A New Configuration Of Irregular Reflection Of Shock Waves

L.G. Gvozdeva and S.A. Gavrenkov
Joint Institute for High Temperatures of the Russian Academy of Sciences (JIHT RAS)
125412, Izhorskaya st. 13 Bd.2, Moscow, Russia

Abstract

A new configuration of shock waves has been found in the reflection of shock waves in a stationary supersonic gas flow in addition to the well-known regular and Mach reflections. This new three shock configuration occurs with a negative angle of reflection and Mach numbers greater than 3 and an adiabatic index smaller than 1.4.

It has been shown that this new configuration is unstable and leads to a radical change of the total flow pattern. The emergence of this new kind of instability can negatively affect operation of aircraft and rocket engines due to the failure of the flow to be as conventionally predicted.

1. Introduction

The reflection of shock waves is a fundamental element of external and internal aerodynamics [1,2]. In a stationary supersonic stream of gas two types of reflections are known: the two shock (regular reflection) and the three shock wave configurations (irregular or Mach reflection). One can see these for example at the entrance of an aircraft air inlet (Fig. 1 a, b) or at the over-expanded jet from a rocket engine nozzle (Fig. 1 c). In this work the three-shock configuration is considered; it consists of a incident wave IA, a reflected wave AR and a Mach wave AM. These waves intersect in a triple point A. The arrangement of waves is defined by the Mach number of flow M_1, the angle of incidence ω_1 and the effective adiabatic index of gas γ.

Typically, the predicted and observed flow pattern looks as in Figure 1 (b, c), i.e., the reflected wave AR is located above the line of incoming flow. This configuration we will call a configuration with a positive angle of reflection ($\omega_2 > 0$). Gvozdeva [3] has hypothesized that in a steady supersonic flow of gas at high Mach numbers and small values of the adiabatic index γ, the reflected wave must be located below the direction of inflow stream ($\omega_2 < 0$); this constitutes a new configuration with a negative angle of reflection (Fig. 2 a, b). This configuration would be unstable and its appearance would lead to a radical change in the entire flow pattern. For example, let's consider the case in Figure 2 (a). If the reflected wave, which is going at the negative reflection angle ω_2, were to intersect the line of symmetry OO, then the gas would start to accumulate in the closed area ARM. That would lead to choking of the flow and to the movement of the Mach wave upstream. So there would be a radical change in the entire flow pattern. A similar process would occur in the over-expanded jet if the angle of reflection becomes negative (Fig. 2, b). This idea was developed in [4-9] where preliminary analyses and testing of the calculation program have been presented.

In the present work two questions have been answered:

1) Under what initial and boundary conditions could the new configuration exist?
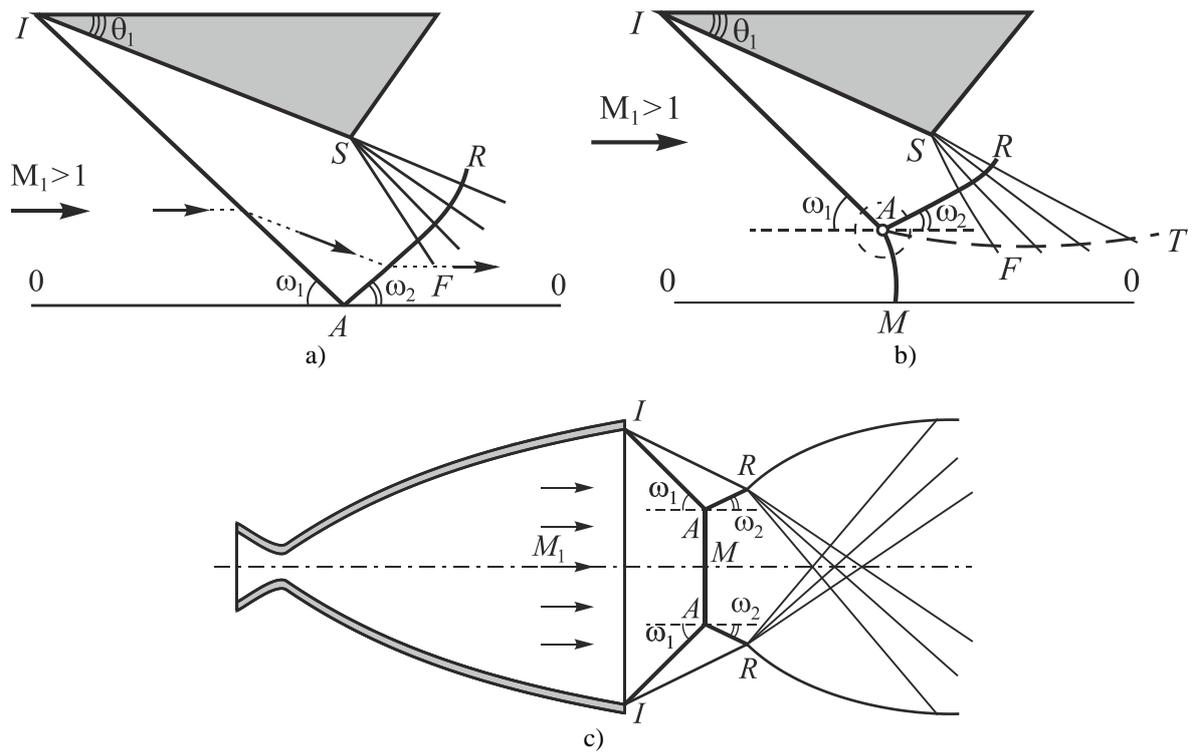2) What restructuring of the flow might result from the emergence of this new configuration?

