Numerical Model to Analyse Nozzle Termochemical Erosion in Hybrid Rocket Motors

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Abstract

Nozzle erosion has a significant impact on rocket motor performances. The objective of the present work is to study the erosion behavior of graphite nozzles in hybrid engines at different operating conditions. The adopted approach relies on a thermochemical ablation model that takes into account heterogeneous chemical reactions at the nozzle surface and heat conduction inside the nozzle material. The model is coupled with the heat flux and the diffusion rate of the species through the boundary layer calculated with the use of the Reynolds analogy. The model has been validated with experimental data from the BATES solid motor test campaign and previous full Navier-Stokes simulations. The numerical model provides good accuracy with a low computational time. The model can be used to perform parametric analysis in order to assess the impact of various parameters that affect the nozzle erosion rate, taking into account various combinations of fuels and oxidizers operating at different conditions. Finally, the erosion model has been coupled to a 0-Dimensional model of the hybrid motor to assess the impact of nozzle erosion on motor chamber pressure, thrust profile and specific impulse.

1. Introduction

HRMs¹ generally use a liquid oxidizer stored in a tank and a solid fuel grain placed inside the combustion chamber. The oxidizer flow is controlled through a regulation valve that makes the system able to throttle and abort. Moreover the hybrid is potentially restartable and so well suited as a multi-purpose engine.

To avoid significant liquid flow pressure drop or premature oxidizer decomposition no regenerative technique is usually used, hence the nozzle and combustion chamber are protected from the thermal environment the same way as SRMs². As a result HRMs are much cheaper than liquid rocket engines and slightly more complex than SRMs. For their heterogeneous nature hybrid motors are less prone to explosion and fire hazard, this intrinsic safety allows to reduce both development, manufacturing and operational costs. In addition to that, typical HRMs propellant combinations are more environment friendly than the commonly used SRMs propellant grains.

There are few drawbacks anyway, mainly due to the diffusive flame mechanism that characterize the HRMs: reduced regression rate, low combustion efficiency, difficulties in scaling up the results from labscale tests and, in general, the presence of a peculiar time-varying combustion chamber thermochemical environment. The objective of different research efforts worldwide is to find a way to mitigate/solve these major HRM disadvantages and obtain a more consistent design procedure, since a mature HRM technology could be a promising solution for the future commercial launch systems.

In order to contain the cost of the motor, the thermal protection system is analogous to the one adopted in SRMs. This means that the combustion chamber and the nozzle are protected from the harsh thermochemical environment

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¹Hybrid rocket motors

²Solid rocket motors.

using ablative materials. In particular, graphite is used for the manufacturing of nozzle throat inserts, since this material allows to mitigate the incoming heat flux at the expense of a low recession of the surface. This leads to an increase of the nozzle throat area during the motor operative time, with the consequent variation of the performance.

The aim of the present work is the development of a numerical method to predict the throat erosion rate in hybrid rocket motors. This method involves the use of a non-equilibrium thermochemical ablation model, which turned particularly useful in obtaining a good estimate of the throat erosion. This model was limited for the modelling of non-charring ablative materials. The need to develop such method is required by the peculiar application of these ablative thermal protections, in fact the throat erosion rate in hybrid rocket motors results to be higher than those obtained in solid rocket motors operating at the same pressure and it is also characterized by the propellant formulation, which can vary significantly for different motor configurations and within a motor operating time.

2. Aerothemal analysis

In order to get an estimate of the throat erosion rate an aerothermal analysis must be carried out. Aerothermal analysis is a procedure adopted in nozzle design which couples the gas flow behaviour with the thermochemical behaviour of the ablative thermal protection in order to verify if the foreseen thermal design suits the purpose. The steps involved in a nozzle aerothermal analysis are:

- Thermochemical environment characterization;
- Convective heat flux and diffusive mass flux evaluation;
- In-depth thermal response evaluation with a surface thermochemical ablation boundary condition.

Every single step involved in the analysis has been solved using a method that allows a fast but still accurate solution.

2.1 Thermochemical environment characterization

Thermochemical environment characterization is a main feature in understanding the thermal protection system behaviour. In order to characterize the combustion chamber, different thermodynamic and transport properties are required; these are often temperature and mixture dependent.

