# Chance constrained optimization for launcher GNC validation

*G. Maurice*, M. Ganet-Schoeller*, R. Lebrun ** and S. Bennani****
** ASTRIUM Space Transportation*
*66 Route De Verneuil, 78133 Les Mureaux Cedex, France*
***EADS IW*
**** ESA/ESTEC (TEC-ECN); GNC Group*
*Keplerlaan 1, NL-2201 AZ Noordwijk; The Netherlands*

## Abstract
Within a framework for validation of launcher control laws, a method to search for worst case configuration is proposed. Hybrid optimization is used to find the combination of model's uncertain parameter that leads to the worst case for a given specification. A constraint on the input parameters probability density function is used to restrain the search domain to a domain compatible with a probability objective. The proposed method was validated and tested. Optimization based process represent an effort reduction to identify worst cases and the chance constraint allows exploring a more probable domain for the parameters than optimization without constraints.

## 1. Introduction

This joint work between ASTRIUM Space Transportation (ST) and EADS Innovation Work (IW), was carried out within the scope of the SAFE-V project (Space Application Flight control Enhancement of Validation Framework), an ESA TRP under contract number 4000102288, and the support of EADS Upstream Research Projects. The objective of the SAFE-V project was to analyze, develop and demonstrate effective design, verification and validation strategies and metrics for advanced Guidance Navigation and Control (GNC) systems for launcher applications.

Launcher GNC traditional validations are based both on linear analysis (frequency domain response) and on Non Linear (NL) analysis (simulation for time domain performance). Validations shall be made on a wide domain of variation of influent parameters; leading to robustness issues since robust analysis methods and algorithms are complex to implement into efficient engineering processes and are often limited by their computational complexity. Traditional validation includes on the one hand Worst Case (WC) validation that is based on gridding based worst cases defined using analytical techniques and simplified launcher model, (see [1]). On the other hand Monte Carlo (MC) simulations are run for probabilistic requirements verification. Main drawbacks of these methods are that they are either local and deterministic (worst case methods) or global and probabilistic but very time consuming (statistical methods).

To enhance traditional validation plan, a widely developed approach (see [2], [5]) is to use optimization based methods to ensure better coverage of the parameter space and to identify worst case configurations in terms of uncertainties, flight conditions and perturbations. In this approach, the search domain is delimited by the identified worst case bounds of each parameter. Optimization methods were successfully applied in frequency and time domain (see [2]). However, in case of multiple parameters following a non uniform probabilistic law, this approach is pessimistic since the probability for all the parameters to be around their bounds at the same time is low (assuming independent parameters).

Another method for worst case identification and robust stability analysis in frequency domain is mu-analysis (see [3], [4] and [6]). It was successfully tested for robust stability of flexible launcher (see [7]). An interesting improvement to reduce worst case bounds conservatism was proposed in [8] where probabilistic gain analysis could complement the worst case analysis by providing a guaranteed bound of the cumulative probability distribution of the input/output gain as a function of the parameter uncertainty (and fault level). However, the computational burden of this approach is very heavy and it requires the use of LFR formalism which despite its elegant formulation could not replace the direct use of non linear simulations.

The aim of this paper is to propose and apply a novel method that includes a chance constraint of the domain of variation of the inputs during the optimization process for worst case identification. Optimization represents an effort reduction to identify worst cases. Furthermore, the chance constraint allows exploring a more probable domain thus reducing the pessimism of optimization based worst case search. This method's development and evaluation was done on benchmarks representative of next generation launchers.

This paper is organized as follows; the benchmarks are described in section 2, followed by an overview of SAFE-V validation framework in section 3. Then chance constrained optimization principle and mathematical definition are defined in section 4. Finally, results obtained for launcher applications are presented in section 5 justifying the method and demonstrating the interest of the developed method.

## 2. Launcher benchmark – models and objectives

All the benchmarks developed during SAFE-V project are described in ([9]). The launcher benchmark that was used for testing chance constrained optimization method deals with the robust performance analysis of a one axis LPV launcher model during atmospheric phase through time domain simulation. The major validation issue of this benchmark is linked to simultaneous impact of uncertainties, external perturbations and time varying aspects on loads requirements.

### 2.1 Model

The launcher model is composed of a rigid body dynamics (linearization around the reference trajectory), flexible modes (four modes are considered) modeled as an additive perturbation, a second order actuator model with saturation and bias, a controller and a measurement noise model. A wind model is also included in the benchmark.