Figure 1. Two-shock (a) and three-shock wave configuration (b) at the entrance of an air intake. Three shock wave configuration at the outlet of the nozzle (c).

IA, incident shock; AR, reflected shock; AT, slip stream; AM, Mach wave; SF, rarefaction fan; ω_1, incidence angle; ω_2, reflection angle; θ_1, angle of the wedge; A, triple point; 0-0, line of symmetry; M_1, Mach number of flow.
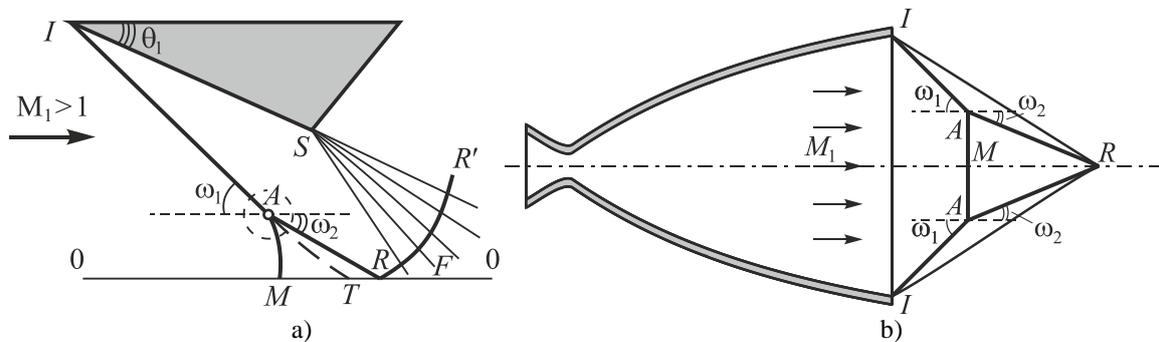

Figure 2. Three shock wave configuration with a negative reflection angle $\omega_2<0$ at the inlet of the air intake (a) and in over-expanded jet (b). The notation is the same as in Figure 1.

2. Analytical studies

2.1 Analytical calculations of the boundaries of the existence of the three shock configuration with a negative angle of reflection

To answer the first question, the analytical calculations have been done in accordance with the three shock wave theory [10,11].
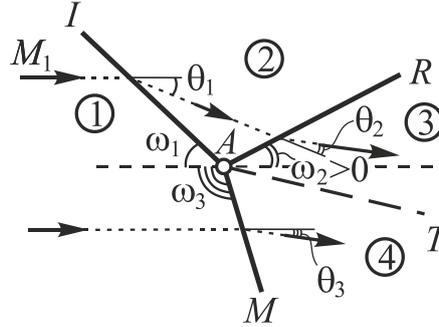

Figure 3. The location of shock waves in the area of triple point A

We consider the steady supersonic flow of gas with adiabatic index γ, moving with a Mach number M_1 (Fig. 3). The incident wave IA is located at an angle to the incident flow ω_1. The reflected wave AR is located at an angle ω_2, and a Mach wave of AM, located at an angle ω_3. It is assumed that there exists a certain neighborhood of the triple point where all three waves are straight. The three shock theory suggests that the gas flow behind the incident wave is deflected by an angle θ_1, the gas behind the reflected wave is deflected in the opposite direction at a smaller angle θ_2. In the Mach wave gas is deflected by an angle θ_3. The flow behind the incident and reflected wave must be parallel to the flow passing through the Mach wave, and pressures on the tangential surface AT must be the same. So the compatibility conditions can be written as follows: $\theta_3 = \theta_1 - \theta_2$; $p_3 = p_4$.

For each wave we can write the system of equations:

$$\rho_1 u_1 = \rho_2 u_2 \,,$$
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \,,$$
$$\frac{u_1^2}{2} + h_1 = \frac{u_2^2}{2} + h_2 \,,$$
$$p_2 \mu_2 = \rho_2 R T_2 \,,$$

$$h_2 = C_{p2} T \,,$$
$$C_{p2} = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{\mu_2} \,,$$
$$\mu_1 = \mu_2 = const \,,$$
$$\gamma = const \,,$$

where ρ - density; u - velocity component normal to the shock wave; p-pressure; h - enthalpy; μ - molecular weight; R - universal gas constant; T - temperature, C_p - specific heat at constant pressure; γ - ratio of specific heats. Indices 1, 2 correspond to the regions brfore and after the shock wave respectively.

and the relation of the angles in an oblique shock wave:

$$\tan(\theta) = \frac{2}{\tan(\omega)} \cdot \frac{M^2 \cdot \sin^2 \omega - 1}{M^2 \cdot (\gamma + \cos(2\omega)) + 2}$$

where ω — angle between the wave and the flow line, θ — the angle of deflection of the flow passing through the shock wave; M — Mach number of flow.

