make UAV follow the desired trajectory for the accomplishment of the given mission. Especially, multiple UAVs are involved to perform a very complicated mission. Various guidance laws for the multiple UAVs, thus, have been developed.

Recently, Ryoo et al. applied nonlinear guidance laws to solve impact time and impact angle constraints[1-3]. To solve the terminal impact angle constraint, optimal guidance laws with terminal impact angle controller was derived using the concept of energy minimization[1]. The optimal guidance command was given by a linear combination of the step and the ramp responses of the missile's lateral acceleration. Jung and Kim utilized a geometry analysis to set up waypoint passing angles to design impact angle guidance law based on the backstepping control method. Impact time guidance law was also introduced based on Proportinal Navigation Guidance(PNG)[2]. Harl and Balakrishnan proposed a second order sliding mode control method to track the desired Line-of-sight(LOS) rate satisfying the constraints of the impact angle and impact time[3].

Lyapunov stability theorem has been widely used to prove the stability of the nonlinear guidance laws. Lyapunov candidate function is used to find the time derivative of a sliding surface sending the sliding surface to zero[3]. A quadratic Lyapunov candidate function was used to design the nonlinear guidance laws with free-singularity[4]. Lyapunov stability was demonstrated to make UAV follow the curved paths generated by using a nonlinear guidance law[5]. Asymptotic Lyapunov stability of the nonlinear guidance law was demonstrated when the UAV is following circular paths[6]. The stability analysis was also performed to analyze the robust stability of the nonlinear guidance law. Conventional nonlinear guidance law was designed based on the Lyapunov stability theorem on stability for finite time convergence in both two dimensional and three dimensional environments[7].

Lyapunov vector fields were also applied for the coordinated standoff tracking of targets[8-11]. Lyapunov vector fields were constructed for the multiple UAVs to produce a stable convergence to a circling limit cycle behavior[8,9]. Lyapunov approach producing vector fields by considering contraction and circulation terms in the vector field separately also guarantees global stability[10]. Lyapunov vector fields approach was also used for coordinated standoff tracking of stationary or moving targets in UAV formations using a rigid graph theory[11].

Distance error dynamics can be also used to design a nonlinear guidance law for the stationary target observation. Error dynamics-based nonlinear guidance law is designed to make the distance error go to zero exponentially[12]. However, the analysis for selecting the guidance gains was not systematic.

In this paper, guidance law is analyzed to provide a suitable guideline for selecting gains in the consideration of the UAV turning direction. In order to analyze the guidance law, distance error dynamics is treated as oscillation motion. It is assumed that the overdamped motion can be achieved by analyzing the characteristic of guidance law and selecting proper guidance gains. The relation between the turning direction and the rate of LOS change is also considered to provide the guideline of the gain selections for determining the turning direction.
This paper is organized as follows. In Section 2, the distance error dynamics is first formulated to derive the error dynamics-based guidance law. The relation between the guidance gains and the turning direction of UAV is analyzed in Section 3. Numerical simulations for a point mass UAV model is presented in Section 4 to verify the performance of the proposed guidance law. Finally, conclusions are made in Section 5.

2. Error dynamics-based guidance law

2.1 Point mass model

Let us consider a point mass model UAV as shown in Fig. 1. It is assumed that the autopilot and sensor dynamics of the UAV are much faster than UAV dynamics so that they can be neglected. Also, the velocity of the UAV is assumed to be constant.

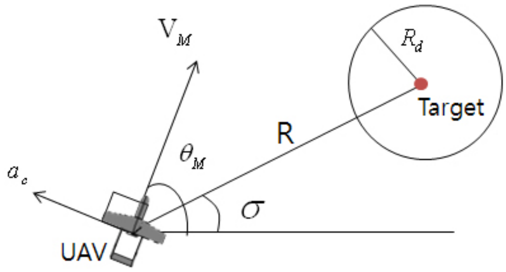

Figure 1: Point mass model geometry

It is also assumed that the angle-of-attack(AOA) of USV is very small enough to be omitted. Then, the acceleration vector is perpendicular to the velocity vector, and therefore the equations of motion can be represented as

$$\dot{R} = -\mathrm{V_M}\cos(\sigma - \theta_\mathrm{M}) \tag{1}$$

$$\dot{\sigma} = \frac{\mathrm{V_M}}{R}\sin(\sigma - \theta_\mathrm{M}) \tag{2}$$