The dynamic of the launcher rigid body movement is given by the following state-space representation

$$\left.\begin{array}{l}
\begin{bmatrix}\ddot{\theta}\\ \dot{\theta}\\ \dot{Z}p\end{bmatrix}=\begin{bmatrix}0 & A6(t) & A6(t)/V(t)\\ 1 & 0 & 0\\ 0 & A1(t) & -A6(t)\alpha 3(t)\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \theta\\ Zp\end{bmatrix}+\begin{bmatrix}K1(t) & -A6(t)/V(t)\\ 0 & 0\\ K2(t) & A6(t)\alpha 3(t)\end{bmatrix}\begin{bmatrix}\beta_R\\ W\end{bmatrix}\\
A1(t)=-\dfrac{Pc(t)+Ps(t)}{m(t)}+\dfrac{Q(t)Sref}{m(t)}(C_A-C_{N\alpha})\\
A6(t)=\dfrac{Q(t)SrefC_{N\alpha}L_F}{I(t)}\\
K1(t)=\dfrac{P_c(t)l_{tu}}{I(t)}\\
K2(t)=-\dfrac{P_c(t)}{m(t)}\\
\alpha 3(t)=\dfrac{I(t)L_F}{m(t)V(t)}
\end{array}\right\} \quad (1)$$

With the following notations :

| | |
|---|---|
| $\theta$ | : Attitude error |
| Zp | : Drift speed |
| $\beta_r$ | : Realized deflections (actuator output) around nominal deflection to follow reference trajectory. |
| W | : Crosswind speed (perpendicular to the trajectory part) |
| $C_A$ | : Axial aerodynamic force coefficient expressed in body frame |
| $C_{N\alpha}$ | : Normal aerodynamic force coefficients derivative wrt angle of attack expressed in body frame |
| $I$ | : Launcher inertia expressed in body frame |
| $L_F$ | : Distance between CoG and Centre of Pressure (on longitudinal axis) |
| $L_{tu}$ | : Position of nozzle CoG wrt nozzle rotation point |
| m | : Launcher total mass |
| $Q$ | : Dynamic pressure |
| $S_{ref}$ | : Reference area. |

$V$ : Absolute velocity
Pc : Controlled thrust
Ps : Uncontrolled thrust

The wind perturbation is modeled by a synthetic crosswind according to the model described by NASA in [10]. For simulation a synthetic wind is built from a wind envelope, wind shear and gust. Various wind profile can be generated by changing the altitude at which the wind gust occurs. Therefore either a fixed or random wind profile can be used. For the application presented here, a fixed wind profile with a wind gust at 50s was used.

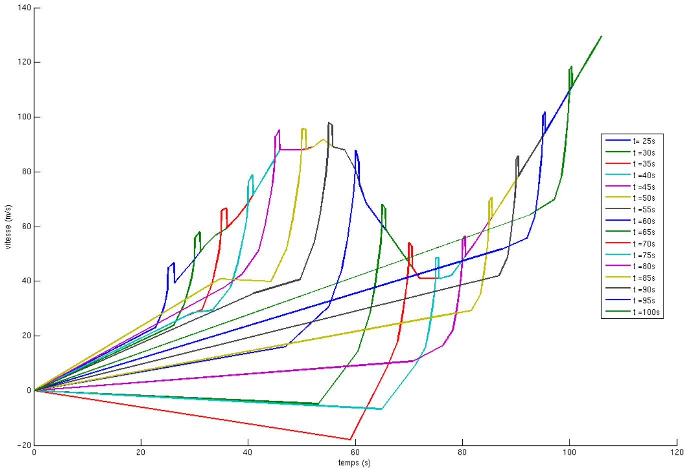


Figure 1 : examples of wind profiles

### 2.2 Uncertainties

For robust performance analysis, variations of the main parameters shall be taken into account. The parameters that can change are the following:

**Rigid parameters:**

- Inertia: MI
- Thrust: MP
- Aerodynamic coefficient Cz: MCz
- Aerodynamic focus position Xf: MXf
- Dynamic pressure: MQ
- Centre of gravity Xg: MXg
- Mass: MM
- Deflection Misalignment : $\Delta\beta$

**Bending modes**:

- Pulsation: Mpuls
- Deformation at nozzle point: Mhtu
- Slide of deformation at nozzle point: Mhptu
- Deformation at IMU location: MhpIMU
- Slide of deformation at gyrometer location: MhpIMU

The variations of these parameters are composed of two contributions (that will be more detailed in section 4):

- An “uncertainty” value that defines a range of variation with uniform probability
- A “dispersion” value that is the standard deviation of the gaussian probability function associated with the parameter.

