Simulation of unsteady aerodynamic characteristics of aircraft at high angles of attack using neural networks.

Ignatyev D.I.
Central Aerohydrodynamic Institute (TsAGI),
Zhukovsky, Russia

Abstract
Adequate modeling of unsteady aerodynamic characteristics is required for investigations of aircraft dynamics and stability analysis at high angles of attack. Recurrent neural networks of NARX type are applied in the paper. The efficiency of the approach is demonstrated with two examples, namely, delta wing and aircraft with canard surface. To improve accuracy the fact that models of unsteady aerodynamic characteristics are developed using various dynamic tests is taken into consideration and neural network training algorithm is proposed. The algorithm based on Bayesian regularization for the different-type initial data uses Gauss-Newton method to approximate Hessian matrix with modified Levenberg-Marquardt optimization algorithm to locate the minimum point. The algorithm is shown to improve the mathematical model accuracy.

1. Introduction.

Significant extension of the angles-of-attack range used in modern flight leads to necessity of more adequate modeling of aircraft unsteady aerodynamic characteristics. This problem is urgent for both maneuverable and commercial airplanes. The first ones use high-angle-of-attack range in the air fight, the latter ones use high-angle-of-attack range during take-off and landing. In addition, according to EASA (European Aviation Safety Agency) a loss of control in flight became the main cause of the commercial airplane fatal accidents during the last decade [1]. That is why it is necessary to model the aerodynamics in the extended flight envelope to provide realistic pilot training in upset conditions and to study more thoroughly the aircraft dynamics in critical flight regimes [2, 3].

Aircraft aerodynamics at high angles of attack is determined to a large degree by detached and vortical flows. The account for the effects associated with these processes is important for the correct description of aircraft dynamics. A breakdown of vortexes generated by the forebody and wing leading edge is the key physical effect for the maneuverable aircraft at high angles of attack. Dynamics of the vortex breakdown position while changing the angle of attack and sideslip angle cause nonlinear variation of the aerodynamic characteristics, stability and controllability characteristics. As concerns transport airplanes with high wing aspect ratio, the important role for the aerodynamics at high angles of attack is played by the dynamics of the wing flow separation. A complicated character of the aerodynamic interference upon the detached flow conditions is a significant source of nonlinearity of aerodynamic characteristics at high angles of attack for the full aircraft configuration. Particularly, interaction of the detached wing flow with the flow over horizontal tail affects significantly the stability and controllability characteristics. The flow detached from the vertical tail, and the vortices generated by the fuselage nose part and interacting with the tail make contribution to nonlinear behavior of aerodynamics upon further increase in the angle of attack.

The unsteady effects are very important since aircraft usually do not use high angles of attack in the normal flight but go beyond the normal flight envelope in consequence of dynamic maneuvers, pilot's mistakes, and wind gusts. The problem of modeling of unsteady aerodynamics at high angles of attack is directly connected with ensuring the flight safety.

Development of computers and numerical techniques has recently caused significant progress in the direct numerical simulation (DNS) of aerodynamic loads using Navier-Stokes equations [4]. Nevertheless, at the present state of art the equations of fluid mechanics and aircraft motions cannot be solved simultaneously in the certain flight mechanics problems. Simulation of aircraft dynamics and control design problems require a large number of parametric studies that is possible only while using simple and real-time aerodynamic models. DNS cannot also be applied for semi-realistic real-time simulation of the aircraft flight using pilot simulators.

For solution of the important flight dynamics problems, the simplified mathematical models of the unsteady aerodynamics that consider complex effects of the detached and vortical flows allowing real-time simulation are required. These models should be capable of describing non-linear phenomena important for the flight dynamics in the wide range of kinematic parameters. In practice, such models are developed using experimental data obtained from a set of wind-tunnel dynamic tests with various test rigs.

At the present moment, the representation of the aerodynamic coefficients using aerodynamic derivatives is widely used for flight dynamics engineering applications [5]. For the small angles of attack and sideslip angles, the aerodynamic forces and moments are supposed to be represented as the linear terms of Taylor series expansion of the aerodynamic coefficients in the motion parameters

$$\begin{aligned} C_L &= C_L(\alpha_0) + C_{L_\alpha}(\alpha - \alpha_0) + C_{L\bar{q}}(q\bar{c}/V) + C_{L\bar{\dot{\alpha}}}(\dot{\alpha}\bar{c}/V) \\ C_m &= C_m(\alpha_0) + C_{m_\alpha}(\alpha - \alpha_0) + C_{mq}(q\bar{c}/V) + C_{m\bar{\dot{\alpha}}}(\dot{\alpha}\bar{c}/V) \end{aligned} \quad (1)$$

The most prevailing way to obtain expansion coefficients is the dynamic tests in wind-tunnels. This method can be successfully applied in the range of linear variation of aerodynamic characteristics, i.e., for the flows without separation. Application of this method in the range of non-linear variation of the aerodynamic characteristics can lead to significant errors.