NASA's CEA³ computer program was used to obtain combustion chamber temperature (T_{cc}) and molar fractions (X_i) of the species present in the flow starting from a given combustion chamber pressure (p_{cc}) and oxidizer to fuel ratio (o/f). Since this procedure requires a certain amount of time and an active input from a user, a CEA interface with MATLAB has been developed in order to generate a look-up table from which a second code will upload the required data more quickly than the standard procedure.

At normal operating conditions the propellant gasses can be considered made up of H_2O , CO_2 , CO, N_2 , O_2 , OH, O and NO, which are present in major quantity. For these species the investigation of thermodynamic and transport properties is required since they are further needed. These properties have been evaluated for every single species using different temperature dependent functions. The constant pressure specific heats $(c_{p,i})$ were obtained using *Chase M.W. Jr.* interpolating function described in [⁵]. As far as the species transport properties are concerned, they were obtained using *Sutherland's law* for the viscosity and ad hoc piecewise polynomials for the thermal conductivity. The physical data necessary to obtain the constants in the functions previously cited were taken from the database [⁶].

Once all these properties for every single species are obtained it is necessary to compute those of the overall gaseous mixture. In order to achieve this the *mixing laws* reported in $[^3]$ is used.

2.2 Heat and mass fluxes evaluation

Once all the thermodynamic and transport properties are available, it is possible to evaluate the heat and mass fluxes involved in the nozzle, focusing on the throat region since the evaluation of the erosion in this particular zone is the aim of this work.

The main heat fluxes involved in the gas side are the convective and the radiative ones, whilst in the wall side only the conductive flux is taken into account. The radiative component is the sum of the flux from the hot gas and the energy re-radiated back from the wall. In HRM the exhaust gas emissivity could vary significantly for different propellant formulation. Moreover treating with a gaseous emitter is not straightforward⁴ and the radiative heat flux

³Chemical Equilibrium and Application

⁴It require advanced methods and knowledge of nozzle wall temperature along the contour.

contribution is only a fraction of the convective one, so its evaluation is generally considered an advanced step in nozzle design. Hence in this work only the convective contribution to the overall gas side heat flux has been evaluated.

There are different ways to estimate the convective heat flux transferred from the flow to the nozzle wall. But we must keep in mind that we are looking for quick methods, so this excludes an accurate CFD^5 analysis. A method that maintains an acceptable level of accuracy without an excessive computational cost is the *Bartz equation*^{[8}]:

$$\alpha = \left[\frac{0.026}{D_{th}^{0.2}} \left(\frac{D_{th}}{R_{c,th}}\right)^{0.1} \frac{\mu_0^{0.2} c_p}{Pr^{0.6}} \left(\frac{p_{cc}}{c^{\star}}\right)^{0.8}\right] \left(\frac{A_{th}}{A}\right)^{0.9} \sigma \tag{1}$$

where α is the *convective heat flux coefficient* used in the well known equation:

$$q_{conv} = \alpha (T_{aw} - Tw) \tag{2}$$

where T_w is the *actual wall temperature* and T_{aw} in the *adiabatic wall temperature* which is correlated with the fluid stagnation temperature via the following formula: $T_{aw} = T_e + R(T^\circ - T_e)$. Here *R* is defined as *recovery factor* and can assume different values between 0 and 1 depending on the *Prandtl number* and on the correctional formula adopted⁶.

This method proved to successfully fit data from turbulent boundary layer calculations and motor test experiments. In order to validate the method the following procedure has been carried out. Figure 1 shows the difference between the calculation performed using the *Bartz equation* (Eq.1) and a commercial CFD computer program (ANSYS[®] Fluent). The nozzle geometrical configuration adopted in our simulations was that of USAF's BATES⁷ test motor, in order to easily compare our results with those obtained by *D. Bianchi & F. Nasuti* in [¹²]. This comparison will be shown further on.



Figure 1: Comparison between the results obtained using the *Bartz equation* and an CFD analysis

The main differences that can be found in figure 1(a) are:

- The *Bartz equation* maximum occurs at the throat, while the CFD analysis shows that the maximum occurs upstream of the nozzle throat. The relative difference between the peaks is 1.2%, while at the throat (x = 0) the spread is about 15.6%, with respect to the CFD maximum;
- The Bartz method over-predicts the film coefficient in the divergent section, and it's about 1.7 times greater than the CFD calculation;
- Approaching the convergent section, the CFD solution shows higher values than the *Bartz* solution due to boundary layer growth.