The value of the parameter is then obtained by drawing two random values associated to the uncertainty ($X_U$) and the dispersions ($X_D$) and then combining them with the nominal value $X_0$ either:

- in a multiplicative way, in that case the uncertainty is expressed as a percentage of the nominal value

$$X=X_0*(1+X_U)*(1+X_D) \quad \textbf{(2)}$$

- in an additive way, in that case the uncertainty is expressed as an absolute value

$$X = X_0 + X_U + X_D \tag{3}$$

The dispersions on bending mode are set to 0. Only the centre of gravity position and aerodynamic focus position are modeled with additive uncertainty. The numerical values are gathered in the following table.

Table 1: Uncertain characteristics

| Parameters | MI | MP | MCz | MXf | MQ | MXg | MM | Δβ | Mpuls | Mhtu | Mhptu | MhpIMU | MhpGY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Uncertainty | 10% | 3% | 20% | 1.79m | 20% | 0.3m | 5% | 1° | 20% | 30% | 30% | 30% | 30% |
| Dispersion | 3% | 1% | 10% | 0.2m | 4% | 0.05m | Ø | Ø | Ø | Ø | Ø | Ø | Ø |

In total, there are 28 uncertain parameters (the four bending modes have different uncertain values) including 6 parameters with dispersion.

## 2.3 Objectives

The performance of the controller is evaluated with the following criteria:

**Guidance tracking constraints**
At the end of the simulation, the control function shall be able to control the attitude with the following accuracy (true attitude wrt attitude commands from Guidance).

- Attitude ≤ 2° (3.10-3 with confidence level 95%)
- Attitude rate ≤ 0.8°. (3.10-3 with confidence level 95%)

**Disturbance rejection**

- Wind rejection: the control function shall keep the induced aerodynamic angle of attack compatible with general load specification: Qα < 500 kPadeg. (3.10-3 with confidence level 95%)
- Thrusters' misalignment impact on load shall not exceed 10% of the specification (for information, not analyzed within this study).
- Noise filtering: cumulated thruster deflection shall remain lower than 200° during the 100s of the flight phase (10-5 with confidence level 95%).

# 3. SAFE-V framework

This work is part of a more global project dealing with enhancement of launcher GNC validation process. In this section we remind the context of enhanced SAFE-V framework (Fig.2) and place chance constrained optimization in the context of the overall process. The goal of SAFE-V project was to propose an enhanced validation framework for launcher GNC, covering:

- *metrics* for better definition of design and validation objectives. Among the wide range of metrics, we have for instance: stability margins, alpha stability for risk analysis, RPE and Lyapunov exponent for performance and transient analysis and finally Sobol indices for sensitivity analysis.
- *specifications,* gathering all traditional stability and performance requirements (deterministic or stochastic depending on high level requirement) with the addition of enhanced stability margins based on real functional needs that allow giving the priority to performance requirement.
- *Enhanced Toolbox* that was developed during first part of SAFE-V activities. It contains in particular WCAT ([12], [13], [14] and [15]) based on either global, local or hybrid optimization algorithms to perform worst case analysis. The key points of these methods are the selection of the cost functions and the parameters envelops of variation. WCAT includes both mono and multi-objective methods.
- *control design* with multi-objective optimization of existing controller, leading to both optimization of control performance and cost reduction of iterative control design ([9]).
- *validation plan* which improvements cover general stability analysis process depending on model characteristics and on specifications, worst case analysis with chance constraints, and enhanced sensitivity analysis tools.

This enhanced validation plan contains both frequency domain validation part and time domain validation part. For time domain validation we define various types of test with various objectives:

- Nominal simulation for reference purpose
- Worst case simulations for predefined worst cases and also, worst cases defined by optimization based methods to ensure better coverage of the parameter space and identify worst case configurations in terms of uncertainties, flight conditions and perturbations. Optimization outputs could be used directly for validation (worst case requirements) or as warnings (stochastic requirements). For stochastic requirement, optimization could be used with chance constraints as described in section 4.
- Limit test simulations using predefined directions or optimization methods.
- Failure case simulations
- Monte Carlo simulations

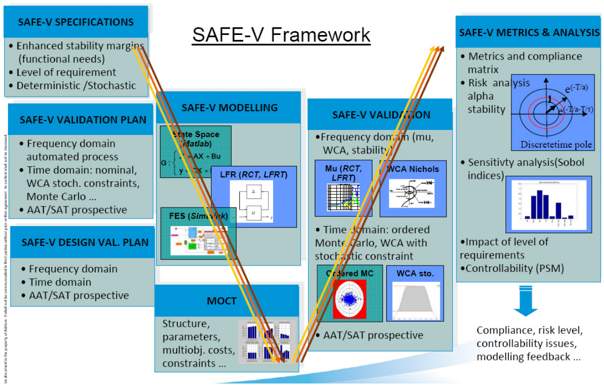


Figure 2 : SAFE-V Framework

## 4. Chance constrained optimization

In this section, we give an overview of uncertainty modeling and worst case definition used within SAFE-V project, and then we present the principle of worst case approach and details on chance constrained optimization.

