Numerical investigation of regular and Mach reflections at the expiration of the gas jet from the nozzle

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Abstract

A numerical investigation of a structure of the supersonic jet at the nozzle exit, in dependence on the Mach number and the specific heats ratio is presented. The simulations have been performed on a two-dimensional unstructured polyhedral mesh with smoothing at the nozzle walls and adaptation to the shock wave. The structures of jets corresponding to a regular and Mach reflection have been derived for the different specific heat ratio. These calculations can serve as a basis for the study of a new type of the configuration (irregular reflection with a negative angle) at the exit of the jet from the nozzle.

1. Introduction

The study of supersonic jets is important in many engineering applications. Issuing of the supersonic jet from the nozzle may occur in overexpanded mode, when the pressure in the gas flow from the nozzle outlet is less than the ambient pressure. In a planar overexpanded nozzle flow two oblique shocks are created that start at the nozzle lips and are directed towards the symmetry plane. These incident shocks can reflect either the regular reflection or the Mach reflection [1]. These types of reflection are shown in Fig. 1. At Mach reflection triple shock configuration appears, with three shocks and a slipstream.

Location of shocks in triple-shock configurations, and their intensity depend on the Mach number M_1 of the gas issuing from the nozzle, the initial angle of incidence ω_1 and the ratio of specific heats γ. For the calculation of the triple shock configuration at Mach reflection three-shock theory may be used [2].

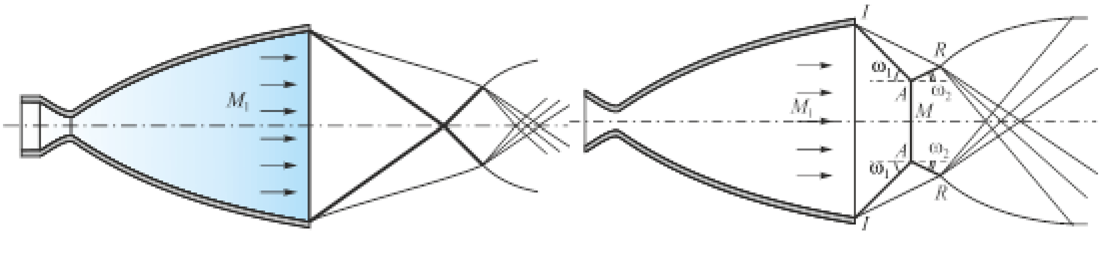


Figure 1. Supersonic nozzle flow. Left: regular reflection, right: Mach reflection. IA - incident wave. AR - reflected wave. AM - Mach wave. RF – rarefaction fan. ω1 - angle of incidence. ω2 - angle of reflection, A - triple point, M1 - Mach number.

The triple-wave configurations can be calculated independently of boundary problems of their origin using the triple-shock theory [3-7]. Let us consider a plane supersonic flow with the gas specific heats ratio γ and the Mach number M1 (Fig. 2) It is assumed that a triple-shock system exists and transforms a uniform flow into two flows – the first of them propagates through incident (IA) and reflected (AR) shocks, and the second of them crosses Mach stem AM. Those flows are divided by the contact discontinuity (the slipstream) AT. It is also assumed that the conservation laws for shock waves (Hugoniot conditions) are fulfilled at each of the discontinuities; the consistency conditions are fulfilled at the slipstream (a flow propagating through an incident wave and a reflected wave is parallel to a flow downstream, through a Mach wave; pressure values at the both sides of the slipstream are equal).

Shock strengths and slope angles in perfect gas depend on the Mach number M1 of an incoming flow, the incident shock inclination angle $\omega 1$ and the ratio γ of gas specific heats.

It is generally acknowledged in the problems of a shock reflection that the configurations presented in Fig. 2a (type II according to the classification in [6]) is typical. This conclusion is based on the calculations which deal with the specific heat ratio 1.4 primarily, because the steady reflections are usually studied experimentally in wind tunnels where a working gas is an air. The figures 1a show the example of such flow. In both triple configurations shown in Fig. 1 the reflected wave is situated in the same half-plane that the incident wave. Further we call such configuration “triple configuration with positive angle of reflection”.

Some dependencies of shock disposition in a triple configurations on the values of $\omega 1$ and M1, and the specific heats ratio γ have been studied recently in [8-10,19]. New triple configuration with a negative reflection angle has been found (see Fig. 2b). Its distinctive feature is that the reflected wave is situated below the initial flow line (angle $\omega 2$ is negative).

