Design method of channel blades
(Spline for blade grids design)

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Abstract

Methods of designing blade grids of power machines, such as equal thickness shape built on middleline arc, or methods based on target stress spreading were invented long time ago, well described and still in use. Science and technology has moved far from that time and laboriousness of experimental research, which was involving unique equipment, requires development of new robust and flexible methods of design, which will determine the optimal geometry of flow passage.

This investigation demonstrates simple and universal decision in methods of creating blades, which adequately responds to the initial data, and amount of that data is significantly reduced.

Introduction

Since conversion of kinetic energy to potential energy happens mainly on the diffuser and the wheel of compressor stage, less attention has been paid to designing of return channel blades. However, it has been established, that by low-consuming compressor stages energy conversion is being performed on every element including return channel blades. Their importance can also be judged from the increasing amount of recent works [1,2] dealing with influence of the meridional section geometry on the blade efficiency. In this work the detailed step-by-step designing of the return channel blades has been described.

General explanations

By low volume flow rate compressor stages the b_5/b_4 ratio can reach rather big values (up to 10). In consequence of this, the meridional width of the return channel increases and the value of α_5 angle decreases to 15-20 degrees (in Russian notation angles are defined between vector of flow and tangent to a circle), and by ultra-high-pressure compressor stages it can decrease even more in low volume flow rate mode. Since the gas flow has to have no twist on the next stage entrance (that is, the velocity component, also referred as tangential projection of flow velocity, $C_u=0$), the return channel blade must be able to turn the gas flow almost 90 degrees.
To decrease surface friction, it is recommended to design channel segment with a square section which would minimize the surface of the channel.

The blade grid is an important part of the low volume flow rate compressor stage. It has to be designed in such a way, that the flow without separation would be possible on any compressor operating mode. To accomplish that, the blade width has to be increased (this kind of blade is called body blade). Body blades are currently used by such industrial compressor manufacturers as “Dresser Rand” and “Nuovo Pignone”.

One aim of the present work was to reduce the number of control parameters without sacrificing the accuracy of the blade geometry under given conditions. As a result of the current work, the number of parameters has been reduced to 12 (s. appendix A).

Due to the fact, that a return channel blade grid has complex geometry, it would be unpractical to set a middle line of the blade like it is commonly done for blades with uniform width. In this case, a concave side of the blade, i.e. its pressure side, was chosen as a base line for blade design. A cubic spline-function was used to design the convex side of the blade because of its controllability – since the cubic spline-function is constructed of piecewise third-order polynomials, it is a twice differentiable function, which allows one to control its behavior on a given interval effectively.

The convex side of the blade consists of two adjoining areas:

Segment I - entrance region (*from leading edge to the section of the throat, i.e., to the beginning of the channel segment of the blade grid*)
Segment II – channel region *(from the section of the throat to the exit of the blade grid)*