The most general technique of modeling of unsteady aerodynamic characteristics is to use the nonlinear indicial functions [6]. To develop the model based on the nonlinear indicial functions the unsteady aerodynamic data are used. Nevertheless, it requires a set of serious simplifications when applied to real experimental data, so that final mathematical models are formulated in a simple form of first-order linear differential equations.

The phenomenological approach [7] takes into account delays of separation and recovery of flow without separation. The aerodynamic loads are separated into linear and nonlinear components while using this approach. Ordinary differential equations are used for modeling of the nonlinear components of the aerodynamic characteristics. The equations contain characteristic time constants corresponding to the times of flow separation development. The dynamic wind-tunnel tests are also used to identify these constants. This approach enables the dependence of aerodynamic characteristics on frequency and amplitude of oscillations, and aerodynamic hysteresis to be modeled quite precisely. Unfortunately, application of phenomenological approach in an arbitrary case can cause a series of difficulties associated with selection of nonlinear components of unsteady aerodynamic characteristics.

Neural networks have been used recently for identification and modeling of nonlinear aerodynamics in a number of papers [8-14]. Such an active introduction of neural networks is mainly connected with their universal approximation properties [15], which enable the neural networks to be used for an arbitrary aircraft without significant simplifying assumptions. Peculiarities of solving real-world problems cause necessity of more in-depth study of neural-network techniques and their adaptation for the problems of unsteady aerodynamic modeling.

An approach for neural-network modeling of unsteady aerodynamic characteristics in the wide angle-of-attack range using wind-tunnel dynamic tests is presented in the paper.

2. Description of neural network configuration and learning techniques.

2.1. Neural network configuration.

Recurrent neural networks are used recently for modeling of dynamic systems; therefore, such type of neural networks is preferable for flight dynamics problems. Thanks to their properties they can be used for any type of aircraft motion. The recurrent neural networks were used for modeling of unsteady aerodynamic coefficients of a delta wing aircraft at high angles of attack in [13].

A recurrent neural network of NARX type (nonlinear autoregressive network with exogenous inputs) is used in the paper. Its configuration is shown in figure 1. For modeling variable *y* at the time *t* the state vector $x(t)$ and a series of its former values $x(t-1), x(t-2)...x(t-T_{in})$ are inputted into the neural network. The values of the modeling variable $y(t-1), y(t-2)...y(t-T_{out})$ calculated by the neural network earlier are also used.

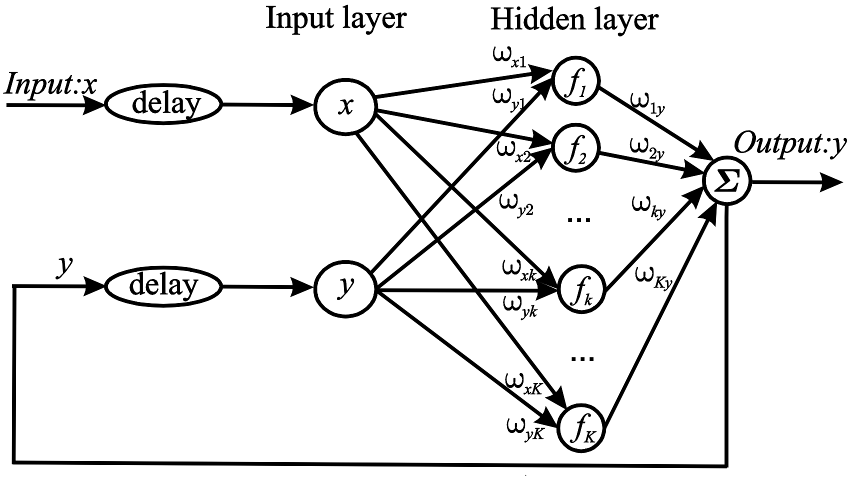

Figure 1: Neural network configuration

A neural network can be considered as a directed graph with neurons placed in it nodes. Neuron is an elementary calculating unit. The neurons of the first layer do not execute calculations but distribute input signals between neurons of the hidden layer. After a set of signals from the input layer have been obtained each neuron of the hidden layer multiplies each signal by its own weight factor ω_{ik} corresponding to the signal transfer connection, sums derived products, and adds bias b_k. The result undergoes nonlinear transformation through the neuron activation function f_k. The operational diagram of hidden layer neuron is presented in figure 2. The transformation of input signal $s = (x, y), \{x = x(t), x(t-1)...x(t-T_{in}), y = y(t-1)...y(t-T_{out})\}$ into the output signal φ_k can be presented in the following form