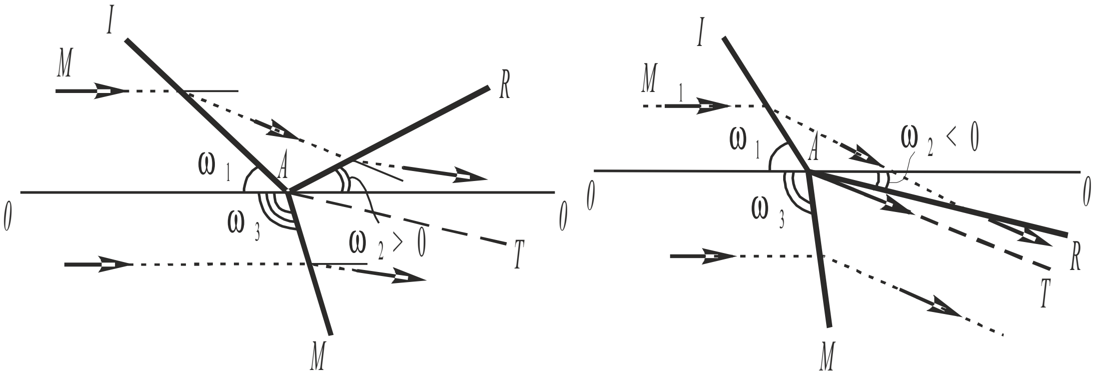


Figure 2. Scheme of three-shock configuration: Left at $\omega_2 > 0$, right at $\omega_2 < 0$.

To define the region of flow regimes where triple configurations with negative reflection angles are generated, the analytical method of shock polars has been used

. Fig. 3 presents the boundaries of the configuration with a negative reflection angle ($\omega 2=0$) in (M1, $\omega 1$) coordinates for different values of the specific heats ratio. The negative reflection angles are formed inside the regions bordered by those curves.

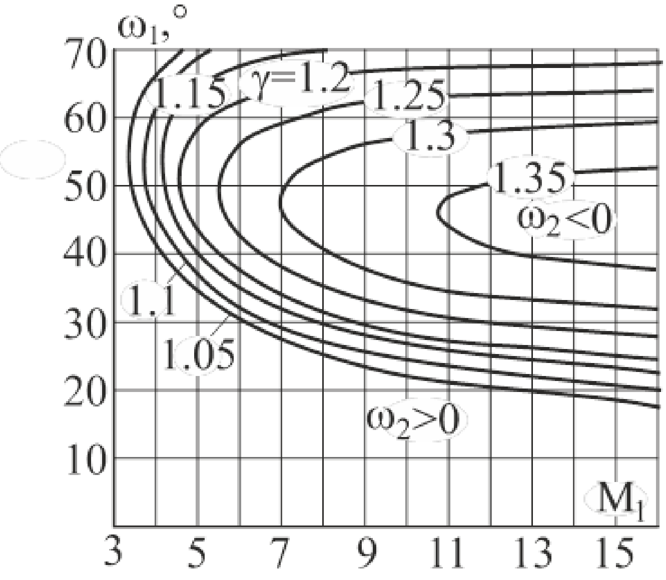


Figure 3. Boundaries of triple configurations with negative angle ($\omega 2=0$) in (M1, $\omega 1$) coordinates and various γ’s. Angle $\omega 2$ is negative in the regions inside the corresponding curves

Thus, boundaries and regions of the new configuration with negative reflection angle are found. The configuration with a negative reflection angle is formed when Mach number M1 of the undisturbed flow exceeds 3 and the specific heats ratio is less than 1.4. The effect of the value of γ value on the regular/Mach reflection (RR/MR) transition criteria can be also found and regions of double-shock and triple-shock configurations can be constructed as a function of the specific heats ratio and flow Mach numbers.

Let us superimpose the RR/MR transition boundaries and the boundary ω_G of the negative reflection angle configuration. Fig. 4 illustrates the complete pattern of all types of reflections for γ=1.2. The diagram includes all three boundaries of existence: regular reflection (*AB)*, irregular (*CD)* and the configuration with a negative reflection angle (*EGH)*. The picture reveals that, except for the dual solution region *AGHCD*, where both Mach and regular reflections are possible, there is a new dual solution region *BGH*. Both RR and MR with a negative reflection angle can exist there. Note, that only one region of double solution exists at γ≥1.4 because the negative reflection angle configuration is not possible then.