⁵Computational Fluid Dynamics

⁶Since the *Prandtl number* is not unity different corrections to the *Renynolds' analogy* have been presented

⁷Ballistic Test and Evaluation System

As far as the driving force $(T_{aw} - T_w)$ is concerned, figure 1(b) shows three different ways to get the adiabatic wall temperature trend. The classic *Prandtl number correction* (dash-dot line) which consists in using a recovery factor of $Pr^{\frac{1}{3}}$ gives too high adiabatic wall temperature. This turns into a higher erosion rate, that could not be a problem during the design phase but could be undesired in preliminary performance evaluation. Anyway a difference of 20 *K* at the throat can be insignificant. The CFD solution (solid line) shows a similar behaviour in the convergent and throat region to the one obtained with the use of recovery factor, but then a change happens short after the throat. This is due to the presence of an oblique expansion wave shortly downstream of the throat. The dashed line shows the trend obtained with a less classic method. This consists in the use of a recovery factor equal to 0.9 regardless of the effective *Prandtl number*. Here this solution sticks to the one obtained from CFD until the step change of the latter (expansion wave).

With the classic method ($R = Pr^{1/3}$) temperature differences up to 90 K are reached while at the throat the spread is reduced to 19 K. These values are not so critical in the design phase. By imposing R = 0.9 the adiabatic wall temperature results more accurate than the one obtained with the previous method but in the divergent section the spread becomes negative, and this can lead to underpredict the required insulator thickness.

Results of this validation, shown in figure 1, are analogous to those reported in $[^{10}]$. The CFD trend has to be considered more accurate than the one obtained using *Bartz equation* and *Prandtl number correction* but since the differences at the throat are relatively low and the computational time is much shorter, the use of the *Bartz equation* is the best choice for the present work.

Mass injection in the boundary layer can reduce the convective heat transfer, this is known as *blowing effect*. This phenomena occurs in ablative thermal protections and it's a benign effect since it reduces the incoming heat. This effect is commonly accounted using a *Stanton number* correction, which is computed with the following equation (if the blowing parameter is set to 0.5):

$$\frac{C_H}{C_{H_1}} = \frac{\ln(1 + \dot{m}/\rho_e \, u_e \, C_{H_1})}{\dot{m}/\rho_e \, u_e \, C_{H_1}}$$

here:

- *m* is the gaseous mass being injected into the boundary layer;
- $\rho_e u_e C_{H_1}$ is the non-blowing heat transfer coefficient.

The film coefficient obtained using the Bartz equation is then corrected using the previous relation.

The study of chemical species diffusion through the boundary layer is a main matter in thermochemical erosion evaluation since diffusion regulates the amount of gaseous reactants that reach the solid thermal protection. Even if this topic seems unrelated with the convective heat transfer analysis it is introduced here for reasons that will be further revealed. There are many ways to model the mass diffusion flux, one of this is the *Fick's law*:

$$\dot{m}_{diff} = \rho D_i \nabla Y_i$$

where D_i is the species diffusion coefficient. This equation, with the hypothesis of equal diffusion coefficients at the wall becomes:

$$\dot{m}_{diff} = \rho D \frac{\partial Y_i}{\partial y} \Big|_{W}$$

Unfortunately the determination of the diffusion coefficient (D) is not immediate, it requires the use of an analogy between chemical concentration and total enthalpy profiles through the boundary layer, in order to estimate the species mass fluxes to the wall. The Reynolds' analogy between the diffusive mass flux and the convective heat flux undergoes two main hypotheses:

- *Lewis number* close to unity, $(Le \approx 1)$;
- Similitude between the species concentration and the total enthalpy profiles through the boundary layer.

Under these hypotheses the Reynolds' analogy leads to the following form of the diffusive mass flux equation:

$$\rho D \frac{\partial Y_i}{\partial y} = \frac{\alpha}{c_{p_{prop}}} \frac{T_{aw} - T_w}{T_e^\circ - T_w} \left(Y_e - Y_w \right) \tag{3}$$

If the Lewis' number is not unity the following more general equation should be used:

$$\rho D \frac{\partial Y_i}{\partial y} = \rho_e \ u_e \ C_M \left(Y_e - Y_w \right)$$

where C_M is directly linked to the Stanton number C_H , via different corrective relations.