Solving this system of equations for a given M_1, γ, and ω_1 one can completely calculate the parameters of the flow behind the shock waves and determine the angles in the configuration. Calculations based on the three shock theory give good agreement with experimental data for the strong shock waves when the flow of gas behind the reflected wave AR is supersonic, because only in this case the waves are straight in the vicinity of the triple point [12, 13]. It should be noted that this theory determines only the angles and the parameters of the flow behind the shock wave

near the triple point, but does not determine the height of the Mach wave AM. Since the flow behind the Mach wave is subsonic, the height of it is determined by the geometry of the flow and conditions downstream.

The calculations have been performed in a wide range of initial parameters of Mach number M_1 from 3 to 20. The effective adiabatic index varied from 1.05 to 1.66, the initial angle ω_1 from 10 to 60 °. The results are presented as a function of the angle of reflection angle ω_2 on the angle of incidence ω_1 for different values of the Mach number M_1, and the adiabatic index γ.
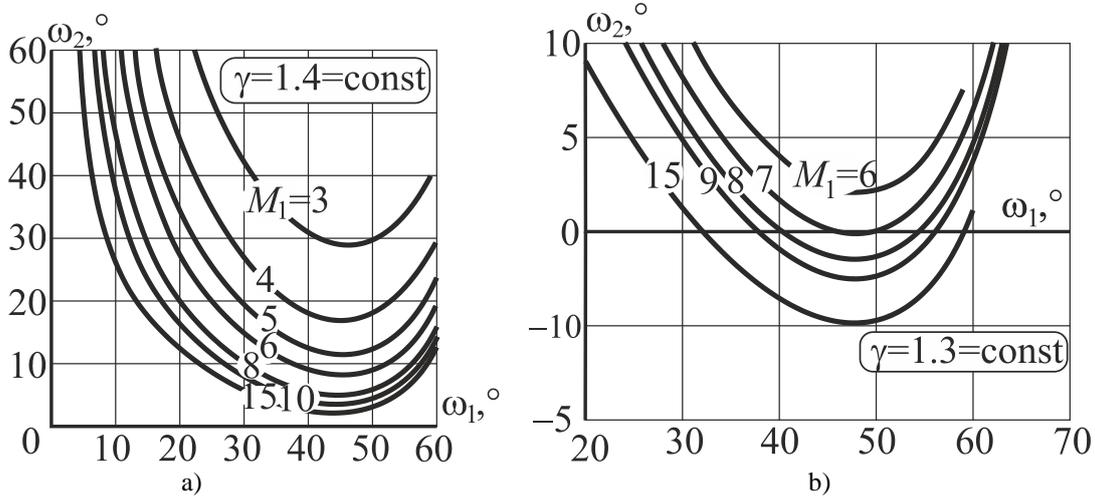

Figure 4. Dependence of the reflection angle ω_2 on the angle of incidence ω_1, for different Mach numbers M_1 at a constant value $\gamma = 1.4$ (a) and $\gamma = 1.3$ (b).

Figure 4 (a) shows that at constant adiabatic index $\gamma = 1.4$ the curves $\omega_2=f(\omega_1)$ lie only in the positive area for any values of the Mach number M_1. The same is obtained for larger values of the adiabatic index $\gamma > 1.4$. This explains the fact that all known three shock wave configurations are displayed only with a positive angle of reflection, because all experiments were conducted in wind tunnels in which the working gas is air with an adiabatic index $\gamma = 1.4$.

When reducing the adiabatic index, for example, down to $\gamma = 1.3$ (Fig. 4, b), the curves begin to shift down and part of the curves at high Mach numbers occur in the negative area. This is due to the fact that with decreasing γ compressibility of the gas increases and the deviation of the flow increases too. The deviation of the flow increases also with increasing Mach number.

The resulting graph of the boundaries of the existence of the configuration with a negative angle $\omega_2 < 0$ is shown in Figure 5 (a) in the coordinates of ω_1-M_1 for different values of the adiabatic index γ. Negative angle appears inside the curves.
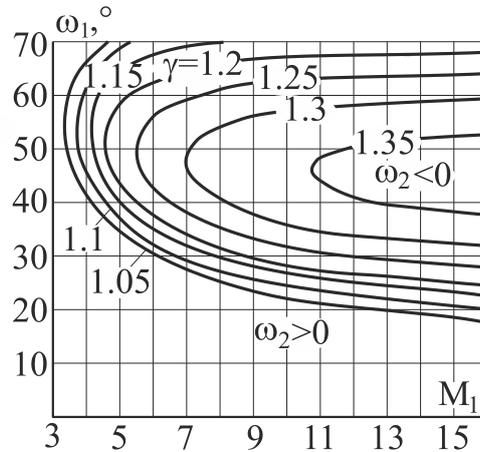

Figure 5. The boundaries of the existence of a three shock configuration with the negative angle of reflection in the plane (M_1, ω_1) in dependence on γ. The angle ω_2 becomes negative for values of the parameters that are inside the corresponding curves.