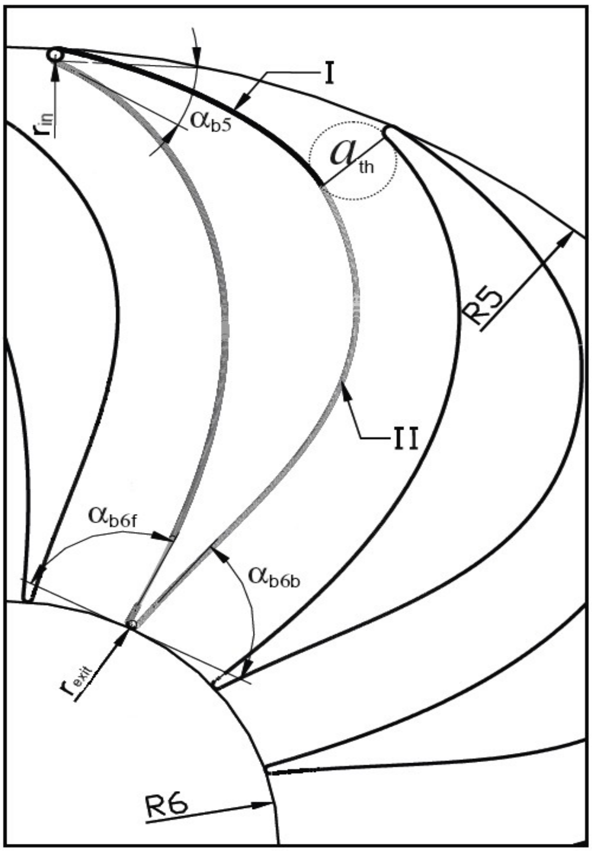

Figure. 1 - Description of blade grid parameters.

Both segments I and II have equal first derivative values at the joining point, that is, the angle of the end of the entrance segment I (beveled cut area) is equal to the angle of the beginning of the segment II (channel part of the convex side of the blade), which allows to get a smooth, kink-free curve of the convex side of the blade.

Due to the rotational symmetry of the centrifugal compressors, spline-function equations were transformed into cylindrical coordinates. The article also contains mathematical description of blade geometry in the Cartesian coordinates, which allows to design axial-flow compressors and turbines as well.

Concave side of the blade

In order to design a blade grid, one can use flow parameters on inlet of blades, which can be obtained either experimentally or numerically. In his work [3], Carl Pfleiderer proposed a following method for designing of the blade middle line for radial flow compressors, which can be represented as:

$$\frac{d\varphi}{dr} = \frac{1}{r\, tg(\alpha)} \tag{1}$$

The alpha-parameter in Pfleiderer's work is defined as a constant. In this case, the solution of the equation (3.1.2.) would be a logarithmic spiral, i.e., the path of the undisturbed flow of the ideal liquid without taking friction into account. Later, in [4] the following equation was proposed:

$$\frac{d\varphi}{dr} = \frac{1}{r\ tg(\cos(Ar^2 + Br + C))} \quad (2).$$

This equation was taken as the basis of calculations for designing of the concave side of the blade. As an argument, $a(r) = \cos(Ar_2 + Br + C)$ [4] leads to the deformation of the spiral described above. From the physical point of view, such a curve represents angular deflection of the flow from the undisturbed one according to the parabolic law.

$$\begin{gathered} \cos(\alpha_5) = Ar_5^2 + Br_5 + C \\ \cos(\alpha_6) = Ar_6^2 + Br_5 + C \\ D = 2\mathrm{Ar}_6 + B \end{gathered} \quad (3)$$

In order to find the coefficients in the equation (2), it is necessary to build an equation system (3) with boundary conditions and solve it for the said coefficients. As the boundary conditions, the geometrical parameters of the blade being designed are taken, such as r_5 , r_6 , $\cos(a_5)$ and $\cos(a_6)$.

Since there are only two boundary conditions for three required coefficients, it is sensible to add the third equation to the system, which would be the first derivative of the boundary condition for r6. That, in its turn, leads to the adding of the supplementary parameter *D*. This parameter defines the sign and the velocity of the tangential angle change along the length of the blade (Fig. 2).

If the parameter *D* is positive, the angular length of the blade increases; if negative, an inflection point appears on the concave side of the blade. Its exact position on the blade is also defined by this parameter – the smaller is the value of *D*, the closer the inflection point gets to leading edge. In case of *D=0* the inflection point sits exactly on the diameter D_6 .

Coefficients A, B and C can be found by solving the system of equations (3), the rest of the system members are boundary conditions for blade designing.