### 4.1 Uncertainty, dispersion and worst case definition

We describe here the definitions of uncertainties and dispersion which are currently used for European launchers.

**Uncertainties (unknown type)** correspond to the variation domain with the current knowledge of the system (not known in advance). It corresponds to conception uncertainties. In theory uncertainty value on the parameters should reduce during development phases with system knowledge improvement and even be cancelled after qualification flights and post flights analysis. However, sometimes, this value could not be reduced since we were not able to improve the knowledge of the parameter (for instance, no measurement available). Uncertainty is a bias on the data and it is represented by a uniform distribution since every value in the domain is equiprobable.

**Dispersions (alea type)** correspond to non deterministic phenomena**;** they can vary from one flight to another. They are usually defined by Gaussian densities (even if other distribution law can be used).

For Worst case approach (predefined worst cases or optimization based approach), the following principle shall be applied to obtain a worst case envelop on the parameters (see [1]):

- Define worst case bounds on each parameter by $X_{WC}=+/-(X_U+N*X_D)$ (additive uncertainty for Xg and Xf) or $X_{WC}=(1+/-X_U)*(1+/-N*X_D)-1$ (multiplicative uncertainty for the other parameters); N is the number of sigma associated to the quantile corresponding to the probability objective. This is illustrated on Figure 3.
- Then consider the domain D defined by these bounds for the uncertain parameters, gathered in the vector U. It could be used directly to define specific worst case or inside a worst case optimization process.
- Finally, if necessary probabilistic aspects may be taken into account using chance constraint as proposed in next section.

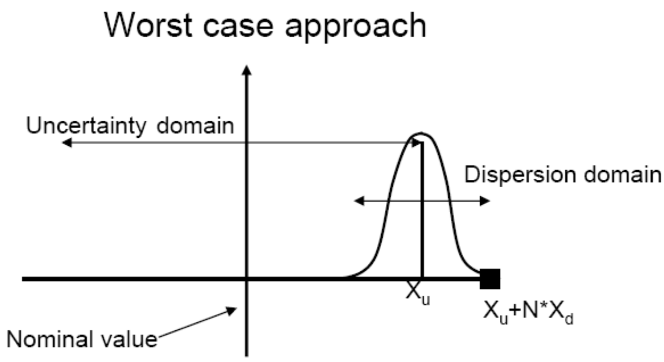


Figure 3 : Worst case value definition

For instance, N= 2.81 sigma is considered to reach 99.7% probability and 95% confidence level with bilateral hypothesis (this value is obtained with the inverse of the cumulative density function of the normal law, see [17]).

### 4.2 Worst case simulations

In traditional validation plan, worst case simulations are run for predefined worst cases (based on mission analysis, stability analysis and/or engineering experience). These worst cases are selected on the bounds of the envelops defined in previous subsection. In *SAFE-V* validation plan, optimization based method is used to ensure better coverage of the parameter space and identify worst case configurations that could be non intuitive.

The optimization-based approach reformulates the problem as an equivalent norm maximization problem, for which various optimization algorithms may be chosen to determine the solutions [11]. Optimization based methods may be used both for control design optimization and for robust stability and performance analysis.

Optimization-based worst-case analysis aims to find the combination of real parametric uncertainties that gives the worst-case values of a defined analysis criterion. Note that this means that not only parametric uncertainty, but also fault and failure scenarios can be included in the analysis, as long as the effect of such faults and failures can be appropriately parameterized in the simulation model.

These methods can handle any analysis criteria that can be expressed in a mathematical form and can be used with any kind of simulation model, no matter how complex they are. However, the reliability and efficiency of this approach strongly depends on a number of factors, including appropriate choice of cost-function, optimization algorithm, and tuning parameters for a given algorithm. In particular, since the parametric search space encountered in complex applications is typically nonlinear and non-convex, the use of global nonlinear optimization algorithms is necessary to ensure that the true worst-case has been found.

Some of these algorithms (local optimization) use the gradient information of the cost function to find the search direction while determining the optimum. Other algorithms (global optimization) use only the cost function value. Many of the non-gradient-based search and optimization techniques make use of heuristic search directions, in an efficient and intelligent way. The search space, or design space, can be convex or non-convex. Depending on this, the optimization algorithms provide a global or a local optimum. Obviously, if the search space is convex, in principle both local and global optimization algorithms will converge to the true global solution. In the case of a non-convex search space, gradient-based optimization algorithms provide a local solution, rather than the true global solution. However, as shown in [12] and [13], both the reliability and efficiency in finding the worst-case solution of global optimization methods can be significantly improved by augmenting the global optimization with local gradient-based optimization algorithms using a deterministic or probabilistic hybrid switching scheme.