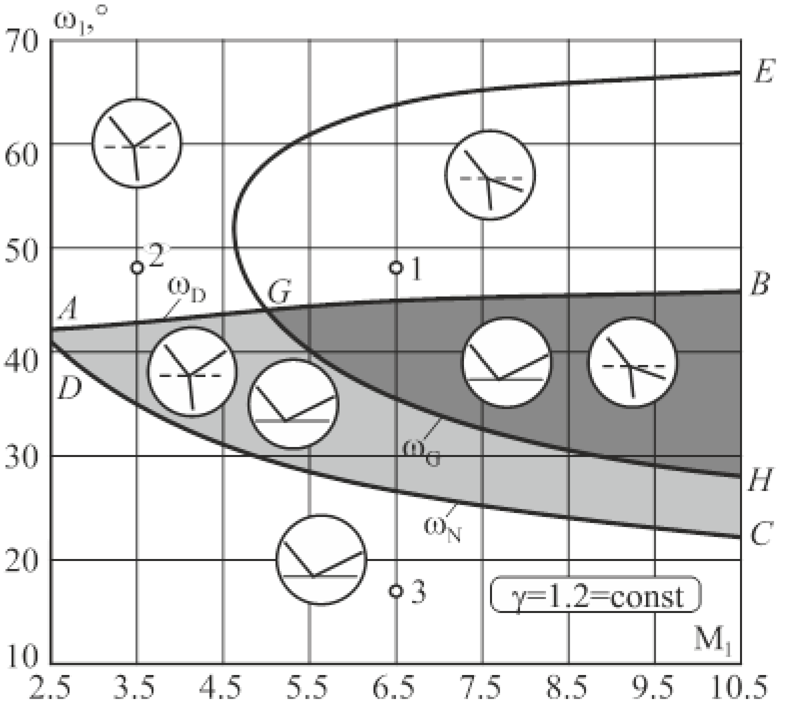
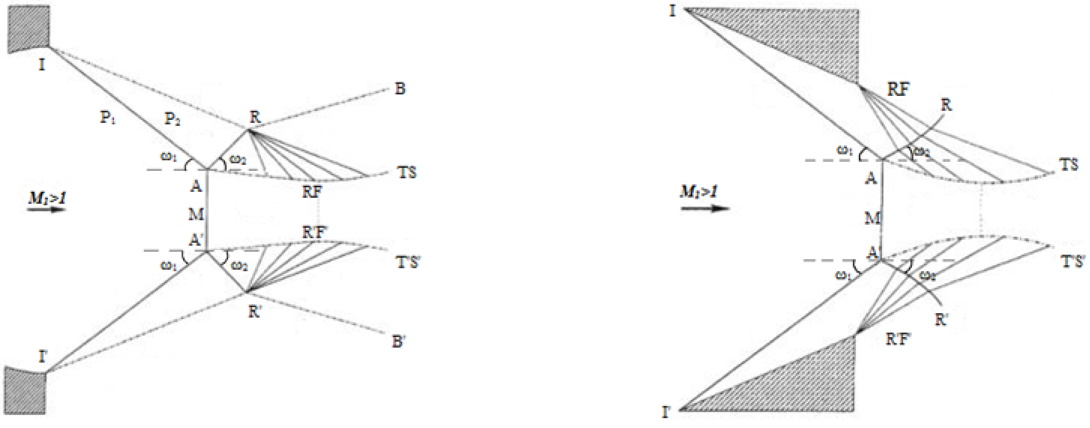


Figure 4. Transition angles ωN, ωD, and ωG for γ=1.2. AGHCD is the dual reflection solution domain (regular and irregular reflections are both possible). GBH is the region where both RR and MR with negative reflection angle can exist

A problem of shock wave reflection from a symmetry plane at the interaction of oblique shocks generated by two symmetrical wedges in a supersonic flow (Fig. 5b) was assumed as a model problem. Calculations were performed at a different values of the wedges angles (ω1=10-50º) and for a wide range of flow parameters (M1=3-10, γ=1.1-1.4). The numerical study has revealed that there is no steady solution in region EGB. An unstable configuration climbing upstream the flow is formed. Its form depends on the previous type of reflection.



a) b)

Figure 5: a) overexpanded jet flow from the plane nozzle and formation of the triple-shock configuration; b) plane flow at the between two wedges modeling the supersonic inlet entrance and formation of the triple configuration at the channel. IA – incident wave, AR – reflected wave, ATS – slipstream, RF – rarefaction fan, IRB – jet boundaries.