$$\varphi_k = f_k(w_{xk}x + w_{yk}y + b_k). \tag{1}$$

The resulting neural-network model can be presented in the form

$$y(t) = \Phi(x(t), x(t-1), ..., x(t-T_{in}), y(t-1), ..., y(t-T_{out})), \tag{2}$$

where Φ is the function of neural network operation.

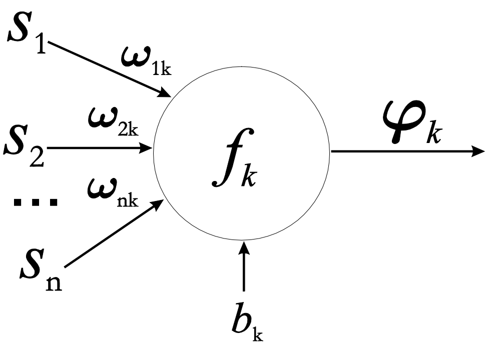

Figure 2: Artificial neuron

2.2 Neural network training

The connection weights ω_{ik} and biases b_k are adjusted during neural network training when the examples of learning set are presented. The weight coefficients are adjusted through minimization of the difference between neural network operation results y_i and target data a_i for each example from the learning set $i = 1..N$

$$E_D = \frac{1}{2}\sum_{i=1}^{N}(y_i - a_i)^2, \tag{3}$$

One of the problems during neural network training is called overfitting. The error on the training set is driven to a very small value, but when new data is presented to the network the error is large. The network has memorized the training examples, but it has not learned to generalize to new situations. A part of the whole initial data can be used for model testing. The test set error should be gained to be as small as possible and not to be significantly higher that the training set error. When this condition is valid the neural network is considered to have good generalization performance.

Regularization is one of the techniques for improving generalization. According to this technique, a term penalizing the neural network for increase of weights is added in the objective function besides the error measure E_D (3). Particularly, the sum of the squares of weights can be used.

$$E_W = \frac{1}{2}\sum_{j=1}^{K}\omega_j^2 , \tag{4}$$

where K is a number of neural network weights. The objective function takes the form

$$F = \beta E_D + \alpha E_W, \tag{5}$$

where α and β are objective function parameters. To define the objective function parameters, Bayes' rule can be used [16], particularly, Gauss-Newton approximation to Bayesian regularization (GNBR) algorithm was used in [17] to train neural networks.

GNBR algorithm is the effective tool for training neural networks, but it supposes that initial data are of a single type. The unsteady aerodynamic models for flight dynamics problems are developed using different dynamic rigs, in the various ranges of kinematic parameters, for various types of motion, and with different accuracies. More accurate models can be obtained while considering that data are of different types. In the present paper, the GNBR algorithm was modified for the case of different-type data and Bayesian Regularization to the training of neural networks considering Different-type Data (BRDD) was proposed.

2.3 Bayesian Regularization to the training of neural networks considering Different-type Data.

Let us suppose that experimental data to be approximated are obtained in n different experiments $(\mathbf{x}_1,\mathbf{a}_1),(\mathbf{x}_2,\mathbf{a}_2),...,(\mathbf{x}_n,\mathbf{a}_n)$, where $\mathbf{x}_i = \left(x_{i_1}...x_{i_{N_i}}\right)$ is the vector of values of the controlled phenomenon parameter, obtained in i-th experiment, $\mathbf{a}_i = \left(a_{i_1}...a_{i_{N_i}}\right)$ is the vector of values of the observed variable obtained in i-th experiment. The errors in each experiment are supposed to have normal distribution with zero statistical expectation but with different standard deviations σ_i.