The algorithm that we used includes such a switching technique. It is called Hybrid Differential Evolution (HDE) and is based on the Differential Evolution (DE) algorithm and is described in [12] and [16]. The DE algorithm is an evolutionary algorithm: at each step, a population of elements in the parameter space is generated from the previous population through either mutation (change of one element) or cross-over between two elements. The new population is then evaluated using the cost function and only the best elements are kept. To this DE algorithm was added a local algorithm (gradient based SQP local optimization scheme, 'fmincon' function in matlab) in order to ensure final convergence. The HDE algorithm includes also a switch mechanism from global to local algorithm for intermediate improvement ([12]).

### 4.3 Worst case with chance constraints

One key features of the worst case search are the bounds of the uncertain parameter space. Indeed, if parameters uncertainties are modeled by non uniform probabilistic laws, these bounds are not straightforward. In particular, if there are multiple parameters, a point in the parameter space for which all the parameters are far from their nominal value is less likely than a point for which only one parameter is far its nominal value(assuming independent parameters). This is due to the multiplication of the probabilistic density functions.

Therefore, for SAFE-V validation we propose using a more realistic method that includes during the optimization process a chance constraint on the domain of variation of the inputs. This method is described hereafter followed by the definition of the worst case process.

In worst case approach with chance constraints, the extreme value of the cost function is searched inside a reasonable domain of variation $D_U$ included in the domain D of the input variables vector U, defined in 4.1.
This domain definition takes into account the requested probability level for the cost function ($\eta$) and information about the probability density function of the inputs (pdf : $p_U$). A reasonable domain of variation $D_U$ is defined by $P(U \in D_U) > \eta$ where $\eta$ corresponds to the level of requirement of the cost function (e.g. maximal loads with $\eta$=99.7% probability level for our benchmark).
In practice, we build this domain by keeping only the points that have a high likelihood: the global constraint of the domain $D_U$ is translated into a local constraint on a neighborhood: the probability density function. The neighborhoods that are too improbable are rejected, meaning that the points in space that have a pdf below a given threshold $\delta$ are rejected (see figure 4). This method yields the smallest domain with respect to the Lebesgues measure that has the probability $\eta$. The domain is bounded by an iso-density curve and defined as follows:

$$D_U = D_U(\delta) = \{u \in \Re^n / p_U(u) \geq \delta\} \quad \textbf{(5)}$$

The value of $\delta$ is related to the probability objective $\eta$ by the following relation:

$$E_U\left[1_{U \in D_U(\delta)}\right] = \eta \quad \textbf{(6)}$$

This relation allows finding d either analytically (for instance if the pdf is a Gaussian law) or by a numerical integration using Monte Carlo simulation that has a very low computing cost and an precision higher than other sources of digital noise.

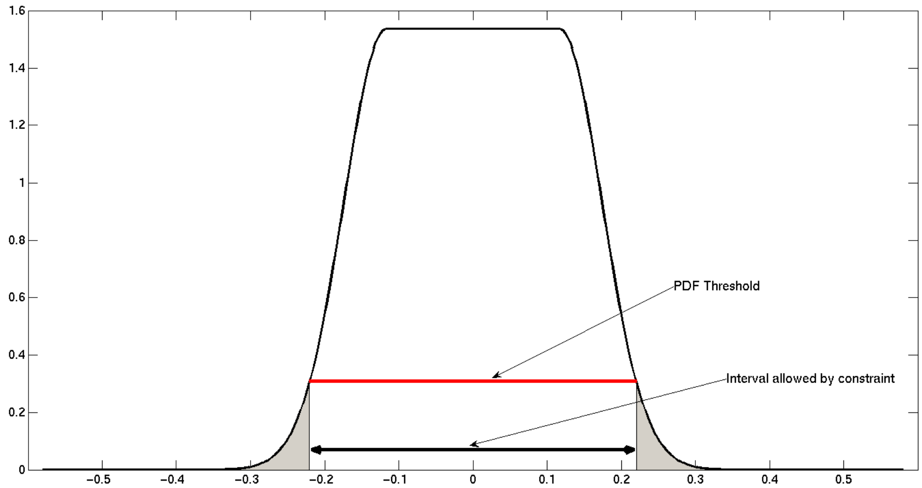


Figure 4 : Example of iso-density bounded domain

The following steps are used to evaluate the iso-density bounded domain and test whether an element pertains to this domain or not:

- Each uncertain parameter Ui is characterized by a marginal probability density function $p_{Ui}$ convolution of uniform pdf and normal pdf for additive uncertainty and logarithm convolution for multiplicative uncertainty)
- All the inputs are supposed independent, and the global pdf is the product of the marginal pdf
- The value of $\delta$ is computed according to the desired probability level
- Using this definition each element of the population is tested during optimization by computing its pdf value. If the element is inside the domain it will be kept, otherwise it will be rejected.