Thus, the boundaries and regions of existence of the new configuration have been found, in answer to the first question. The new configuration occurs when Mach number of flow M_1 is greater than 3 and the adiabatic index γ is less than 1.4. Thus a very large impact have been revealed of the reality of gas on the irregular reflection of shock waves. In this regard, it is interesting to determine the influence of a change in the adiabatic index on the boundary of transition from regular to Mach reflection, and to build a complete picture of the reflection of shock waves as a function of the adiabatic index.

2.2 The complete picture of the types of reflection as a function of the adiabatic index.

Location of shock waves depends on three parameters: the Mach number of free-stream M_1, incident angle ω_1 and the effective value of the adiabatic index γ. Figure 6 shows the well known transition boundaries of the regular and irregular reflection, depending on the Mach number M_1 and the angle of incidence ω_1 with the effective value of the adiabatic index equals to 1.4 [14]. We see that there is a region of double solution where it is possible to get a two-shock and three-shock configurations. This fact explains the so-called hysteresis phenomenon. It turned out that the type of configuration depends on the transition to this area. At a very slow transition from the regular reflection it is delayed and exists in this area up to the angle ω_D. If one returns back from Mach reflection, than Mach reflection is delayed and the transition to regular reflection occurs at the von Neumann angle ω_N. This conclusion follows from theoretical considerations and numerical experiments, but in physical experiment Mach reflection is more stable in the area of double solution than regular one. Hysteresis phenomenon was studied in numerous works, but only for a constant adiabatic index $\gamma = 1.4$ [14-16].

Let us find the dependence of boundaries between the regular and Mach reflection on the value of the adiabatic index, and describe the general picture of all forms of reflection including the new form of reflection (Fig. 7).
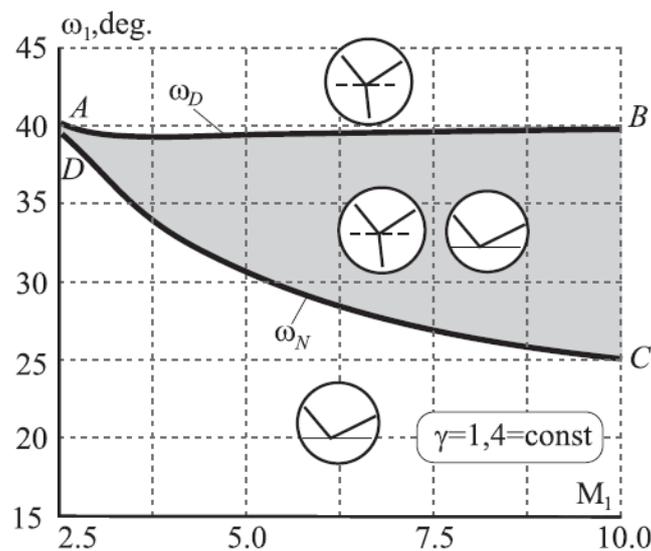

Figure 6. The curves of transition boundaries between Mach reflection and regular reflection ω_D and ω_N depending on the Mach number of the flow at the adiabatic index $\gamma = 1.4$. ABCD - the domain of dual solution, where both regular and irregular reflections are possible.
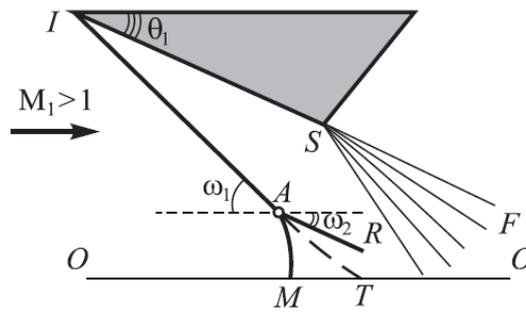
Figure 7. Three-shock configuration of shock waves with a negative angle of reflection $\omega_2 < 0$ (b).

Boundaries and area of regular and irregular reflection have been determined by shock polars method. [10] The calculation has been carried out for a range of Mach numbers from 1 to 15, the incidence angles from 0 to 90 deg. and the adiabatic index from 1.05 to 1.66. The transition angle ω_D for the regular reflection has been determined (at greater angle regular reflection can not exist) in dependence on the Mach number of free-stream M_1 and the adiabatic index γ. For irregular reflection critical angle ω_N has been determined, below which the irregular reflection can not exist. The region of dual solution lies between these two curves. It is known that the area of dual solutions for a constant adiabatic index $\gamma = 1.4$ increases rapidly with increasing Mach number [16]. It has been shown that the same relationship exists for the other values of the adiabatic index. The effect of the adiabatic index (at constant Mach number) on the value of the domain of dual solutions has been studied. The calculations show (Fig. 8), that with a decrease in the value of the adiabatic index the region of double solution increases. For example, for a constant $M_1 = 5$ and changes in γ from 1.4 to 1.05 the domain of double solution increases by 14 °.
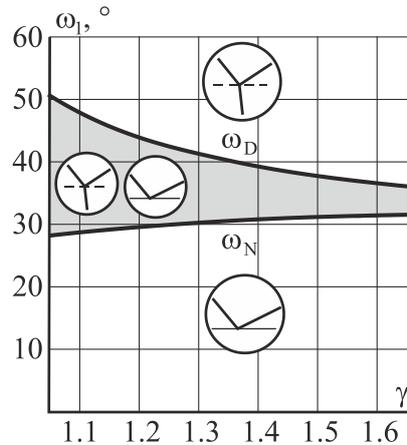

Figure. 8. The curves of critical angles ω_D and ω_N in dependence on an adiabatic index γ for flow Mach number $M_1 = 5$. In the area ABCD both regular and irregular reflection is possible.