The problem is to identify function y that should describe the obtained experimental data

$$\begin{gathered} a_{1_{m_1}} = y\left(x_{1_{m_1}}\right) + \nu_{1_{m_1}},\ m_1 = 1..N_1, \\ a_{2_{m_2}} = y\left(x_{2_{m_2}}\right) + \nu_{2_{m_2}},\ m_2 = 1..N_2, \\ \dots \\ a_{n_{m_n}} = y\left(x_{n_{m_n}}\right) + \nu_{n_{m_n}},\ m_n = 1..N_n \end{gathered} \tag{6}$$

where $D_i = \left\{x_{i_{m_i}}\ a_{i_{m_i}}\right\},\ m_i = 1...N_i$ is the data set obtained at the one-type experiment, y is the approximating function.

Using Bayes' rule the following objective function can be obtained instead of (5)

$$F = \frac{1}{2}\alpha\mathbf{w}^T\mathbf{w} + \frac{1}{2}\mathbf{e}^T\mathbf{B}\mathbf{e} , \tag{7}$$

where $\mathbf{w}_i = (\omega_1\omega_2...\omega_K)^T$ is the vector of weights, $\mathbf{e}_i = (e_1e_2...e_N)^T$ is the vector of errors, $e_j = \left(y\left(x_j\right) - a_j\right)$ is the approximation error of i-th data pair, $\mathbf{B}$ is the matrix $N\times N$; the objective function parameters β_i are placed on the main diagonal of it, the other elements of this matrix are equal to zero.

$$\mathbf{B}=\begin{pmatrix} \beta_1 & 0 & \dots & & & & & 0 \\ 0 & \beta_1 & 0 & \dots & & & & 0 \\ & & & \dots & & & & \\ 0 & \dots & 0 & \beta_i & 0 & \dots & & 0 \\ 0 & & \dots & 0 & \beta_i & 0 & \dots & 0 \\ & & & & \dots & & & \\ 0 & & & & & 0 & \beta_n & 0 \\ 0 & & & & & & 0 & \beta_n \end{pmatrix}. \tag{8}$$

Using Bayes' rule to determine the parameters of objective function F (7), the following expression can be obtained

$$\alpha \approx \frac{\gamma}{\mathbf{w}^T\mathbf{w}},$$

where $\gamma = K - \alpha\,\mathrm{Sp}(\mathbf{H}^{-1})$ is a so-called effective number of parameters, K is the total number of parameters in the network [17], $\mathbf{H} = \nabla^2 F$ is the Hessian matrix of the objective function.
The following expressions are obtained for β_i:

$$\beta_i = \frac{N_i}{\mathbf{e}^T \dfrac{d\mathbf{B}}{\mathrm{d}\beta_i}\mathbf{e} + \mathrm{Sp}\left(\dfrac{d\mathbf{H}}{\mathrm{d}\beta_i}\mathbf{H}^{-1}\right)},$$

where N_i is the number of patterns of the i-th training subset.
Hereby, the proposed approach enables the parameters of the objective function to be adjusted subject to the error of approximation on corresponding subset.
The algorithm for practical implementation of the training technique was developed in the paper. To obtain values of the objective function parameters it is required to calculate Hessian matrix in the minimum point of objective function F. Gauss-Newton method is proposed to approximate Hessian matrix with modified Levenberg-Marquardt optimization algorithm used to locate the minimum point

$$\mathbf{w}_i = \mathbf{w}_{i-1} - \left(\mathbf{J}^T\mathbf{B}\mathbf{J} + (\alpha+\mu)\mathbf{E}\right)^{-1}\left(\mathbf{J}^T\mathbf{B}\mathbf{e} + \alpha\mathbf{w}_{i-1}\right),$$

where $\mathbf{J}$ is the Jacoby matrix. The proposed modification improves the algorithm convergence in the vicinity of the minimum point.