$$\dot{\theta}_\mathrm{M} = a_\mathrm{M}/\mathrm{V_M} = a_\mathrm{C} \tag{3}$$

where R denotes the distance between the UAV and the target, θ_M represents the heading angle of the UAV, $\mathrm{V_M}$ is the velocity of the UAV, a_M is the acceleration of the UAV, and σ represents the line-of-sight to the target. Note that the target is any object that the UAV is trying to observe. Let us define a distance error as

$$e_R = R - R_d \tag{4}$$

where R_d is the constant radius that the UAV should keep for the stationary target observation. Differentiating Eq.(4) with respect to time once and twice yields

$$\dot{e}_R = \dot{R} \tag{5}$$

$$\ddot{e}_R = \ddot{R} \tag{6}$$

2.2 Error dynamics-based guidance law

Using the equations of motion, the second derivative term of distance error, $\ddot{e}_R$, can be represented as

$$\ddot{e}_R = \mathrm{V_M}\sin(\sigma - \theta_\mathrm{M})\left(\dot{\sigma} - \dot{\theta}_\mathrm{M}\right) = R\dot{\sigma}^2 - R\dot{\sigma}a_c \tag{7}$$

Let us propose a following guidance command considering the centrifugal acceleration satisfying Eq.(7).

$$a_c = \frac{1}{R\dot{\sigma}}\left(k_1\dot{e}_R + k_2 e_R + R\dot{\sigma}^2\right) \tag{8}$$

Substituting the centrifugal acceleration command, a_c, into Eq.(7), the distance error dynamics can be rewritten as

$$\ddot{e}_R + k_1\dot{e}_R + k_2 e_R = 0 \tag{9}$$

In Eq. (9), two guidance gains, k_1 and k_2, should be carefully selected by considering the characteristics of the distance error dynamics. In this study, the second order dynamics is used to analyze the guidance law.
Definition. Using the damping ratio, ζ, and the natural frequency, ω_n, the guidance gains can be defined as

$$k_1 \triangleq 2\zeta\omega_n \tag{10}$$

$$k_2 \triangleq \omega_n^2 \tag{11}$$

Assumption 1. (Overdamped motion) The damping ratio, ζ, is always larger than 1 and constant.
Assumption 1 is very reasonable for the target observation missions, because in order for the multiple UAVs to approach the target asymptotically. After that UAV should keep the desired radius, R_d, from the target. Substituting Eqs.(10) and (11) into the distance error dyanamics, the distance error can be represented as

$$e_R(t) = R(t) - R_d = c_1 e^{-\left(\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)t} + c_2 e^{-\left(\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)t} \tag{12}$$

where c_1 and c_2 are constants related to the initial distance error and the initial value of the first derivative term of the distance error. In Eq.(12), $\xi\omega_n - \omega_n\sqrt{\xi^2-1}$ is always positive, due to **Assumption 1.** Thus, it can be stated that the error dynamics-based guidance law makes the distance error go to zero exponentially.

3. Guidance gains analysis

In this section, the relation between guidance gains and the turning direction is analyzed to provide a proper guideline for selecting the guidance gains. Let us consider the case that the direction of a UAV's LOS is positive counterclockwise.
Proposition. When a UAV turns counterclockwise, the LOS angle of the UAV always increases. However, when a UAV turns clockwise, the LOS angle always decreases.

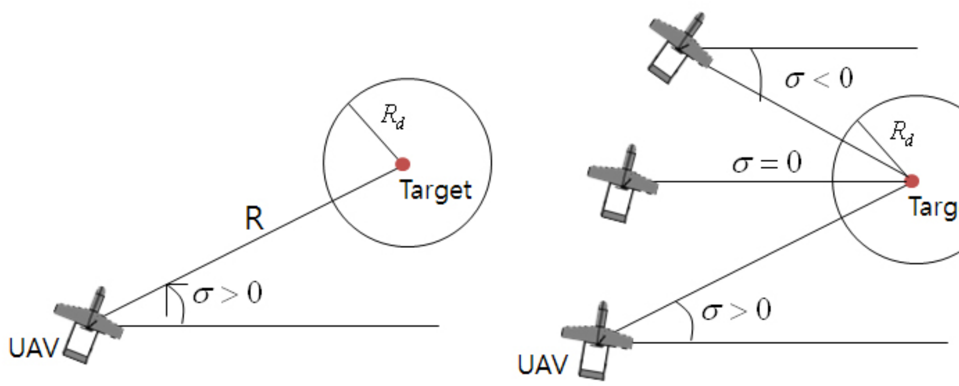

Figure 2: The direction of a UAV's LOS, and the relation between its LOS angle change and its turning direction