2.3 New areas of dual solution.

Let's combine the boundaries of transition between regular reflection and Mach reflection with the boundary of the existence of the new configuration with a negative angle of reflection ω_G. Figure 9 shows the total picture of all types of reflection at the constant adiabatic index $\gamma = 1.28$. It includes all three boundaries of existence: of regular reflection (AB), of irregular (DC) and of the configuration with a negative angle of reflection (EGH). From this figure we can see that except of the region of double solution AGHCD, where it is possible as Mach and as regular reflection, a new region of double solutions BGH has been found. In the region a regular reflection configuration is possible and a irregular with a negative angle of reflection. Note that when the adiabatic index is greater than or equal to 1.4, there is only one area of dual solutions ABCD, as it was shown in Figure 6, since at the adiabatic index greater than or equal to 1.4, a configuration with a negative angle does not occur.
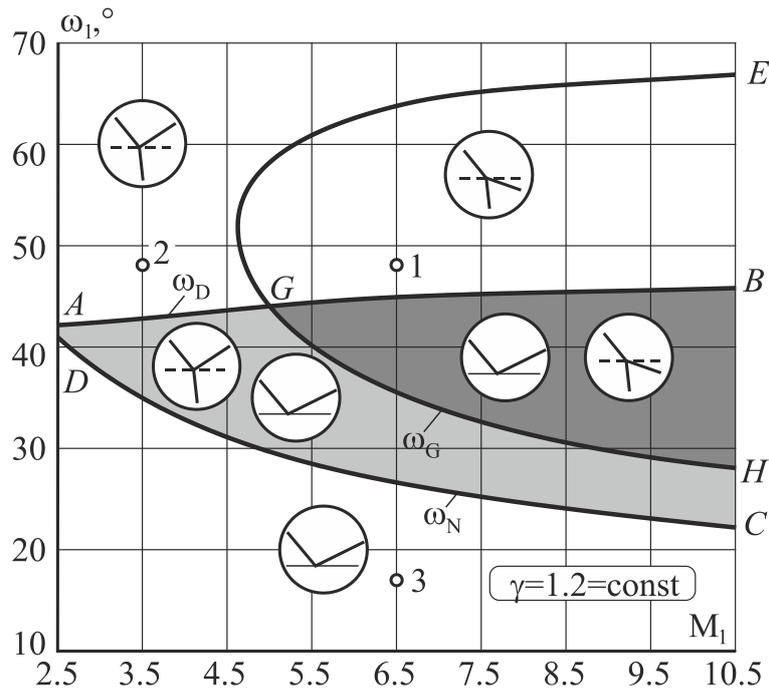
Figure 9. The curves of the transition angles ω_N, ω_D and ω_G in dependence on the Mach number of flow for adiabatic index $\gamma = 1.28$. AGHCD – domain of dual solution, where both regular and irregular reflection may occur. GBH – domain in which it is possible as a regular reflection and the irregular reflection with a negative angle of reflection.

3 The numerical study.

3.1 Description of numerical method of calculation and comparison with experiments.

Analytical theory determines only the location of the shock waves in the close vicinity of the triple point. Therefore, a complete picture of the flow and behaviour configuration with negative angle has been investigated numerically.

Computing program STAR-CCM v. 6.06 has been applied for the numerical calculation. It is designed for solving problems of continuum mechanics [17]. The program incorporates a method of averaging the Navier-Stokes equations (RANS - Reynolds-averaged Navier-Stokes equations). To close the system of equations due to the appearance of new features the model Spalart-Allmaras has been applied to characterize the turbulent stresses [18]. Reflection of the shock wave from the plane of symmetry has been studied as a model. The shock wave was generated by a wedge placed in a supersonic flow (Fig. 1, a, b). The calculations were carried out at different angles of wedges, θ_1 varied from 10 to 50 º in and with a wide range range of flow parameters, $\gamma= 1.05$-1.4, $M_1 = 3$-10.

For the numerical calculation a rectangular area was chosen which is extended in the horizontal plane with a wedge located at the left part of it. Due to the symmetry problem, the solution was carried out for only one-half of the geometry with a symmetry condition at the lower boundary. On the left boundary the Mach number and the static pressure were set; at the top and the right boundary only the static pressure was set. The calculation was performed on a multi-faceted two-dimensional grid. On the wedge boundary prismatic layers were built, necessary for calculation of the turbulence.