## 5. Results

### 5.1 Explored parameter space

To have a better idea of the parameter space coverage allowed by the chance constraint we compared aerodynamic efficiency and control efficiency (respectively A6 and K1, see (1)) values obtained from a Monte Carlo draw of the rigid parameters and values obtain with rigid parameters combinations that fulfill the constraint. These two coefficients were analyzed since they depend on most of the rigid uncertain parameters and are the most important of the launcher model.
Figure 5 shows the results obtained with a Monte Carlo drawing of 1000 values that are compared to points that are kept or rejected according to the chance constraint ($10^6$ points generated uniformly over the worst case domain were tested with the threshold set for a probability objective consistent with the 1000 simulations). Figure 5 shows in green the points that are kept and in red the rejected points. We can see that points far from the “Monte Carlo cloud” (in blue) are rejected and that the domains occupied by points generated by Monte Carlo and points that respect the chance constraints match pretty well. The A6/K1 ratio extreme values obtained are also similar.
Based on A6 and K1 values, the chance constraint seems promising in reducing the explored domain by optimization to the most probable case and thus reducing the pessimism.

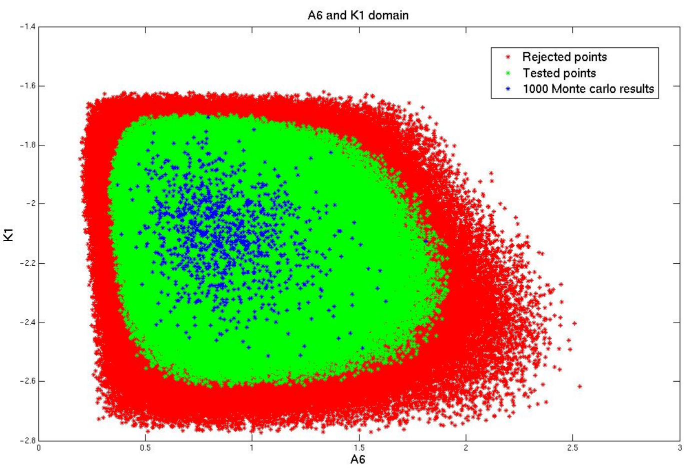


Figure 5 : Comparison of A6 and K1 values obtained by rejection of low pdf points and by Monte Carlo.

### 5.2 Optimization results and analysis

After constraint calibration (determination of $\delta$), the constrained optimization was run. The results of the following methods were compared:

1 Chance constrained optimization calibrated for 99.7% success probability (without confidence level consideration)
2 Unconstrained optimization over the worst case domain (maximal uncertainty and dispersion at 2.8 sigma value)
3 Monte Carlo with 1000 simulations (998 simulations necessary for counting method with 99.7% success probability and 95% confidence level).

All the results were obtained for a fixed wind profile (instant for generation: 50s) and all optimization were run with the same parameters.

**Worst cases values**

Table 2 compares the results for all specifications. First, one can notice that the unconstrained optimization found divergence cases that are not seen by the constrained one. We have checked that the worst cases found by optimization without constraint did not fulfill the probability constraint, which means that the probability to be in such cases is lower than 99.7%. Furthermore, we have launched a Monte Carlo process of 10000 simulations and observed a similar divergence case. There was only one occurrence and therefore it confirms that this case is highly improbable and should not be taken into account considering the given probability objective. Thus, optimization with worst case bounds gives worst case results which are too pessimistic with respect to the requirements. Then one may see that, as expected, chance constrained optimization is less pessimistic than optimization without constraint and constrained optimization results are close to Monte Carlo's ones with a little less simulations.
However some results are lower than Monte Carlo ones which should be due to the fact that final convergence of optimization algorithm is not yet obtained. This could be improved by removing the flexible modes parameters, which have a low influence on the outputs, or by increasing the number of iterations. The results are not presented here.

Table 2: Comparison of the results of the three methods

| | **1000 Monte Carlo** | **Optimization on domain D** | **Chance constrained optimization** | | |
|---|---|---|---|---|---|
| | | | worst value | gap wrt Monte Carlo (%) | gap wrt optimization (%) |
| **Qalpha** (kPa°) spec : 500 | **531** | **569** | **541** | **+1.8** | **-3.1** |
| **θ max** (°) | **4.7** | **divergence** | **4.9** | **+4.3** | **-** |
| **β max** (°) spec :6° | 6.5 | 6.5 | 6.5 | 0 | 0 |
| **θ final** (°) spec : 2° | **0.14** | **divergence** | **0.17** | 21.4 | - |
| **Consumption** spec : 200° | 134 | 135 | 130 | -3.0 | -3.7 |
| **θ rate final** (°/s) spec : 0.8° | 0.46 | 0.45 | 0.43 | -13.0 | -11.1 |

**Worst cases directions**

Another goal of the optimization process is to find worst case direction. Here, the directions found with all three processes are presented.