Using **Proposition**, the theorem of the relation between the second derivative term of a LOS angle, $\ddot{\sigma}$, and its turning direction can be derived.
Theorem. While a UAV keeps its positive LOS angular acceleration, the UAV turns counterclockwise. On the other hand, while a UAV maintains its negative LOS angular acceleration, the UAV turns clockwise.
Proof. If a UAV's LOS angular acceleration is maintained positive value, although its initial LOS angular acceleration is negative, it becomes positive. This means that the UAV turns counterclockwise. If a UAV's LOS angular acceleration is maintained negative value, although its initial LOS angle acceleration is positive, it becomes negative. This

means that the UAV turns clockwise. Q.E.D

Using **Theorem**, the guidance gains can be determined by considering the turning direction of the UAV.

Lemma. If one guidance gain of two design parameters, k_1 and k_2, is already determined, the turning direction of the UAV can be determined by selecting another guidance gain.
Proof. From Eq. (2), UAV's LOS angular acceleration can be represented as

$$\ddot{\sigma} = -\frac{\dot{R}}{R}(2\dot{\sigma} - a_c) \tag{13}$$

Substituting Eq.(8) into Eq.(13), the UAV's LOS angular acceleration can be rewritten as

$$\ddot{\sigma} = \frac{\dot{R}}{R^2\dot{\sigma}}\left(k_1\dot{e}_R + k_2 e_R - R\dot{\sigma}^2\right) \tag{14}$$

Note from Eqs.(10) and (11) that the guidance gains are defined by using the damping ratio and the natural frequency of the error dynamics. In this study, it is assumed that the damping ratio is always larger than 1 and constant (**Assumption 1.**) for the missions of the target observation. After selecting the damping ratio considering the characteristics of the error dynamics, the natural frequency can be chosen by considering the turning direction of the UAV. One more assumption is needed to simplify the problem.
Assumption 2. Coefficients, c_1 and c_2 , in Eq.(12) are always positive.
To satisfy **Assumption 2**, the natural frequency should be satisfied the following equation. See the proof in the **Appendix A. Coefficients of Eq.(12)**.

$$-\frac{\dot{R}_0}{e_{R_0}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} < \omega_n < -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)} \tag{15}$$

Now, the distance error is always positive, due to **Assumption 2**. The first derivative of the distance, therefore, is always negative. Let us consider two opposite cases, counterclockwise turn case and clockwise turn case.

Case 1: Counterclockwise turn case.
From **Theorem**, $\ddot{\sigma}$ should be positive, and we have
i) $\dot{\sigma} > 0$

$$\left(2\zeta\omega_n\dot{R} + \omega_n^2 e_R - R\dot{\sigma}^2\right) < 0 \tag{16}$$

$$-\frac{\dot{R}_0}{e_{R_0}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} < \omega_n < \min\left(\frac{-\zeta\dot{R} + \sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R}, -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)}\right) \tag{17}$$

ii) $\dot{\sigma} < 0$

$$\left(2\zeta\omega_n\dot{R} + \omega_n^2 e_R - R\dot{\sigma}^2\right) > 0 \tag{18}$$

(19)

$$\omega_n > \max\left(\frac{-\zeta\dot{R} + \sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R}, -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)}\right) \tag{20}$$

Similarly, the conditions for clockwise turn case can be derived as follow.

Case 2: Clockwise turn case.
From **Theorem**, $\ddot{\sigma}$ should be negative, we have
i) $\dot{\sigma} > 0$

$$\omega_n > \max\left(\frac{-\zeta\dot{R} + \sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R}, -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)}\right) \tag{21}$$

ii) $\dot{\sigma} < 0$

$$-\frac{\dot{R}_0}{e_{R_0}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} < \omega_n < \min\left(\frac{-\zeta\dot{R} + \sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R}, -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)}\right) \tag{22}$$

Q.E.D

Using **Lemma**, The guideline in the consideration of the UAV's turning direction is proposed as follows. The upper limit of the natural frequency, $\omega_{n\_\text{lim}_{upper}}$, is defined as

$$\omega_{n\_\text{lim}_{upper}} \triangleq \frac{-\zeta\dot{R} + \sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R} \tag{23}$$

The upper limit of the natural frequency is always larger than zero, and therefore it increases as time increases. See the proof in the **Appendix B. The characteristic of the upper limit of the natural frequency**. This property makes it difficult to calculate the upper limit of the natural frequency, and therefore the initial upper limit of the natural frequency is used to choose the guidance gains.