2.3 Ablative model and in-depth thermal response

There are mainly two types of thermochemical ablation models: the ones which involves the chemical kinetics in determining the wall species concentration (*non-equilibrium ablative models*) and the ones in which wall species concentration is based on the chemical equilibrium regardless of the actual kinetics (*equilibrium ablative models*). Non-equilibrium thermochemical ablation codes are computationally slightly more complex than the equilibrium ones, but they are not so widely used since they require accurate surface chemical kinetics data, that are not so easy to get. General purpose thermochemical non-equilibrium ablation models are still under development but are thought to reduce the intrinsic overestimation of the erosion rate obtained with the equilibrium codes.

However, when a non-charring ablative material is considered, the absence of the pyrolysis gas reduces considerably the amount of chemical kinetic data required. So since in this work a graphite thermal protection has been studied (which is non-charring) a non-equilibrium ablation model has been adopted.



Figure 2: Mass and energy fluxes across the control volumes considered.

In non-charring ablative materials erosion occurs because the free stream reacting species cross the boundary layer, reach the surface and there they react with the ablative material. This phenomenon is described by the species surface mass balance. Let's consider a movable control volume tied to the receding surface and with a small extent to the gas and solid sides as shown in figure 2(a). The i^{th} species mass fluxes balance in such control volume is represented by the following equation:

$$\rho D \left. \frac{\partial Y_i}{\partial y} \right|_w = (\rho \, v)_w \, Y_{w_i} + \sum_{reac} \dot{\omega}_i \tag{4}$$

where:

- $\rho D \left. \frac{\partial Y_i}{\partial y} \right|_{w}$ is the species diffusive mass flux at the wall;
- $(\rho v)_w$ is a "blowing" term caused by the surface recession. Assuming that all the base material turns into gaseous reaction products, so that no mechanical erosion or surface melting occurs, this term becomes $(\rho \dot{e})$ which is also equal to the mass flux of consumed base material (\dot{m}_c) ;
- $\sum_{reac} \dot{\omega}_i$ is the overall species reaction rate due to surface heterogeneous reaction with the ablative material. The erosion rate is directly related to these quantities via the stoichiometric coefficients and molecular weight ratios.

The heterogeneous surface reactions considered in this model are the same adopted in other publications ($[^{12}], [^{13}]$ and $[^{14}]$) and are the following:

$$C_{s} + H_{2}O \rightarrow CO + H_{2}$$

$$C_{s} + CO_{2} \rightarrow 2 CO$$

$$C_{s} + OH \rightarrow CO + H$$

$$C_{s} + O \rightarrow CO$$

$$2 C_{s} + O_{2} \rightarrow 2 CO$$

The relative kinetic data and models were taken from [¹³]. For the first four reactions the *Arrhenius equation* was used to obtain the surface reaction rates (\dot{m}_i) whilst for the fifth one the *Nagle & Strickland model* was adopted.

If no mechanical erosion or liquid phase removal occurs then the surface reaction rate is directly related to the erosion rate:

$$\dot{m}_c = \dot{m}_{H_2O} + \dot{m}_{CO_2} + \dot{m}_{OH} + \dot{m}_O + \dot{m}_{O_2} = \rho_c \dot{e}$$

While the species reaction rates $\dot{\omega}_i$ that appear in the SMB are obtainable using the following relation:

$$\dot{\omega}_i = \frac{M_{m,i}}{M_{m,c}} \frac{\nu_i}{\nu_c} \dot{m}_i$$

where $M_{m,i}$ and $M_{m,c}$ are molecular weights of the *i*th reactive species and carbon respectively; v_i and v_c are the relative reaction stoichiometric coefficients. Then for given combustion chamber conditions and local static pressure, the surface mass balance comes out to be a function of the wall temperature and the surface chemical composition. Hence the surface mass balance set of N equations, where N is the number of chemical species considered, contains N + 1 unknown variables: N wall species concentrations and the wall temperature. The missing equation is the energy balance at the surface.