At the continuous variation of flow parameters from the regular reflection, a configuration with the second triple point on the reflected wave appears (so-called double Mach reflection, DMR). The reflected wave should intersect the symmetry line at the very beginning of the transition to DMR, so this configuration is definitely unstable. The triple point climbs the flow. At the transition from regular reflection the DMR leads to the flow steadiness destruction. DMR was recorded earlier in quasi-stationary reflection from plane wedges only. It was stated that this type of reflection can not be realized in steady flows [6,12-15]. However it has been shown that the transition to the flow parameters corresponding to triple configurations with negative reflection angles at a flow around two wedges leads to the disappearance of the steady numerical solution and to the oscillations of the shock pattern of the flow.

In the present paper the boundaries of the triple-shock configurations with negative reflect angle have been found for the different parameters: the Mach number, the specific heats ratio and the nozzle pressure ratio (NPR).It seems that this effect also can be realized in a supersonic jet out a nozzle (Fig. 5a) due to a low value of the specific heats ratio of conventional fuels. Fig. 5a and Fig. 5b show that there is a similarity of shock patterns between a flow in an air inlet and a flow in an overexpanded supersonic jet. It has been shown that if a reflected shock in an air inlet directed at negative angle ω2 intersects symmetry line OO1 a gas is accumulated in an enveloped region ARM. The flow is closed in this region and the Mach wave begins to climb upstream the flow, an unsteady double Mach reflection would be formed. The similar process is anticipated in an overexpanded jet with a negative angle of reflected shock (Fig. 6). Supersonic gas flows should collide in the closed region behind the reflected shocks AR and A’R. Vortices would form in ARA’ region, and the Mach stem would climb upstream the flow up to the nozzle entrance. This can might cause the flow separation in the nozzle and start a self-oscillation process. Thus, the results obtained can be useful for the prediction of in-flight accidents in supersonic jet propulsion engines as well as in the atmospheres of other planets.

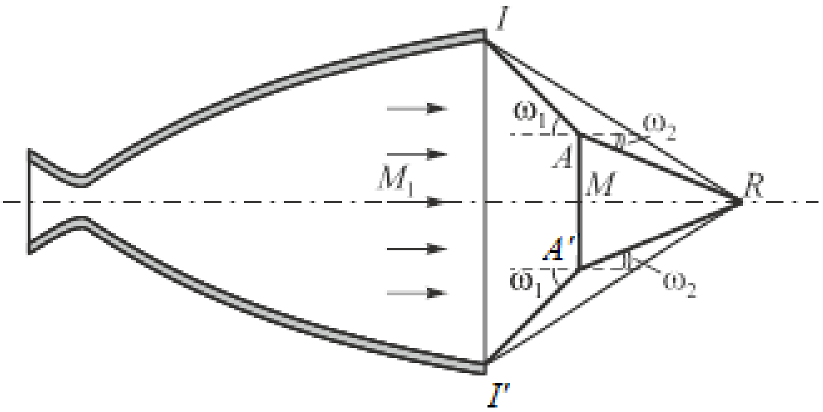


Figure 6. Triple configuration with negative reflection angle (ω2<0) in an overexpanded planejet flow. IA, IA’ – incident shock waves, AA’ – Mach bridge, AR, A’R – reflected wave. AT, A’T – slipstreams.

The main goal of the present study is to develop a numerical method for investigation of the jet flow in the domain in different initial conditions to predict emergency situations at rocket engine operation. In this paper, in the framework of numerical simulation, the structure of the jet issuing from the tapered (constant angle) supersonic nozzle has been investigated for different values of NPR (the ratio between the stagnation pressure in the inlet and the ambient pressure) and different values of γ.

2. Geometry and mesh model

As the object of study a tapered supersonic nozzle is considered. The geometric parameters are presented in table №1. The software for solid modelling SOLIDWORKS is used to construct a solid model of the nozzle and the surrounding area. The computational domain is chosen from the condition that the distance from the nozzle exit to the exit boundary equals to not less than 10 diameters of the output section. Figure 7 presents a solid model of a part of the computational domain. The resulting geometrical model is imported into the software package STAR-CCM +, in which the mesh model was established and further calculations were carried out. To split the geometry on the finite volumes the unstructured polyhedral mesh has been used with smoothing at the solid walls. The advantages of polyhedral cell type over tetra or even to structured hexagonal cells are described in detail [16]. Note that the polyhedral cells allow for the same amount of cells to describe better the gradients than tetra and structured hexa-cells. This is particularly important in the study of supersonic gas flow, which often has to deal with large gradients (shock waves, rarefaction wave, etc.). It should also be noted that the use of polyhedral cells reduces the time for the calculations due to the more rapid achievement of the convergence.