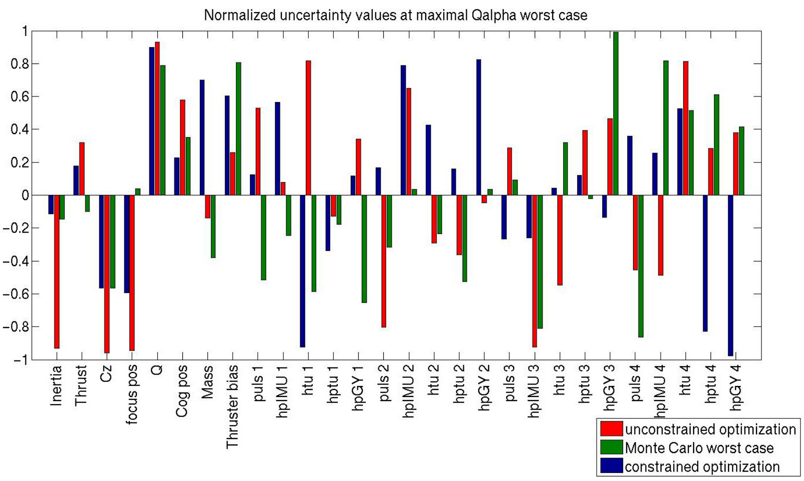


Figure 6 : Worst case direction for Qalpha with Monte Carlo, Optimization and chance constrained optimization

Figure 6 shows uncertainties values corresponding to worst case for Qalpha maximal value. It shows that the various methods do not give the same directions. This can be explained by the low influence of some parameters, e.g. flexible modes parameters that have a low effect on rigid behavior. If we consider that the most influent parameters

are the ones for which all methods give the same direction (rigid mode), these results are consistent with sensitivity analysis using Sobol indices (other results from the SAFE-V study that are not presented here) that found Q, Cz and thrusters' bias as the most influent (in this order).
Figure 7 gives the worst cases directions for maximal theta value. This time, all rigid parameters (except CoG position) go in the same direction with most of the time a higher value for unconstrained optimization.
As a conclusion, if we consider only the most influent parameters, all methods give similar worst cases directions.

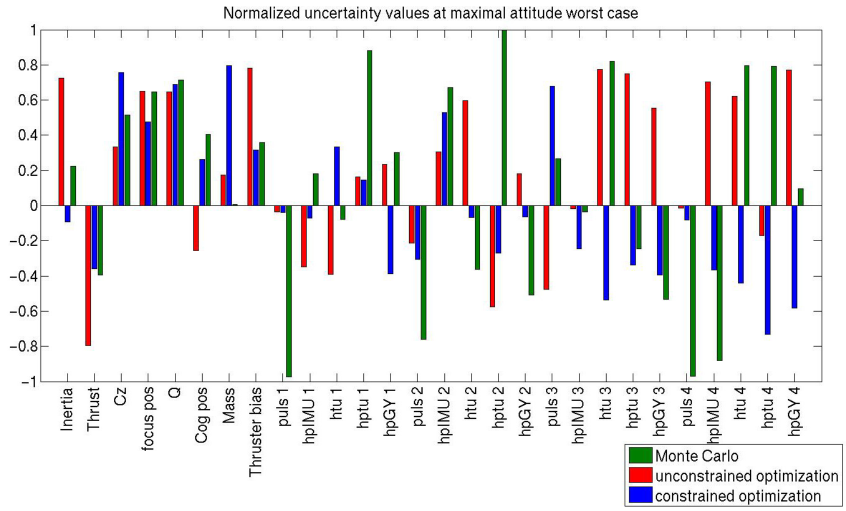


Figure 7 : Worst case direction for Maximal Attitude with Monte Carlo, Optimization and chance constrained optimization

*REMARK - The values on the figures are normalized, thus a value of 1 corresponds in the simulator to a value at the bound of the worst case domain: Xu+ 2.81*Xdisp.*