$$\omega_{n\_\text{lim}_{upper_0}} \triangleq \frac{-\zeta\dot{R}_0 + \sqrt{\zeta^2\dot{R}_0^2 + R_0\dot{\sigma}_0^2 e_{R_0}}}{e_{R_0}} \tag{24}$$

For each case, Eqs.(24) and (26) are used to determine the guidance gains.
a) Case 1 i) and Case 2 ii).

$$\omega_n = \alpha\omega_{n\_\text{lim}_{upper_0}} \tag{25}$$

where

$$\frac{-\dot{R}_0}{e_{R_0}\omega_{n\_\text{lim}_{upper_0}}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} \leq \alpha \leq \frac{-\dot{R}_0}{e_{R_0}\omega_{n\_\text{lim}_{upper_0}}\left(\zeta - \sqrt{\zeta^2 - 1}\right)} \tag{26}$$

b) Case 1 ii) and Case 2 i),

$$\omega_n = \omega_{n\_\text{lim}_{upper_0}} + \beta \tag{27}$$

4. Numerical simulations

Numerical simulations are performed to verify the performance of the proposed guidance law based on analysis of the error dynamics. In this particular example, we consider the case that there exists no wind. Also, a stationary target is considered. The target observation by the coordinated turn with an arbitrary radius is considered as the mission of the UAV.
In this simulation, the velocity of the UAV is 22m/s, and the turning radius is 300m, and the initial position and the initial heading angle of each UAV, and the target position are summarized in Table 1.

Table 1: The initial conditions

	Position	Heading angle
UAV 1	$(0, 0)$	$0deg$
UAV 2	$(1600, 0)$	$90deg$
UAV 3	$(300, 1600)$	$-90deg$
Target	$(800, 600)$	

The damping ratio is chosen as 1.2, the limit of the guidance command is chosen as $2g$, α is chosen as 0.7 to satisfy Eq.(25), and β is selected as 0.05. Figure 3 show the two-dimensional trajectories and the guidance command histories of each UAV in Case 1. The turning directions of all UAVs are counterclockwise. Although the initial guidance commands of UAV1 and UAV3 are negative, after seconds, they become soon positive and maintain positive values. Figure 4 show the two-dimensional trajectories and the guidance command histories of each UAV in Case 2. All UAVs turn clockwise. Although the guidance commands of UAV1 and UAV3 are saturated at the initial stage, they soon become negative and maintain negative values.

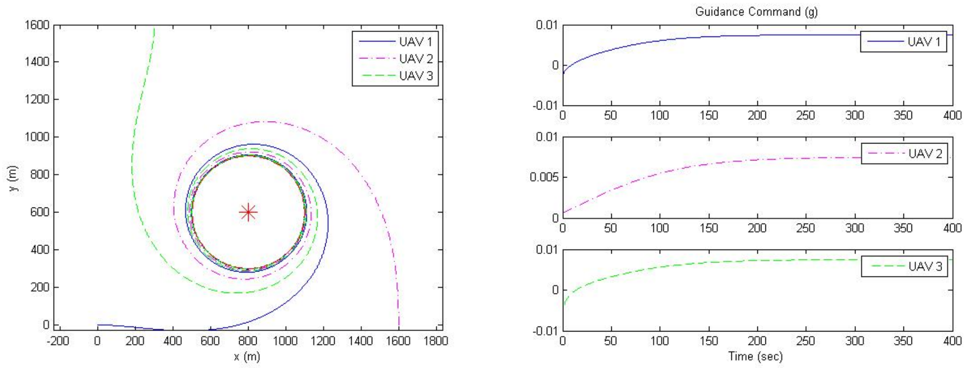

Figure 3: The 2-D trajectory and guidance commands of three point mass UAVs: Case 1

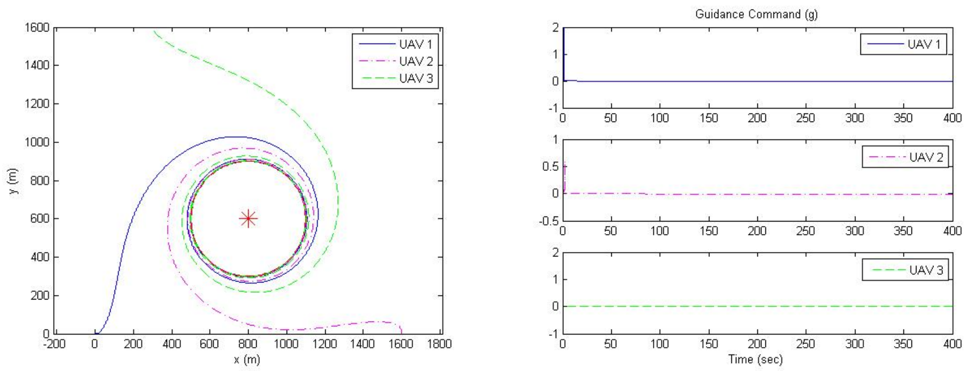

Figure 4: The 2-D trajectory and guidance commands of three point mass UAVs: Case 2