Table 1: Parameters of nozzle

Parameter	**Value**
Throatradius, D_t	16 mm
Area ratio of the exit section of nozzle to the throat A_e/A_t	8
Angle of the nozzle	10^0
Length of the computational domain	2400 mm
Height of the computational domain	1600 mm
Radius of curvature of the convergent part	24 mm

The turbulent flow is modeled in the present study. In the numerical study of turbulent flows it is important to create on the surface of wall not only small but also quite uniform grid with a little stretching in height. This is called the creating the prismatic layer of cells. The size of the first layer is selected basing on the used turbulence model. In this case, a model SST was chosen and y + for this model should not exceed 5. Figure 8 present fragments of the nozzle grid pattern, and the detail of the prismatic layer of the wall.

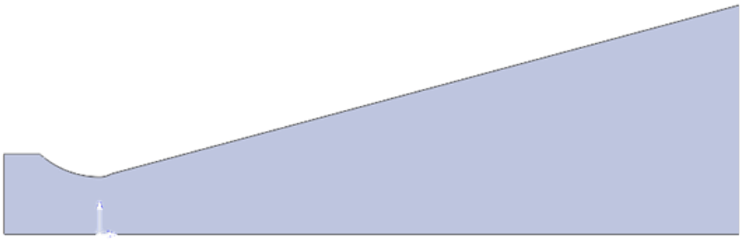


Figure 7. Geometry model of a fragment of the nozzle

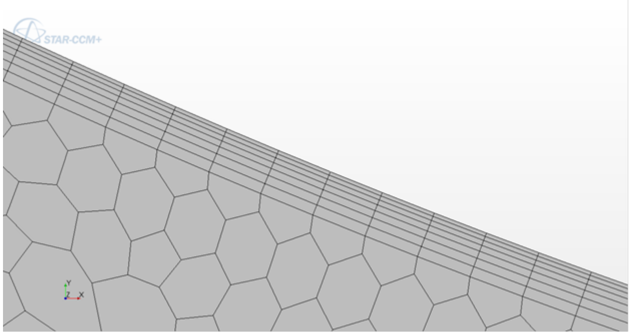


Figure 8. Fragment of the mesh model near a wall

3. Numerical model

A system of Navier-Stokes equations, Reynolds averaged, and the energy equation is used to describe the dynamics of a viscous turbulent gas flow in the nozzle. This system has the following form [17]:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_i}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial \rho \overline{u_i}}{\partial t}+\frac{\partial \overline{u_i u_j}}{\partial x_j}=-\frac{\partial p}{\partial x_i}-\frac{\partial}{\partial x_j}\left(\mu \frac{\partial \overline{u_i}}{\partial x_i}+\frac{\partial \rho \overline{u_i' u_j'}}{\partial x_j}\right) \tag{2}$$

$$\frac{\partial \rho c_P T}{\partial t}+\frac{\partial \rho u_i c_P T}{\partial x_j}=\frac{\partial q_i}{\partial x_i}+\frac{\partial \tau_{ij} u_i}{\partial x_j} \tag{3}$$

Where ρ - density, T –temperature, u_i - the i-th component of the velocity, i = 1,2,3, x_i - coordinates, Cp - specific heat at constant pressure, τ_{ij} - viscous stress tensor, q_i - heat flux due to the thermal conductivity λ. Expression ($\rho u_i u_j$) is called the Reynolds stress tensor, which is used to close the turbulence model SST, the equation for which are given by [11]:

$$\frac{\partial \rho k}{\partial t}+\overline{u}_j \frac{\partial \rho k}{\partial x_j}=\tau_{ij}\frac{\partial \overline{u_i}}{\partial x_j}-\beta^* \rho k \omega+\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{4}$$

$$\frac{\partial \rho \omega}{\partial t}+\overline{u}_j \frac{\partial \rho \omega}{\partial x_j}=\gamma \frac{\omega}{k}\tau_{ij}\frac{\partial \overline{u_i}}{\partial x_j}-\beta^* \rho \omega^2+\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma_\omega \mu_t\right)\frac{\partial \omega}{\partial x_j}\right]+ \tag{5}$$

$$+2\left(1-F_1\right)\rho \sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