Figure 2(b) shows the equilibrium between heat fluxes across the same movable control volume in figure 2(a). The surface heat balance is then:

$$q_{w,conv} + q_{rad} = \sum_{j}^{N_{prod}} \dot{m}_{tot,j} h_j - \sum_{i}^{N_{reag}} \dot{m}_{tot,i} h_i + q_{cond}$$
(5)

where the involved heat fluxes are:

- The wall convective heat flux *q_{w,conv}*;
- q_{rad} the overall radiative heat flux, which is the sum of heat from the semitransparent gas and the emitted flux from the nozzle wall (here neglected);
- q_{cond} is the conductive heat through the base material;
- $\sum_{j}^{N_{prod}} \dot{m}_{tot,j} h_j \sum_{i}^{N_{reag}} \dot{m}_{tot,i} h_i$ is the enthalpy difference between the gaseous products of the surface heterogeneous reactions and their reactants which are both gasses from the free-stream and solid from the nozzle ablative material. This term is often referred to as chemical heat flux q_{chem} .

While convective and radiative heat fluxes have been introduced previously, the thermal conductivity has not been analyzed. In a complete procedure the conductive heat flux value is provided by an in-depth solution calculator which in turn is coupled with the ablation model. In this way it's possible to take into account how much the transient heat conduction and the storage of sensible energy phenomena do influence the surface erosion rate through the nozzle operating time, in particular during the warm up phase. With charring ablative materials the use of this procedure is almost inevitable since other effects occur, but in this work the *heat sink* effect of the non-charring ablative materials under analysis is neglected and the steady state conductive heat flux is used, so to reduce the overall computational time.

The steady state in-depth thermal profile of a solid material with constant thermal conductivity λ , density ρ , specific heat capacity c_p and with a receding surface is described by the following equation:

$$T = T_o + (T_w - T_o) \exp\left(-\frac{\dot{e}}{\alpha}x\right)$$

where T_o is the initial (before ignition) temperature, x is the depth with reference to the movable coordinate system, \dot{e} is the surface erosion and $\alpha = \lambda/(\rho c_p)$ is the thermal diffusivity. Hence the steady state conductive heat flux at the wall is:

$$q_{cond} = -\lambda \left. \frac{\partial T}{\partial x} \right|_{w} = \lambda \frac{\dot{e}}{\alpha} \left(T_{w} - T_{o} \right) = \rho c_{p} \dot{e} \left(T_{w} - T_{o} \right)$$

It must be noted that the quantity $\rho c_p \dot{e} (T_w - T_o)$ is the solid ablative material mass flux $\rho \dot{e}$ multiplied by the enthalpy change from the initial condition to the wall condition $c_p (T_w - T_o)$. This make perfectly sense from the energy point of view.

The last energy term in the steady state surface heat balance (eq. 5) is the chemical heat flux q_{chem} which is the overall chemical energy difference in the control volume, which is also known as heat of ablation since this is the heat

mitigated by the ablation phenomena. In order to get the enthalpies differences and the related mass fluxes the chemical heat flux can be rearranged as the heat obtained from the chemical reactions that take place at the surface. Here this heat was computed referring to the standard reference state ($T^{\circ} = 298.15 \text{ K}$). The *standard heats of reaction* were computed by the heats of formation always available in the NIST database [⁵].

The complete equation set that describes the ablation phenomena is composed by N species surface mass balance and one surface heat balance. The system unknowns are the N considered species concentrations and the operative wall temperature. It's composed by non-linear equations and often the unknown variables are implicitly defined, so this requires the use of numerical methods for the solution of systems of non-linear equations. The *MATLAB*[®] *function* fsolve, developed by *Mathworks*TM, solves such systems, but unfortunately not this system, even using initial conditions close to the right solution, hence an *ad hoc* solver was created.

The key points in developing the solver are the following facts:

- For a given wall temperature the SMB equation set is easier to solve than the complete system.
- In particular if the "blowing" term of the SMBs are neglected the system results uncoupled;
- The SHB could be easily solved if the wall species concentrations were known, this also means that at a given temperature the deviation from zero of the SHB could be evaluated;
- Once an erosion rate is obtained the film coefficient must be updated accordingly to the *blowing correction*.

Noted this, a step solution of the system, based on the assumption of the wall temperature, was adopted.

The SMB equation set has been solved using a bisection method, which is a functions solver and not a systems solver. In order to use this method, which always grantees the solution, at the first iteration the blowing term in equation 4 was considered equal to zero in order to uncouple the SMB equation system, and in the following iterations the overall surface reaction rate estimated in the previous iteration is considered.