3. Results

3.1. Delta wing

First, let us consider the neural network modeling of unsteady aerodynamic characteristics of a delta wing. The dynamics of vortex burst position above the upper wing surface is known to be the main physical effect determining unsteady flow of the delta wing at high angles of attack. The experimental results [18] obtained for the delta wing with aspect ratio $\lambda = 1.5$, mean aerodynamic chord $\bar{c} = 0.725$ m, wing sweep $\chi \approx 70°$ are used in the present paper. The tests were carried out in wind tunnel T-103 of TsAGI with flow velocity $V_\infty = 25$ m/s. Dynamic tests with forced pitch oscillations with small amplitude of 3° were carried out for frequencies of 0.5, 1 and 1.4 Hz. Angle-of-attack range was from 0° to 60°. Large-amplitude forced oscillations were also implemented. The amplitudes were from 15º to 24º, frequencies were from 0,2 to 1,2 Hz.
For the delta wing, the unsteady Lift force coefficient C_L model was developed. The neural network of NARX type with one hidden layer was used. The proposed model of $C_L(t)$ depends on the angle of attack $\alpha(t)$, pitch rate $q(t)$ at the time t, prehistory of motion $\alpha(t-T_{in})$, $q(t-T_{in})$. In addition, the neural network uses the results of modeling on the previous time period $C_L(t-T_{out})$. Initial values of the Lift force coefficient are input in the model. The only GNBR algorithm was used for training of this model.
Let us consider the results of modeling of unsteady Lift force coefficient obtained at forced pitch oscillations. Figure 3 shows the results of modeling in comparison with large amplitude experiments. The given examples are referred to the test subset. The unsteady derivatives $C_{L\alpha}$ and $C_{L\bar{q}} + C_{L\bar{\dot{\alpha}}}$ were also modeled and compared with the experimental results, which are given in figure 4. Figures 3 and 4 show the adequate ability of neural networks to describe the results of dynamic experiments while developing unsteady aerodynamic models. Nevertheless, the

application of BRDD technique enables more accurate results to be obtained. This will be demonstrated in the following section.

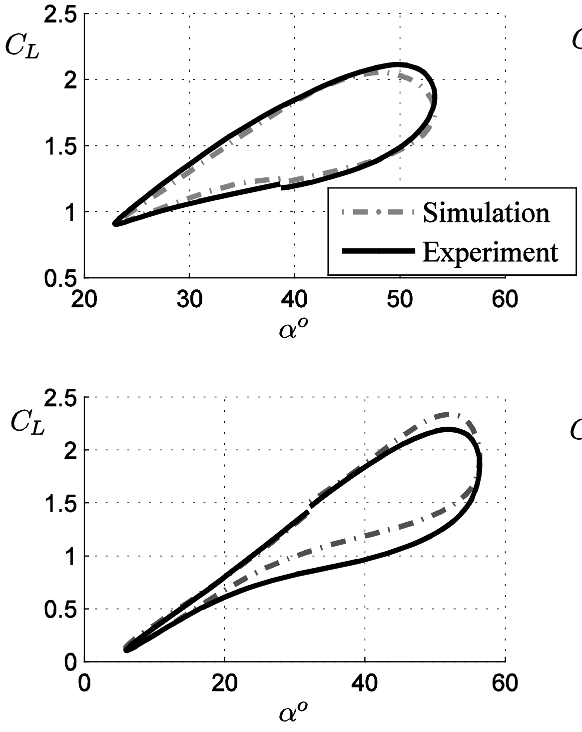

Figure 3: NARX simulation of lift force coefficient C_L compared to large amplitude oscillation measurements - Delta wing