Where k - kinetic energy of turbulent fluctuations, ω - specific energy dissipation rate, β, β *, σ_k, σ_ω – turbulence constants. Turbulent viscosity and Reynolds stresses are determined similarly as in the k-ω model, namely:

$$\mu_t=\rho \frac{k}{\omega} \tag{6}$$

$$\tau_{ij}=-\rho \overline{u_i' u_j'}$$

To close the above system of equations it is necessary to determine the condition of uniqueness. In this problem, the following boundary conditions are used (Table 2):

Table 2: Boundary conditions

Type of boundary conditions	**Value**
At the nozzle inlet – stagnation parameters	P*=50 kPa, T*=200 K
At the Symmetry Plane	$U_n = 0$ $U_\tau = \text{var}$
At the exit – Static Pressure	P=constant
On the wall – no slip	$\vec{U} = 0$

Equations (1) - (6), together with the boundary conditions form a closed system which can be solved by numerical methods.

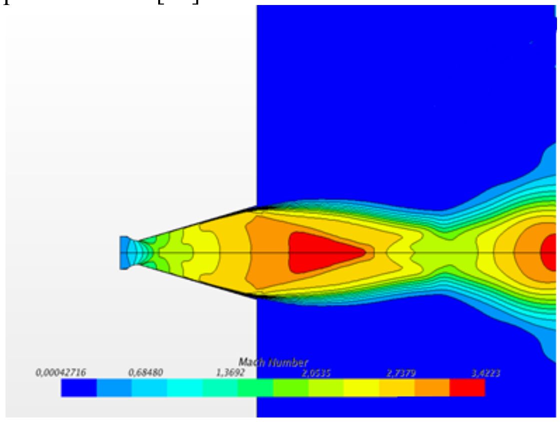
4. Results

The perfect gas is used for the modeling the working fluid with the equation of state:

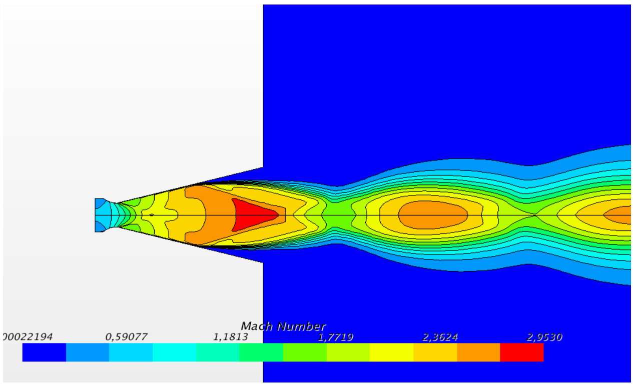
$$P = \rho RT \tag{7}$$

In this case, to assess the impact on the structure of the adiabatic index the studiers have been made for two values of γ – 1.4 and 1.2.

Fig. 9 (a-c) show the calculated jet with γ = 1.4. At NPR = 20 this structure is characteristic of regular reflection. The point of intersection is disposed at certain distance from the outlet of the nozzle. When NPR reducing to 10 the structure is resembled the regular reflection, however, the point of intersection moves inside the nozzle, because there is a small flow separation. With further decrease of the number of NPR to 5, the numbers of "diamonds" inside the nozzle increase. The subsonic region separating the incident wave from the "diamonds" appears. As NPR reduces, the subsonic region is close to the point of reflection. The separation zone is greatly increased. This mode is preceded the formation of Mach reflection. Note that usually in ideal nozzles Mach reflection should appear after regular reflection before the flow separation. The observed inverse sequence is typical for the tapered nozzles [18].



a)