The SHB is solved along with the complete system using the following algorithm:

- 1. Two boundary wall temperature values, are initially assumed to be 300K and 3000K. Then a middle value is computed;
- 2. For these three wall temperature values the relative SMB system of equation is individually solved in order to get the related wall concentrations;
- 3. The deviation of the SHB is evaluated in the three points using the new set of values;
- 4. Based on the deviations obtained, two new wall temperatures boundary values are determined and used in point 1 until the convergence is reached.

With this algorithm it's possible to obtain the wall concentrations and temperature, the convergence is granted since it's based on the bisection method. Once the wall temperature and species concentrations are obtained, the surface reaction rate is once again computed and then the blowing correction parameter can be calculated. With the new convective heat transfer coefficient the procedure to solve the complete system is restarted since the blowing correction variation is within a certain tolerance. Only then the effective erosion rate that accounts for the blowing effect is obtained.

To conclude, this aerothermal analysis for a steady state response of the ablative thermal protection leads to estimate the throat erosion rate and wall temperature using the following main inputs:

- Propellant formulation, that is the kind of fuels and oxidizer used;
- The relative ratio between oxidizer and fuel involved in the combustion (o/f);
- Combustion chamber pressure, *p_{cc}*;
- Nozzle throat diameter, D_{th} ;
- Ablative material density, ρ_c .

So we obtained a $MATLAB^{\mathbb{R}}$ solver for the following function:

 $\dot{e} = \dot{e} \left(p_{cc}, \ o/f, \ D_{th}, \ \rho_c \right)$

3. Validation and Results

In order to understand whether the developed function correctly describes the ablation phenomena, it has been subjected to a series of validation analyses. Most validations involved a comparison with the data in the paper published by *D*. *Bianchi and F. Nasuti* in [¹²]. *D. Bianchi and F. Nasuti* applied the non-equilibrium ablative model previously presented as an ablative boundary condition in a CFD analysis. This surely grants a more accurate prediction of the erosion rate since the method involves less assumptions. In [¹²] the BATES nozzle configuration is adopted as case study. This nozzle configuration is the one adopted to evaluate the main differences between the *Bartz equation* and a CFD analysis in evaluating the convective heat flux (fig.1). Two validation were carried out, one in order to validate the method for one single operating point and the other for different oxidizer to fuel ratios.

3.1 First Validation

The first validation consisted in determining the erosion rate for a given propellant formulation and then comparing it with the one reported in reference [¹²]. The erosion rate was computed for three different propellant formulations resulting from the reaction between *paraf finwax* fuel and 90% H_2O_2 , N_2O and LOX as oxidizer. The relative combustion chamber condition (composition and temperature) were obtained from table 3 in [¹²]. With these data a film coefficient has been calculated at the throat using the *Bartz equation*, with a recovery factor of 0.9 for the adiabatic wall temperature. A density of 1830 kg/m^3 was used for the bulk graphite.

Then the *ablation MATLAB*[®] *function* was used to calculate the erosion and the wall temperature and these were compared with the one reported in the reference. Validation results and comparison are presented in figure 3 and table 1.



Figure 3: Bar diagrams of the thermochemical erosion for the three considered propellant formulations

Bar diagrams that describe the individual species contributions to the total throat erosion rates are reported in figure 3. Every bar diagram consist of three columns (bars), the first one reports the calculated results obtained using a unity recovery factor, the second one shows the calculations made with a recovery factor equal to 0.9 and the third bar is the one reported in the reference, which calculation involved the use of a more accurate CFD computer program. The first thing that catches the eyes is that the difference between the R = 0.9 and R = 1 is very small compared to the difference between the computed results and the ones reported in the reference. The second thing that can be noted is that in most cases the differences between the calculation and the reported results are due to the behaviour of the O_2 and H_2O which are the most influencing species. In fact H_2O gives the highest contribution to the erosion allowing to mitigate part of the incoming heat since the related heterogeneous reaction is highly endothermic. While O_2 , which is normally present in higher concentrations than the atomic oxygen, reduces the effectiveness of H_2O and other species giving an exothermic contribution to the overall endothermic wall reaction. That's why the *WAX*-LOX propellant formulation gives a higher erosion than the others: the residual O_2 free-stream concentration, together with the flame temperature, is higher.

In table 1 the numerical value of the erosions shown in figure 3 are reported. Here the quantitative evaluation of the model accuracy can be made. The average relative difference between the calculation and the paper results is 3.46%, which is quite good considering the discrepancies previously detected in the results. Also the wall temperature results to be accurate, the average difference is 22 K relatively speaking less than 1%.