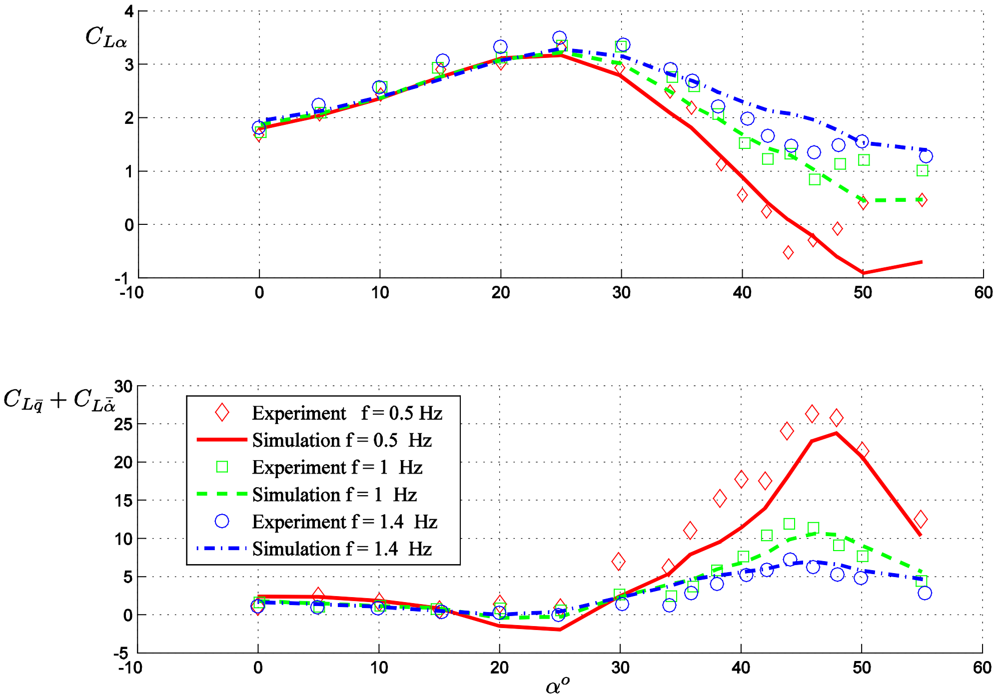

Figure 4: NARX simulation of C_L dynamic derivatives obtained during small amplitude pitch oscillations compared to oscillation measurements - Delta wing

3.2. TCR model

Let us consider the neural network modeling of unsteady aerodynamic characteristics of a passenger aircraft designed for the transonic cruise.

The experimental study of a passenger aircraft designed for the transonic cruise (TCR) was carried out during the participation of TsAGI in European Project SimSAC of the Sixth Framework Program. The aerodynamic configuration is characterized with high-sweep wing and canard surface. Interaction of the flow detached from the canard surface with the wing flow is the crucial physical effect at high angles of attack for this model. The TCR wing span is $b_a = 0.725$, mean aerodynamic chord is $\overline{c} = 0.2943$ m. The experiments were carried out in wind tunnel T-103 of TsAGI with flow velocity $V_\infty = 40$ m/s. Dynamic tests with forced pitch oscillations with small amplitude of 3° were carried out for frequencies of 0.5, 1 and 1.5 Hz. Angle-of-attack range was from -10° to 40°. Large amplitudes were equal to 10º and 20º, frequencies were equal to 0,5, 1, and 1,5 Hz. The general view of the model used in the experiment is given in figure 5. More detailed discussion of the experiment with the TCR model is given in [19].

Figure 5: General view of TCR model

For TCR model the neural, the network technique was applied for unsteady pitch moment coefficient C_m.

The same NARX configuration of neural network with one hidden layer was used. The dependencies of pitch moment coefficient on the angle of attack during oscillations are very complex for such aerodynamic configuration, so the application of the BRDD is desirable to increase the model accuracy. Large amplitude tests and aerodynamic derivatives are correspondingly modeled in figures 6 and 7. Figure 6 shows the hysteresis loops observed in the experiments that are modeled quite precisely. Small amplitude test results, which are used to determine aerodynamic derivatives, are modeled and shown in figure 7. The dependencies of derivatives on oscillation frequency in the angle-of-attack range corresponding to the canard surface flow separation are seen to be modeled by the neural network with acceptable accuracy level.

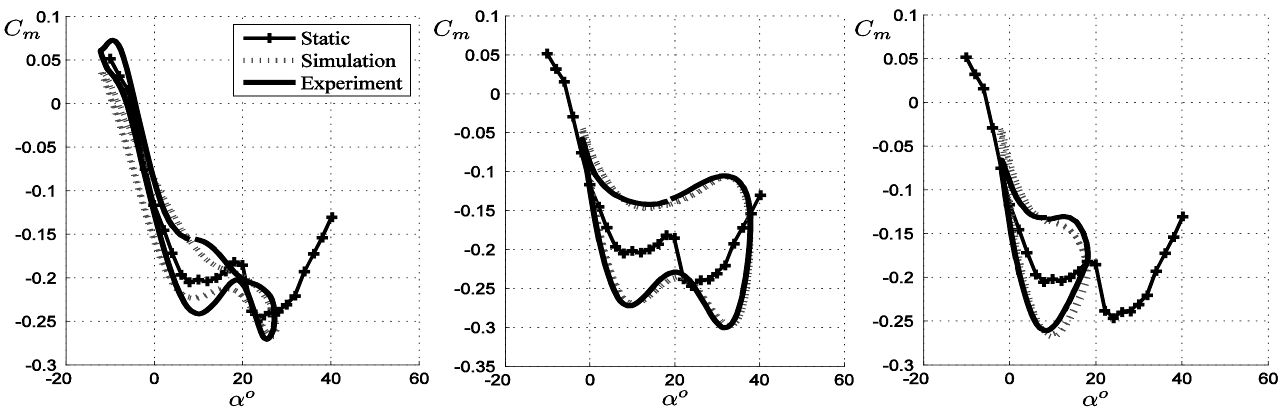

Figure 6: NARX simulation of pitch moment coefficient C_m compared to large amplitude oscillation measurements – TCR model