Numerical calculations simulated the free flow of gas. It was assumed that the initial conditions coincide with the boundary ones. Inlet pressure — $P_0 = 101325$ Pa, temperature — $T_0 = 300$ K. The equation of state of an ideal gas was set. Courant number was set equal to 0.1 initially and was changed up or down depending on the convergence of the solution.
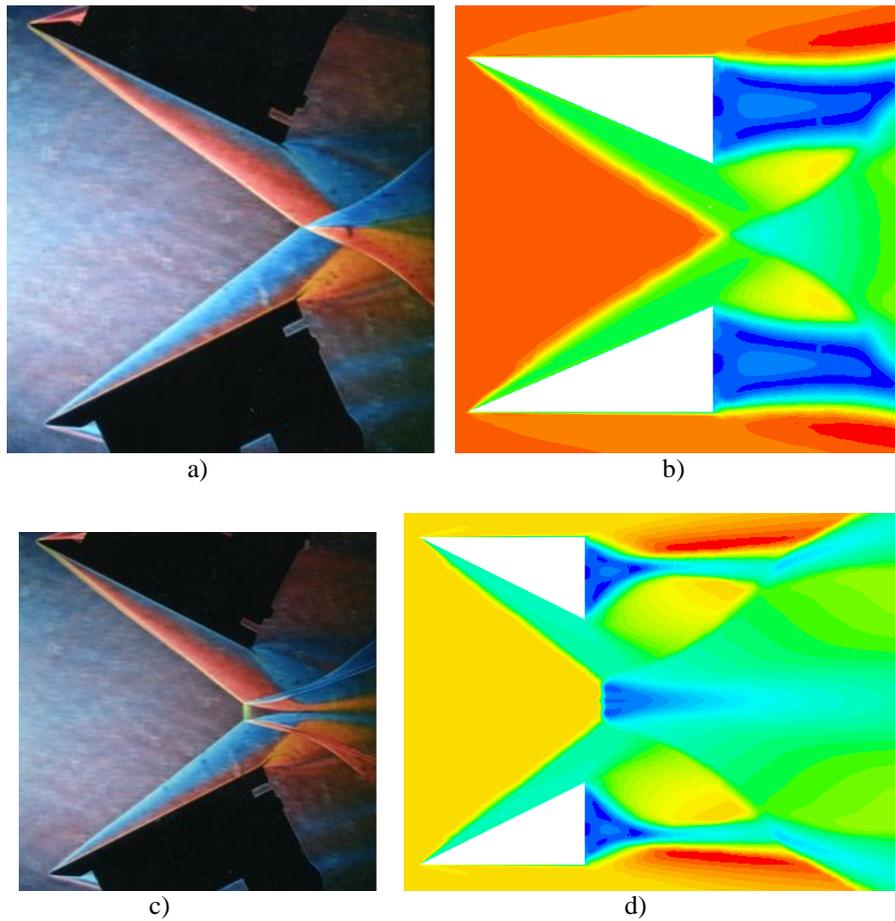
Figure 10. Comparison of the results of experiments in a wind tunnel (a, c) [18] and the numerical calculations (b, d) for regular and Mach reflection. The flow parameters: $M_1 = 4.95$, $\gamma = 1.4$.

To test the program, a series of preliminary calculations for regular and Mach reflection for different angles of the wedge and the parameters of the flow have been made in those areas where only the configuration with a positive angle is obtained. Figure 10 shows the comparison between the results of numerical calculation and the experiment in wind tunnel for regular and irregular reflection. A good visual agreement between the results is seen. Comparison of the gas parameters and location of the waves in the vicinity of the triple point A also gave good agreement with analytic calculations. As it was noted, the three shock theory does not determine the height of the Mach stem. It is determined by the geometry of the wedge and the distance between the wedges, so in the figure there is a slight difference in the value of height of the Mach stem.

The authors have suggested a method to control the height of Mach stem. A system of two small wedges was included downstream. As a result the Mach wave began to move back upstream, as we see in Figure 11. Thus it is possible to vary the height of the Mach stem by changing the size and location of the wedges in the flow.
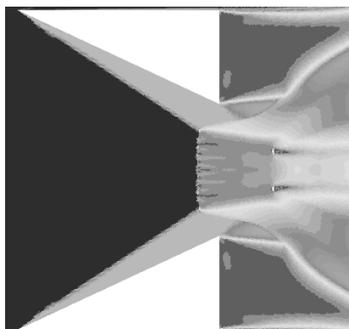

Figure 11. Result of the numerical calculation of Mach reflection with incorporated small wedges. The flow parameters: $M_1 = 4.95$, $\gamma = 1.4$.

3.2. Configuration with a kink on the reflected wave.

The focus of this paper is on the calculation of the region of existence of a negative angle of reflection. Research in this area is very complicated because of the influence of the geometric characteristics of the downstream environment on the wave flow pattern. Analytical calculations can not take this into account, they define the parameters of the flow behind shock waves only in the near vicinity of the triple point.

First of all, the flow pattern can be affected by the rarefaction fan SF emanating from the lower edge of the wedge (Fig. 1, a, b). For example, at the Mach number $M_1 = 6$ and the adiabatic index $\gamma = 1.08$ the analytical calculations show that a three shock configuration with a negative angle must arise. However, the rarefaction fan emanating from the lower edge of the wedge intersects the incident wave, the incident wave is curved, the angle of incidence decreases and we get a complex stable stationary configuration, like regular one, which can not be described with three shock theory because the incident wave is curved (Fig. 12).
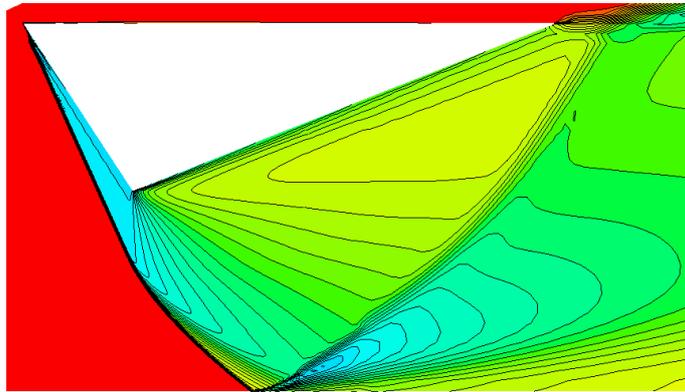