Concluding, given the combustion chamber conditions reported in $[1^2]$, the erosion rate estimates computed using the developed *ablation MATLAB*[®] *function* are really close to the results reported in the reference paper $[1^2]$,

	H_2O_2		N_2O		LOX	
	function	paper	function	paper	function	paper
H_2O	52.8	49.9	36.4	35.6	61.2	61.6
CO_2	0.54	0.51	3.15	2.41	2.52	2.6
ОН	14.4	15.3	10.4	10.6	27.1	28.9
0	1.65	1.6	3.81	4.03	11.7	12.8
O_2	3.77	5.4	6.5	3.36	15.8	9.1
ė [µm/s]	73.2	73	60.2	56	118	115
$T_w [K]$	2183	2191	2500	2478	2405	2441

Table 1: Thermochemical erosion rates during the first validation

nonetheless some discrepancies were found in the individual reactive species contributions, but these are small if compared with the differences between the methods used to determine the throat erosion rates.

3.2 Second Validation

Since the accuracy of the *ablation MATLAB*[®] *function* has been shown for different oxidizers but single operating conditions, the next validation step is to demonstrate the accuracy with variable combustion chamber conditions, in particular varying the oxidizer to fuel ratio. This validation was done for the same three oxidizers discussed in the first validation (H_2O_2 , N_2O and LOX) but the fuel used this time is HTPB, in order to compare, once again, the results with the trends reported in figure 9 of reference [¹²], where the trends of erosion rates for different equivalence ratios are shown. In order to get the variable combustion chamber conditions, a set of CEA based look-up table were used.



Figure 4: Calculated erosion rate trends due to a variation of the oxidizer to fuel ratio for the three different propellant formulations

In figure 4, the erosion rate trends for a variable o/f is shown. The obtained results are similar to the one reported in figure 9 of reference [¹²]. Let's consider the differences between these comparable trends:

- The calculated erosion rate trends are slightly higher than the ones reported in the reference paper;
- For the HTPB- H_2O_2 propellant formulation, the maximum erosion rate occurs at an oxidizer rich mixture just like in the reference, both maximums are close to $\phi = 0.9$. The calculated maximum erosion relative error is close to 8%. For $\phi = 0.5$ a value of $29\frac{\mu m}{s}$ is calculated while the reported one is approximately $27\frac{\mu m}{s}$ which corresponds to a 7.4% over-predictive error. For $\phi = 2$ a value of $18\frac{\mu m}{s}$ is calculated while the reported one is approximately 17 $\frac{\mu m}{s}$ which corresponds to a 6% over-predictive error;

- For the HTPB- N_2O propellant formulation, the maximum erosion rate occurs for the same o/f reported in figure 9 of reference [¹²], close to the stoichiometric one, a relative error lower than the 6% is detected for the maximum erosion rate values. For $\phi = 2$ a value of $27\frac{\mu m}{s}$ is calculated while the reported one is approximately $25\frac{\mu m}{s}$ which corresponds to a 8% over-predictive error. For $\phi = 0.5$ a value of $46\frac{\mu m}{s}$ is calculated while the reported one is approximately $43\frac{\mu m}{s}$ which corresponds to a 7% over-predictive error;
- For the HTPB-LOX, the calculated maximum throat erosion rate seems to occur at a slightly higher o/f than the one reported in figure 9 of reference [¹²], a value of $111\frac{\mu m}{s}$ is calculated, while the reported one is approximately $107\frac{\mu m}{s}$, leading to a relative error lower than 4%. For $\phi = 0.5$ a value of $104\frac{\mu m}{s}$ is calculated while the reported one is approximately $98\frac{\mu m}{s}$ which corresponds to a 6% over-predictive error. For $\phi = 2$ a value of $46\frac{\mu m}{s}$ is calculated while the reported one is approximately $42\frac{\mu m}{s}$ which corresponds to a 9.5% over-predictive error.

Other results obtained during this validation were the concentration trends of the chemical species involved in the ablation mechanism. Figure 5 reports the trends for H_2O and O_2 which are the two major contributors to the erosion. Here the diffusion limited or kinetics limited behaviour of the species diffusion mechanisms can be observed, in particular the molecular oxygen results to be kinetic limited since the free-stream concentration is close to the wall one. The H_2O on the other hand is diffusion limited for the HTPB- N_2O and HTPB- O_2 propellant formulations and kinetic limited for HTPB- H_2O_2 where the free stream amount of H_2O is quite high.