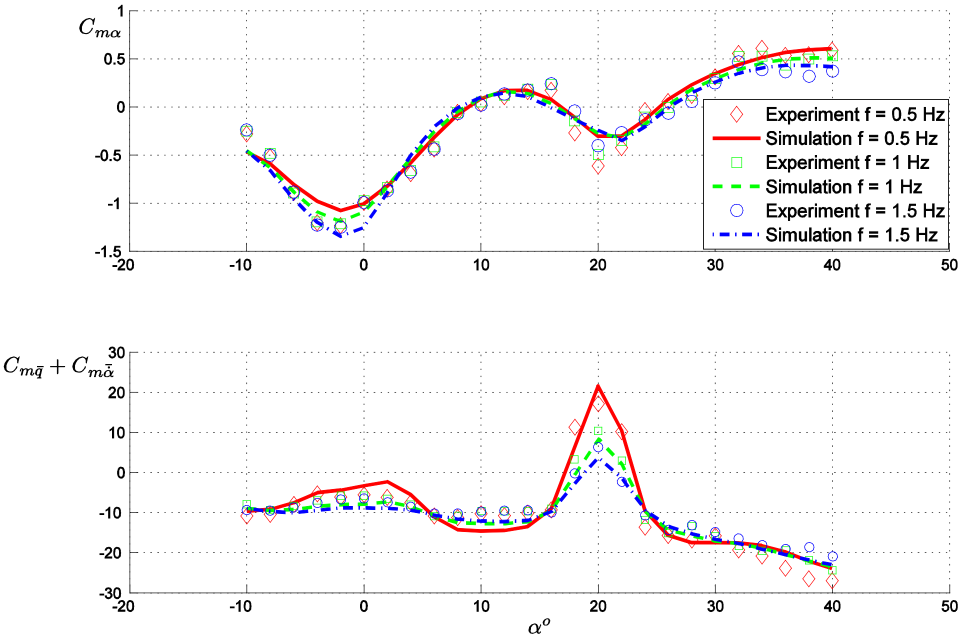

Figure 7: NARX simulation of C_m dynamic derivatives obtained during small amplitude pitch oscillations compared to oscillation measurements - TCR model

Let us carry out an analysis of the obtained results to determine whether the BRDD algorithm helps to improve accuracy of models derived from different type data. For this purpose, the neural network models of TCR pitch moment coefficient obtained while using GNBR and BRDD training algorithms should be compared.

A numerical comparison of the neural network is implemented via calculating the errors obtained for models of pitch moment coefficient C_m and derivative $C_{m\bar{q}} + C_{m\dot{\bar{\alpha}}}$ separately for train and test subsets. The error measure is the mean square error divided by the entire range of the measured value Δy

$$err_i = \frac{\sqrt{\frac{1}{N_i - 1}\sum_{j=1}^{N_i}(y_j^{\text{exp}} - y_j^{\text{sim}})^2}}{\Delta y}$$

Tables 1 and 2 give the errors of neural network models.

Table 1: Model error for GNBR algorithm

Variable	Train subset, %	Test subset, %
C_m , (large amplitudes)	5.59	8.3
$C_{m\bar{q}} + C_{m\dot{\bar{\alpha}}}$	7.09	8.58

Table 2: Model error for BRDD algorithm

Variable	Train subset, %	Test subset, %
C_m , (large amplitudes)	4.53	6.34
$C_{m\bar{q}} + C_{m\dot{\bar{\alpha}}}$	5.65	5.77

One can see a significant accuracy improvement of models. The errors for C_m decreased by 23% and 31% for the train and test subsets, respectively. The errors for $C_{m\bar{q}} + C_{m\dot{\bar{\alpha}}}$ decreased by 25% and 49% for train and test subsets, respectively.

Figures 8 and 9 show the scatterograms for C_m and $C_{m\bar{q}} + C_{m\bar{\dot{\alpha}}}$ obtained on the test subsets. Less scattering is seen to be obtained using the developed technique.