Figure. 12 The changing of the shape of the incident wave due to the influence of the rarefaction fan. The flow parameters: $M_1 = 6$, $\theta_1 = 56.9°$, $\gamma = 1.08$

In order to produce a three shock wave configuration with a negative angle of reflection, it was necessary to find the calculated geometry so that the rarefaction fan interacts only with the reflected wave. This has been done in two ways: in the first case a small wedge was put behind the Mach wave to move it upstream; in the second case, by changing the distance between the bottom edge of the wedge and the line of symmetry. As a result, the irregular reflection pattern with a negative angle of reflection in steady supersonic flow has been obtained for the first time (Figure 13).
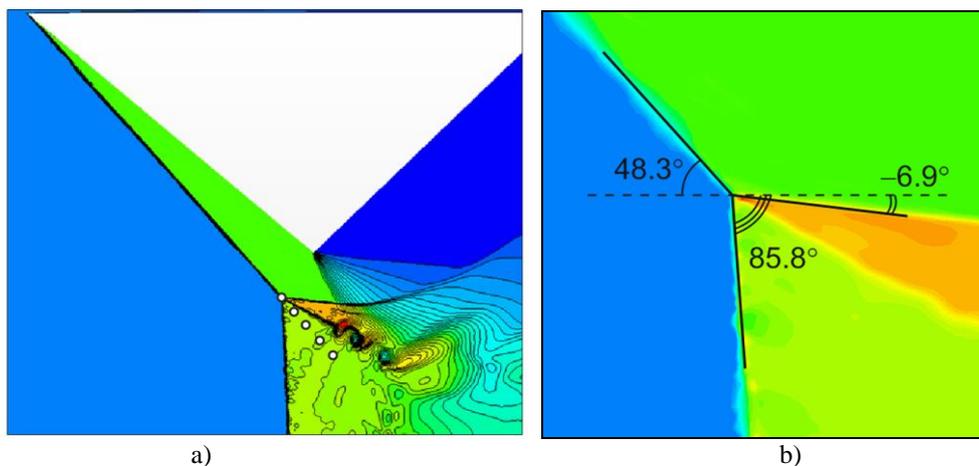

a)   b)

Figure 13. (a) Thee shock wave configuration with a negative angle of reflection and a kink in the reflected wave. Flow parameters: $M_1 = 6.5$; $\theta_1 = 40°$, $\gamma = 1.2$; $\omega_2 = -6.9°$ The curvature of the reflected wave is due to the influence of the rarefaction fan. Dots show the location of the sequence of position of the triple point. (b) Larger image of the triple point. The values of the angles are calculated with the three shock theory.

As can be seen from the results of numerical experiments, the reflected wave $\omega_2 = -6.7°$ in full accordance with the analytical calculation, whose results are shown in Fig. 13 (b). The interaction of the reflected wave with the rarefaction fan led to the change in angle, movement to a positive value.

However, despite the fact that the reflected wave has not crossed the line of symmetry, as shown in Figure 2, the configuration is unstable. They arose in the process of setting and is not stationary. The triple point moves upstream. The intermediate position of the triple point is shown in the figure 13 (a) as open circles. The behaviour of irregular reflection with the negative angle and a kink in the reflected wave needs further investigation. Schematic representation of the new configuration with a bend in the reflected wave is shown in Figure 14.
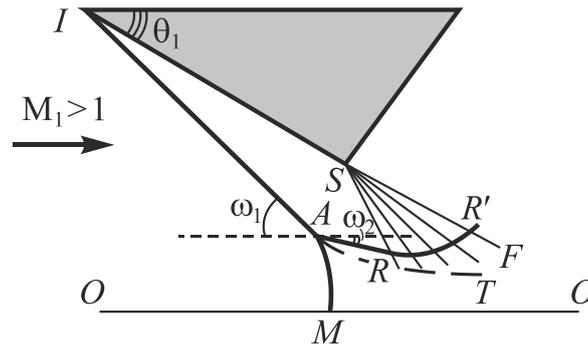

Figure 14. Schematic representation of the new configuration with a negative angle of reflection and a bending in the reflected wave.

3.3 Double Mach reflection in stationary supersonic gas flow.

It has been found that there are several forms of reflection with a negative angle of reflection. Analytical calculations of three-shock configuration have provide only the location of shock waves in the immediate vicinity of the triple point, i.e. the initial part of the reflected wave near the triple point in the configuration with a negative angle of reflection must be directed downward relative to the flow (Fig. 2, a). General view of the resulting configuration and the height of Mach stem will depend on the geometry of the system of wedges and the other conditions downstream.