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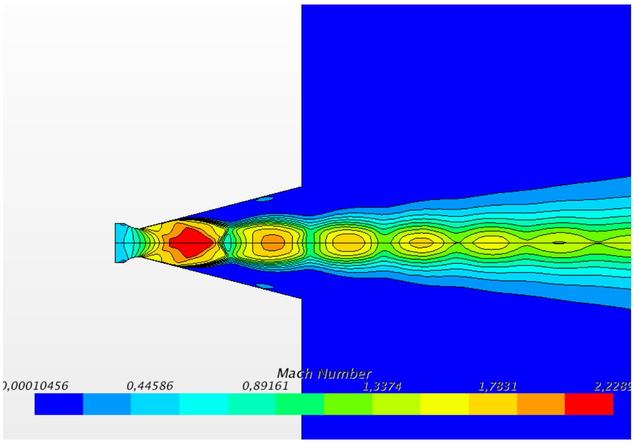
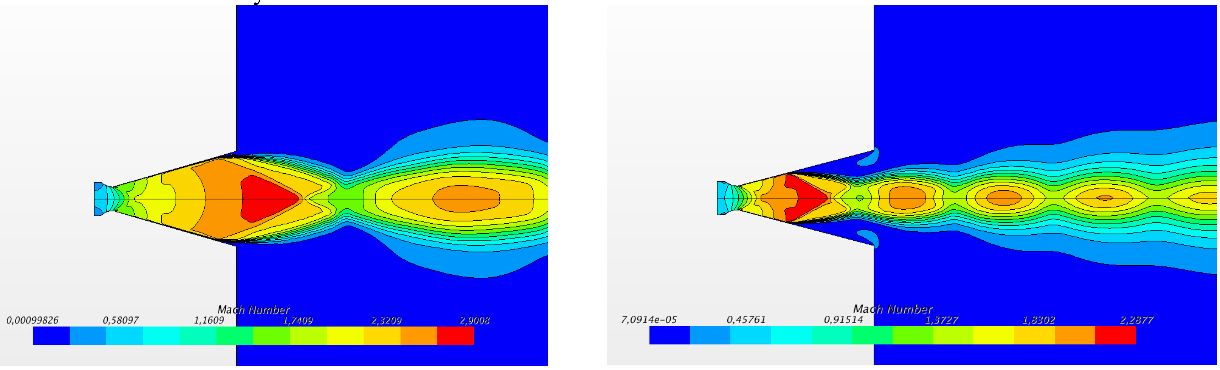


c)

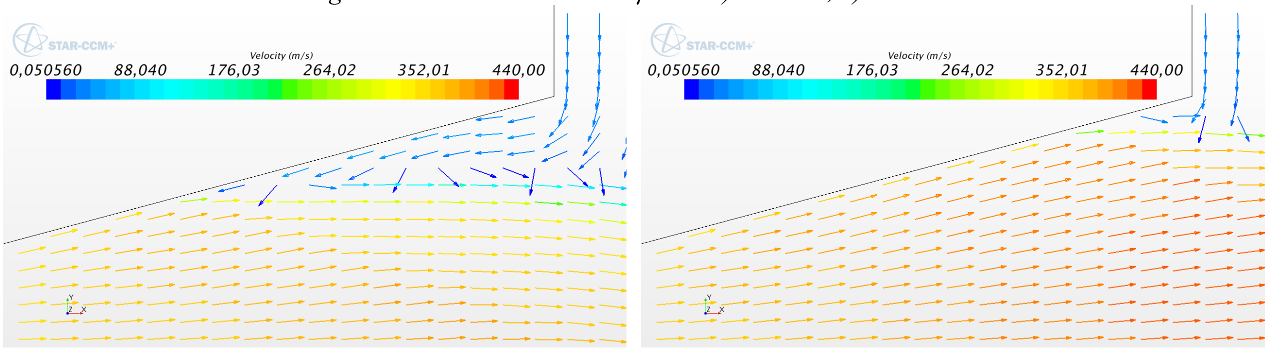
Figure 9.Mach number contours at γ=1.4in a tapered nozzle while reducing NPR. a) NPR=20, b) NPR=10, c) NPR=5

Figure 7-8 show the results of the calculation of the jet with γ = 1.2. Note that for the same NPR, compared to γ = 1.4, the reflection point is closer to the nozzle exit as in the case with γ = 1.4. In addition, the flow separation on the same modes is substantially less.



a) b)

Fig.9Mach number contour at γ=1.2. a)NPR=10, b)NPR=5



a) б)

Fig.10.Compase of velocity vector fields at NPR=5 for different γ. a) γ=1.4, б)γ=1.2

5. Conclusion

The solid state mesh model has been created for the numerical simulation of the flow of the gas inside the jet issuing from the nozzle. The calculation have been carried out for the values of specific heats γ = 1.4 and γ = 1.2, the value of the parameter NPR being 20, 15, 10, 5. With the parameter NPR = 20 there is a typical structure for regular reflection with the intersection point at the axis located near the exit of nozzle. When reducing NPR to 10 the structure resembling the regular reflection remains, however it is shifted into the nozzle, the flow separation occurs. With further decrease of NPR to 5 the number of consecutive normal shock waves in the nozzle increases. Subsonic separating zones appear. This mode is preceded to the formation of Mach reflection. For the same value of the parameter NPR, but at a smaller value of γ, the intersection point shifts further away from the inlet section of the nozzle. The flow separation became substantially less. The investigations are continuous with the aim to study the appearance of configuration with negative reflection angle in a jet flows.