## 6. Conclusion

SAFE-V project ensures closing the loop of validation process from specification to design and validation with cost reduction, effort reduction (automation), confidence improvement and conservatism reduction.
In this paper, the constrained optimization process was presented and we have shown that it was successfully validated on launcher benchmark and that it gives results consistent with Monte Carlo and Sobol indices evaluation.
Optimization based process represent an effort reduction to identify worst cases and the chance constraint allows exploring a domain consistent with the probability objective for the parameters and therefore reducing the pessimism..
A remaining open point is the selection of the size of the population and of the number of iteration that are necessary to obtain accurate results. For the size of the population a heuristic is to select a population larger than the number of varying parameters. For the number of iteration a large number was recommended by WCATT designers (about 500) which could be prohibitive for engineering applications and leads to higher number of simulations than Monte-Carlo. Thus we recommend an application dependent selection with plotting of the cost function along the optimization process.
Optimization computation time remains significant and another remark that could be mentioned for modeling is that multiplicative uncertainties lead to numerical integration of pdf while uncertainties modeling with use of additive uncertainty (or use of other mixed mode) leads to analytical integration of pdf that is faster.
Further work should focus on the relation between the inputs probability density function that is used here and the output probability density function that is the real objective. Another axis of improvement is the introduction of correlation between parameters.

**References**

[1] Rongier I., G. Droz. (1999) Robustness of the Ariane 5 guidance navigation and control. Proceedings of the 4th ESA International Conference on Spacecraft Guidance, Navigation and Control Systems, Noordwijk, 1999.
[2] P. Menon et al, Nonlinear Worst-case analysis of an LPV controller for approach-phase of a Re-entry vehicle, AIAA-2009-5638 AIAA Guidance, navigation and control conference, Chicago, Illinois, Aug 10-13, 2009

[3] G. Balas, R. Chiang, A. Packard, M. Safonov, (2011), “Robust Control Toolbox User’s Guide”, 2011.
[4] Ferreres G., Biannic J.M. “Skew Mu Toolbox (SMT): a Presentation”, 2003, http://www.cert.fr/dcsd/idco/perso/Biannic/toolboxes.html
[5] A. Marcos, H. Garcia de Marina, V. Mantini, C. Roux; S. Bennani, “Optimization based worst-case analysis of a launcher during the atmospheric ascent phase”, to be published in AIAA GNC conference
[6] A. Marcos, V. Mantini, C. Roux, S. Bennani, “Bridging the gap between linear and non-linear worst-case analysis: an application case to the atmospheric phase of the VEGA launcher”, to be published in IFAC conference on Automatic and Control in Aerospace 2013
[7] Ganet-Schoeller M. and M. Ducamp (2010), LPV Control For Flexible Launcher, *AIAA-2010-8193.*
[8] G. J. Balas, P. Seiler, A. K. Packard, Analysis of an UAV Flight Control System using Probabilistic μ, AIAA-2012-4989, AIAA Guidance, navigation and control conference, Minneapolis, Minnesota, Aug 13-16, 2012
[9] M. Ganet-Schoeller et al., “SAFE-V launcher validation framework and controller optimization”, to be published in IFAC conference on Automatic and Control in Aerospace 2013
[10] NASA Technical Memorandum 4511, Terrestrial environment (Climatic) – Criteria guidelines for use in Aerospace Vehicle, 1993 Revision
[11] C. Fielding, A. Varga, S. Bennani and M. Selier (Eds) Advanced Techniques for Clearance of Flight Control Laws, Springer , 2002
[12] P.P. Menon, J. Kim, D.G. Bates, and I. Postelthwaite, “Clearance of nonlinear flight control laws using hybrid evolutionary optimization”, IEEE Transaction on Evolutionary Computation, Vol 10, No 6, 2006, pp. 689-699
[13] P.P. Menon, D.G. Bates, and I. Postelthwaite, “Nonlinear Robustness analysis of flight control laws for highly augmented aircraft, “IFAC Control Engineering practice, Vol 15, pp. 655-662, 2007.
[14] A. Kamath, P.P. Menon, M. Ganet, G. Maurice, S. Bennani, and D. Bates, “Worst case analysis of a Launcher Vehicle Using Surrogate Models”, in Proc of IFAC ROCOND, 2012
[15] A. Kamath, P.P. Menon, M. Ganet, G. Maurice, S. Bennani, and D. Bates, "Robust safety margin assessment and constrained worst-case analysis of a launcher vehicle," In Proc. of IFAC ROCOND, 2012
[16] P. P. Menon, D. G. Bates, I. Postelthwaite, A. Marcos, V. Fernandez and S. Bennani, “ Worst-Case Analysis of Flight Control Laws for Re-Entry Vehicles”, in Proceedings of the IFAC Symposium on Automatic Control in Aerospace, Toulouse, July, 2007
[17] G. Saporta, Probabilites analyse de donnees et statistique, 2eme edition, Edition Technip, 2006