5. Conclusions

Guidance law is proposed in the consideration of the UAV. The equations of motion and error dynamics are formulated to derive a nonlinear guidance law. To reduce the distance error while maintaining the observation radius, the centrifugal acceleration is used. To select the guidance gains, error dynamics is considered as a second-order vibrating system. When multiple UAVs are used to perform the observation mission, the turning direction of each UAv is very important to avoid colliding with each other. Assuming that the motion is overdamped to prevent the occurrence of overshoot, the natural frequency becomes a design parameter for determining the guidance gains, and the rate of distance change becomes always negative. Numerical simulations are performed to verify the performance of the proposed guidance law.

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Appendix A. Coefficients of Eq.(12)

Coefficients, c_1 and c_2, in Eq. (12) can be calculated using the equations of motion and the initial values. From Eq.(12), the initial distance can be represented as

$$R_0 = c_1 + c_2 + R_d \tag{A.1}$$

The initial value of the first derivative term of the distance can be rewritten by using Eq. (12).

$$\dot{R}_0 = -c_1\left(\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right) - c_2\left(\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right) \tag{A.2}$$

Using Eqs.(A.1) and (A.2), c_1 and c_2 can be represented as

$$c_1 = -\frac{\dot{R}_0 + (R_0 - R_d)\left(\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right)}{2\omega_n\sqrt{\zeta^2 - 1}} \tag{A.3}$$

$$c_2 = \frac{\dot{R}_0 + (R_0 - R_d)\left(\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right)}{2\omega_n\sqrt{\zeta^2 - 1}} \tag{A.4}$$

For positive c_1 and c_2, the following conditions should be satisfied.

$$\dot{R}_0 + e_{R_0}\left(\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right) < 0 \tag{A.5}$$

$$\dot{R}_0 + e_{R_0}\left(\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right) > 0 \tag{A.6}$$

where $e_{R_0} = R_0 - R_d$. Therefore, we have

$$-\frac{\dot{R}_0}{e_{R_0}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} < \omega_n < -\frac{\dot{R}_0}{e_{R_0}\left(\zeta - \sqrt{\zeta^2 - 1}\right)} \tag{A.7}$$

Appendix B. The characteristic of the upper limit of the natural frequency

Lemma B.1 The upper limit of the natural frequency is always larger than zero.
Proof. According to Assumptions 1 and 2, the damping ratio is larger than 1 and the first derivative of the distance is always negative. Therefore, $-\zeta\dot{R}$ is always positive, and the distance error is also positive. The upper limit of the natural frequency, thus, is always positive. Q.E.D
Lemma B.2 The upper limit of the natural frequency increases as time increases.
Proof. The term $\zeta\dot{R}/e_R$ converges zero, as time increases. Let us analyze the term of, $\sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}/e_R$. The distance error converges zero exponentially and the rate of the distance, $\dot{R}$, can be represented as an exponential function. Therefore, $\sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}$ is always larger than $\zeta\dot{R}$. Because of $R\dot{\sigma}^2 e_R$, the upper limit of the natural frequency increases, as time increases.

$$\lim_{t\to\infty} \frac{\sqrt{\zeta^2\dot{R}^2 + R\dot{\sigma}^2 e_R}}{e_R} = \infty \tag{B.1}$$

Q.E.D

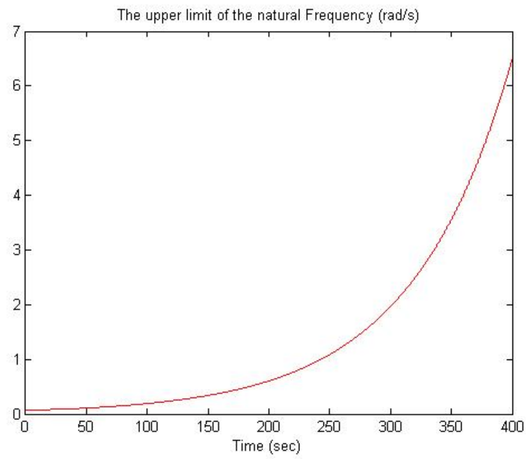

Figure 5: The time history of UAV1's upper limit of the natural frequency in Case 2

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