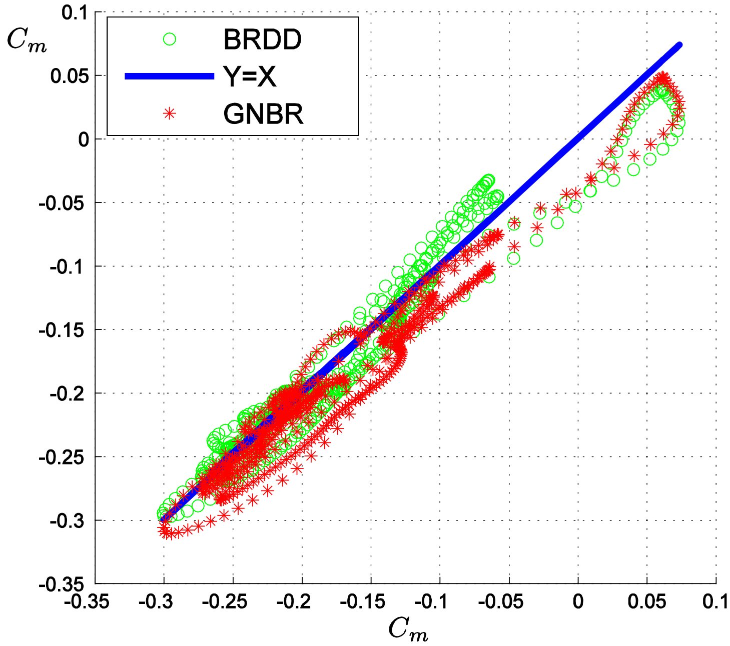

Figure 8. Scatterogram for test data - C_m

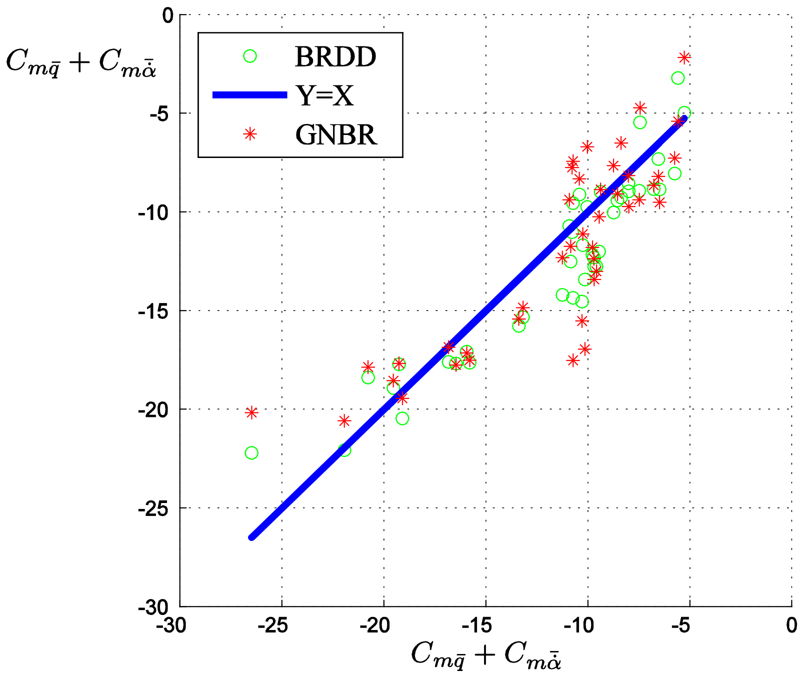

Рисунок 9. Scatterogram for test data - $C_{m\bar{q}} + C_{m\bar{\dot{\alpha}}}$

4. Conclusions

Mathematical modeling of unsteady aerodynamic characteristics at high angles of attack is an urgent problem for improvement of aircraft safety. Simplified and real-time unsteady aerodynamic models that are able to describe a series of phenomena associated with detached and vortical flows are required for an adequate modeling of aircraft flight dynamics.

The recurrent neural networks of NARX type, which were usually applied in dynamic system modeling, were used in the paper for unsteady aerodynamics modeling. Two configurations were considered, namely, delta wing and passenger aircraft with canard surface, the flows over models are characterized with different physical effects. The neural networks were shown to be an effective tool for modeling of unsteady aerodynamic characteristics in a wide range of kinematic parameters regardless of the nature of the observed nonlinear phenomena. This simplifies significantly modeling in case of arbitrary aircraft.
To improve accuracy of neural network models of unsteady aerodynamics the fact that different dynamic tests were used while developing models was taken into account. The effective training algorithm based on Bayesian regularization with the initial data of different type was suggested. The algorithm uses Gauss-Newton method to approximate Hessian matrix with modified Levenberg-Marquardt optimization algorithm to locate the minimum point. The paper shows that the proposed technique enables the model error to be decreased on both the train and test subsets.

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