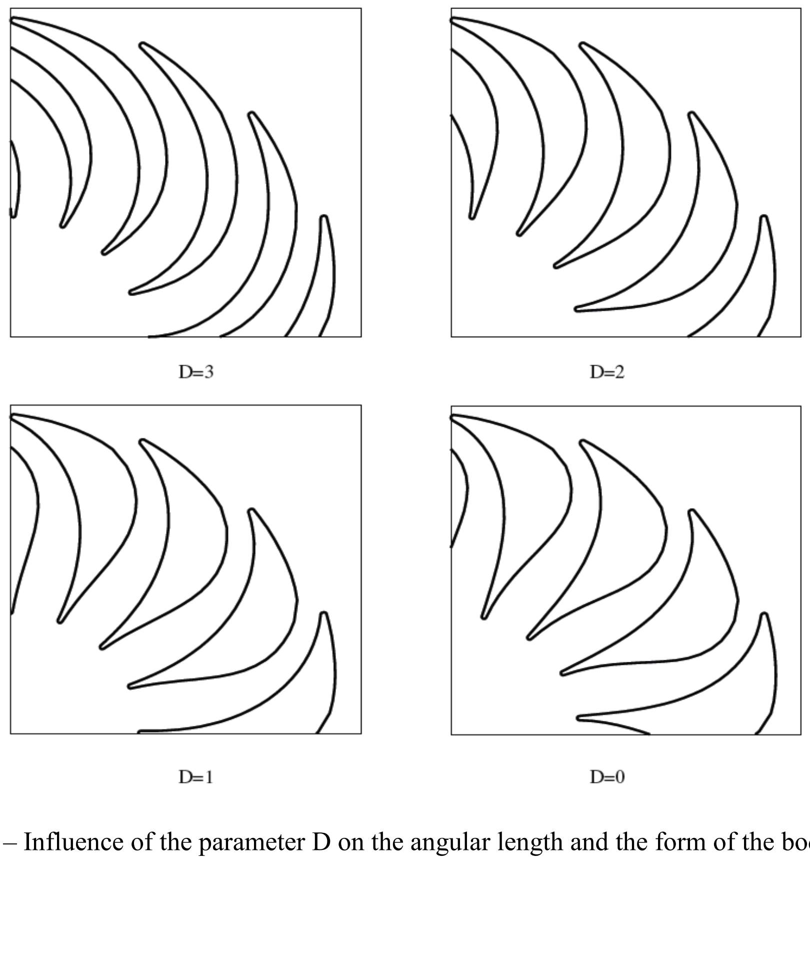

Figure. 2 – Influence of the parameter D on the angular length and the form of the body blades

Convex side of the blade

In order to design the convex side of the blade one can use spline-function in polar coordinates, defined by two third-order polynomials:

$$\varphi = Ar^3 + B\,r^2 + C\,r + D \tag{4}$$

Coefficients of the cubic equation (4) can be defined by the use of first-type boundary conditions [5]. As a first step, coordinates of both ends of the curve must be set, then the values of the first derivative of the desired function on the ends of the given interval. This leads to the smooth connection between the inlet and the channel segments of the spline. As a result, the equation system for defining coefficients A,B,C and D of the cubic equation (4) has following form:

$$\begin{aligned}
\varphi_0 &= Ar_0^3 + Br_0^2 + Cr_0 + D\\
\varphi_1 &= Ar_1^3 + Br_1^2 + Cr_1 + D\\
\frac{d\,\varphi_0}{dr_0} &= 3\,Ar_0^2 + 2\,Br_0 + C\\
\frac{d\,\varphi_1}{dr_1} &= 3\,Ar_1^2 + 2\,Br_1 + C
\end{aligned} \tag{5}$$

To define the position of the inflection point on the inlet segment of the convex side of the blade (Segment I on Fig. 1), it is necessary to add a new condition: the second derivative of the equation (4) with respect to r equals zero.

The following relation describes position of the inflection point:

$$\begin{aligned}
\frac{d\,\varphi}{dr} &= 6\,Ar + 2\,B\\
r &= \frac{B}{3\,A}
\end{aligned} \tag{6}$$

Best results were obtained, when the inflection point sits on the inlet radius of the blade grid *R5*. The channel segment of the blade (Segment I on the pic.13) agrees in coordinates and in values of the first derivatives with the inlet segment, described with spline-function.