Numerical studies have shown that the forms of three-shock configuration with a negative angle of reflection will depend on the way of transition to this domain: from Mach reflection or from regular reflection. It has been found that if the transition from Mach reflection will occur than irregular reflection with the negative angle and a kink in the reflected wave occurs. If the transition from regular reflection takes place than irregular reflection with the second triple point on the reflected wave occurs, i.e. double Mach reflection. The double Mach configuration is absolutely unstable. The triple point moves upstream.

Figure 13 and 15 show the results of calculations of three shock configuration at the same geometry and the same flow parameters (Mach number – 6.5, adiabatic index – 1.2, incident wave angle — 48.3°), in the domain of the existence of negative angle of reflection (point 1 in the domain EGB, in Fig. 9). Two completely different results have been obtained. One can come to the point 1 in two different ways, depending on the initial conditions. If we first get stable configuration with a positive angle of reflection (point 2, Fig. 9) and if we change the conditions of the incoming flow so that a configuration with a negative angle of reflection occurs (point 1), then we get three shock configuration with the kink in the reflected wave. In another case, if at the initial time of the calculation there is a regular reflection (point 3) than for rapid transition from regular reflection (changing the angle of the incident wave ω_1) to a configuration with a negative angle of reflection (from point 3 to point 1), we get double Mach reflection (DMR).

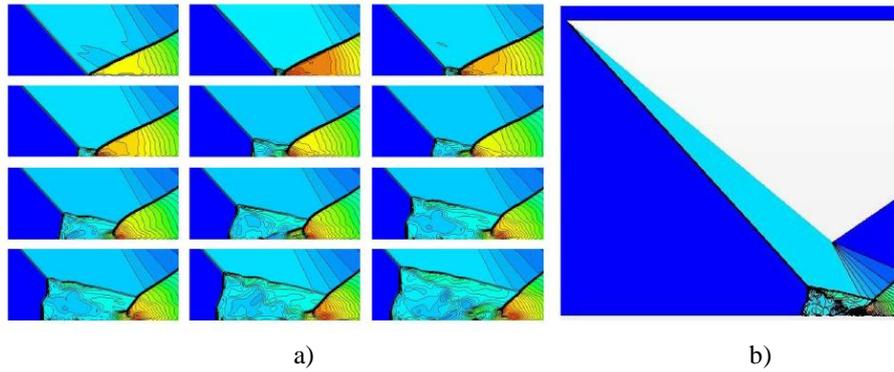a)   b)

Figure 15. Sequence of pictures of the unstable double Mach reflection (a). The overall look (b). Mach number M_1 — 6.5, adiabatic index γ — 1.2, wedge angle θ_1—40º, incident wave angle ω_1—48.3º

Figure 13 shows the configuration with a negative angle of reflection and in the kink in the reflected wave at the transition from Mach reflection with a positive angle of reflection to a configuration with a negative angle of reflection. The dots mark the position of the triple point. The triple point moves upstream.

Figure 15 shows the result of the transition from regular reflection to a configuration with a negative angle of reflection. As you can see, there is a double Mach reflection, which leads to disruption of steady state flow.

Figure 16 gives the scheme of double Mach reflection with a negative reflection angle. Double Mach reflection was observed previously only in quasi-steady reflection from plane wedges. It is noted that this kind of reflection does not exist in stationary gas flows [14].
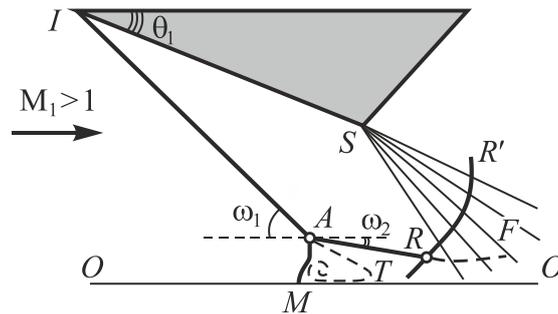

Figure 16. The scheme of double Mach reflection with a negative reflection angle.

Conclusion

A new configuration of shock waves has been found in the reflection of shock waves in a stationary supersonic gas flow in addition to the well-known regular and Mach reflections. This new three shock configuration occurs with a negative angle of reflection and Mach numbers greater than 3 and an adiabatic index smaller than 1.4. Boundaries and areas of existence of this new configuration have been analytically defined.

The complete picture of the different types of reflections is given, depending on the value of the adiabatic index. It has been shown that the domain formerly known as the domain of dual solution, where there is a regular and Mach reflections, increases with the adiabatic index decreases. It was also has been found that at Mach numbers greater than 3 and the adiabatic index less than 1.4, there is another domain of dual solution. In this domain either the regular reflection is possible or configurations with a negative reflection angle.

It has been shown that the configuration with a negative angle of reflection may take different forms depending on the transition. In the transition from Mach reflection — a three-shock configuration has a kink in the reflected wave. The reflection with a second triple point on the reflected wave (double Mach reflection) appears in the transition from regular reflection. The behaviour of both configurations needs further investigation.

The appearance of new configurations may lead to the disruption of the stationary flow pattern. This result may be useful in predicting emergency situations in aircraft flight and rocket engine operation.

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