Figure 5: Computed free-stream and wall concentrations for H_2O and O_2 chemical species

3.3 Results

The developed *ablation MATLAB*[®] *function* results to be slightly less accurate than the CFD based model presented by *D. Bianchi and F. Nasuti* in [¹²], but it is surely faster. These two characteristics fit well with preliminary design throat erosion rate evaluation, in particular for the evaluation of the performance variation due to throat area increase and a thermal protection preliminary design. The computational time required to evaluate the erosion rate for a given set of variables (p_{cc} , o/f, D_{th} , ρ_c) is about fifteen seconds, which is incredibly lower than the time required for a CFD analysis, but if introduced in a preliminary rocket design iterative procedure could rise intensively the overall computational time.

The most general and valid procedure is the creation of a look-up table, whose data can be interpolated in different ways, or a dimensional analysis and the definition of dimensionless groups like in [¹⁷]. In traditional erosion evaluation methods, used in SRM environments, this is achieved by scaling experimental data using simply : $\dot{e} = K \frac{p_{cc}^{0.8}}{\rho_c D_{th}^{0.2}}$ which involves a proportional dependence of the erosion rate with the convective heat flux. For HRM obtaining such a simple scaling equation is quite more complicated because of the contribution due to o/f variations. Anyway it is possible to obtain a simple interpolating function for a given oxidizer to fuel ratio. Considering for example a 90% H_2O_2 -paraffin propellant combination operating near optimum conditions (o/f = 7.35) a possible interpolating function is:

$$\dot{e} = 1.0732 \cdot 10^{-6} \frac{p_{cc}^{0.795}}{\rho_c^{1.01} D_{th}^{0.16}} \tag{6}$$

This function has been obtained by considering the individual contribution to the erosion for the p_{cc} , D_{th} and ρ_c independent variables. Even if no combined contributions were considered, the obtained interpolating function does not lead to excessive errors. The nominal values⁸ for the combustion chamber pressure, throat diameter and material density are 15 *bar*, 0.1 *m* and 1900 kg/m^3 respectively, these values then were varied from [5bar 50bar], [1cm 1m] and [1500kg/m³ 2200kg/m³] in evaluating the relative effects. In this ranges, considering the stoichiometric oxidizer to fuel ratio, function 6 shown to fit the data obtained from the *ablation MATLAB*[®] function with a relative error lower than 2%. It was also shown that if the proportionality constant of equation was assumed to vary following this relation $K(o/f) = -2.264 \ 10^{-6} + 8.802 \ 10^{-7} \cdot o/f - 6.830 \ 10^{-8} \cdot o/f^2 + 1.388 \ 10^{-9} \cdot o/f^3$, with $o/f \in [3.7 \ 11]$ and if the same exponents were kept, then such a function would differ at most⁹ of 12% from the data obtained with the *MATLAB*[®] function, which could be acceptable.

Then the ablative $MATLAB^{\mbox{\ensuremath{\mathbb{R}}}}$ function has been coupled with a 0-Dimensional model, as the one reported in [¹⁸], of a hybrid motor for sounding rocket applications with an initial throat diameter of 36mm in order to estimate the effects of the throat erosion on the motor performance. These particular effects are most evident in small motors with high characteristic velocity and long burning time as shown in figure 6 where the combustion chamber pressure drop for the sounding rocket is reported. The thrust and specific impulse penalty can be also easily calculated.



Figure 6: Combustion chamber drop in a sounding rocket hybrid motor due to throat erosion

4. Conclusion

In this work a numerical method to predict the throat erosion rate in hybrid rocket motors was implemented. This method involves the use of a non-equilibrium thermochemical ablation model, which turned particularly useful in obtaining an accurate estimate of the throat erosion. This model was limited to the modelling of graphite ablative materials, which are quite diffused in the manufacturing of throat inserts. In order to further reduce the computational time required by the numerical method a simple interpolating function is determined for different propellant formulations. Finally the look-up table can be used in lumped-parameters models in order to predict hybrid global performances.

⁸Here nominal values means the values used in obtaining equation 6

⁹This value was obtained for a throat diameter of 1 meter and a pressure of 50 bar, i.e. boundary values for the considered ranges

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