Designing of axial blades

In order to design an axial blade (Fig. 3), it is recommended to set concave side of the blade as a base line, as in the case of radial blades, described above. Because of the drastic changes in tangential angle values along the blade length, it is necessary to use parametric spline-functions.

$$\begin{aligned}
y_1 &= A_1\,t_1^3 + B_1\,t_1^2 + C_1\,t_1 + D_1 & \frac{dy_1}{dt_1} &= 3\,A_1\,t_1^2 + 2\,B_1\,t_1 + C_1\\
y_2 &= A_1\,t_2^3 + B_1\,t_2^2 + C_1\,t_2 + D_1 & \frac{dy_2}{dt_2} &= 3\,A_1\,t_2^2 + 2\,B_1\,t_2 + C_1\\
x_1 &= A_1\,t_1^3 + B_1\,t_1^2 + C_1\,t_1 + D_1 & \frac{dx_1}{dt_1} &= 3\,A_1\,t_1^2 + 2\,B_1\,t_1 + C_1\\
x_2 &= A_1\,t_2^3 + B_1\,t_2^2 + C_1\,t_2 + D_1 & \frac{dx_2}{dt_2} &= 3\,A_1\,t_2^2 + 2\,B_1\,t_2 + C_1
\end{aligned}$$

where $t_2 = ((y_1 - y_2)^2 + (x_1 + x_2)^2)^{1/2}$

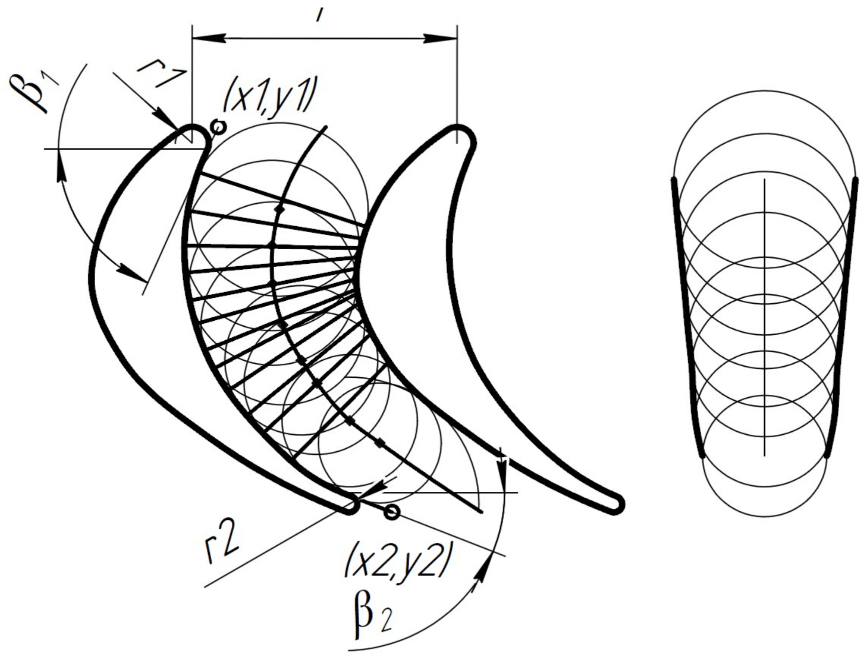

Figure. 3 - Example of turbine blade grid design.

Conclusion

As a result of the current studies, a designing method for return blade grid had been developed. This method allows to design blades with constant thickness as well as body blades and can be applied for both axial and radial kind of blades. Another significant advantage of this method is its simplicity, which, on the other hand, does not sacrifice the accuracy of the blade geometry and its physical adequacy.

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Appendix A

Table 1: Control parameters for blade greed design

Control parameters	
Inlet angle on concave side	α_{b5}
Exit angle on concave side	α_{b6}
Exit angle on convex side	α_{b6b}
Inlet radius	r_5
Exit radius	r_6
Inlet channel depth	b_5
Exit channel depth	b_6
Leading edge radius	$r_{in.}$
Exit edge radius	r_{out}
Number of blades	Z
Absolute value of throttle size	a_t
Parameter	D