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References

[1] Courant R., Friedrichs K. O. Supersonic Flows and Shock Waves. NY:Intersicence. 1948.
[2] Von Neumann J. Collection of works. Oxford: Pergamon Press. 6: 239-299.
[3] Landau L.D., Lifshitz E.M. Course of Theoretical Physics: Vol. 6. Fluid Mechanics. Oxford: Butterworth-Heinemann, 1987.
[4] von Neumann J. Oblique reflection of shock waves // Collected Works. London: Pergamon Press, 1963. Vol. 6. P. 238-299.
[5] Uskov V.N., Chernyshov M.V. Special and extreme triple shock-wave configurations // Journal of Applied Mechanics and Technical Physics. 2006. Vol. 47. No. 4. Pp. 492-504.
[6] Bazhenova T.V., Gvozdeva L.G., Nettleton M.A. Unsteady Interaction of Shock Waves. Moscow: Nauka, 1977. (in Russian)
[7] Hekiri H., Emanuel G. Shock wave triple point morphology // Shock Waves. 2011. Vol. 21. Issue 6. Pp. 511-521.
[8] Gvozdeva L.G., Gavrenkov S.A. Formation of triple shock configurations with negative reflection angle in steady flow // Technical Physics Letters. 2012. Vol. 38. Issue 4. Pp. 372-374.
[9] Gavrenkov S.A., Gvozdeva L.G Numerical investigation of the onset of instability of triple shock configurations in steady supersonic gas flows // Technical Physics Letters. 2012. Vol. 38. Issue 6. Pp. 587-589.
[10] Gvozdeva L.G., Gavrenkov S.A. Influence of the adiabatic index on switching between different types of shock wave reflection in a steady supersonic gas flow // Technical Physics. 2013. Vol. 58. Issue 8. Pp. 1238-1241.
[11] Ivanov M.S., Gimelshein S.F., Beylich A.E. Hysteresis effect in stationary reflection of shock waves // Physics of Fluids. 1995. Vol. 7. Issue 4. Pp. 685-687.
[12] Bazhenova T.V., Gvozdeva L.G., Lobastov Y.S., Naboko I.M., Nemkov R.G., Predvoditeleva O.A. Shock Waves in Real Gases. Moscow: Nauka, 1968. (in Russian)
[13] Bazhenova T.V., Gvozdeva L. G., Nettleton M.A. Unsteady interactions of shock waves // Progress in Aerospace Sciences. 1984. Vol. 21. Issue C. Pp. 249-331.
[14] Ben-Dor G. Shock-Wave Reflection Phenomena, 2nd Edition. N.Y.: Springer, 2007.
[15] Nel N., Skews B., Naidoo K. Schlieren techniques for the visualization of an expansion fan / shock wave interaction // Journal of Visualization. Article in press (accepted 09 December 2014). 11 p.
[16] M. Peric: Flow simulation using control volumes of arbitrary polyhedral shape, ERCOFTAC Bulletin, No. 62, September 2004.
[17] Belov I.A., Isaev S.A. Modelirovanie turbulentnyh yechenij [Simulation of turbulent flow]. St.Petersburg, Baltic State Technical University Publ., 2001. 108p.
[18] Shimshi, E., Ben-Dor G. and Levy A. Viscous simulation of shock reflection hysteresis in overwxpanded planar nozzles, J.Fluid Mech., Vol.635, pp.189-206, 2009
[19] Gvozdeva L.G., Gavrenkov S.A. A new configuration of irregular reflection of shock waves // EUCASS 2013, pp. 471-486
[20] Chulyunin A.Yu., Gvozdeva L.G. Numerical study of the gas flow in the nozzle and a supersonic jet for different values of specific heats// XXX International conference on Interaction of Intense Energy Fluxes with Matter. Book of abstracts. p. 127. Moskow&Chernogolovka&Nalchik, 2015