Model-Based Control Design for a Free-Floating Space Manipulator Capturing Debris

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Abstract
The paper presents designing *model-based* tracking controllers for a free-floating space robot. It may perform maintenance tasks, e.g.: such as: servicing satellites, moving them to graveyard orbits and capturing space debris. The researched task is capturing objects in space.
A free – floating operation mode implicates that a robot is *underactuated*. In control setting, underactuation makes a robot second order nonholonomic, so a free – floating space robot is a multi – constraint nonholonomic control system. 3D robot dynamics, a theoretical control development for approaching, capturing and acquisition of an object is analysed. The theoretic development is illustrated by the simulation results.

1. Introduction

The paper presents results on designing model – based tracking controllers for a free – floating space robot. The robot may perform maintenance tasks in space, such as: servicing communication satellites, moving them to graveyard orbits and capturing space debris.

1.1 Motivation

Motivations for taking this research are then the potential significance of its results in the future, in the face of constantly growing interest in conquest and exploration of space. The growing space exploration by a man results in generation of more space debris and requires sophisticated services [3]. The latter ones are often delivered by astronauts, like in *Extra Vehicular Activities* (EVA). Debris moving in space may cause danger for satellites and need to be removed successively. Both, services in space and debris removal need to be performed by specialized space robots. Also, this research, through a development of new control strategies for space robots may provide a new insight into non-linear control methods for missions in space.
This research is focused on the design of control strategies for capturing objects in space. Such missions are of a significant interest due to the growing number of debris and other space objects needed to be removed from space. Also, due to the asteroids which are promising sources of raw materials.

1.2 Scope

A free – floating operation mode requires spacecraft thrusters to be turned off and the system linear and angular momenta to be conserved. The condition of linear momentum conservation generates the holonomic constraint on a robot. However, due to the angular momentum conservation space robots are nonholonimic control systems. The free – floating mode implicates that a robot is underactuated. In control setting, the underactuation is treated as a second order nonholonomic constraint, so the free – floating space robot is a multi – constraint control system [1, 2].
Control properties of space robots, as mentioned earlier, and various kinds of missions for them make the control design interesting and quite challenging. Although there already exist algorithms which allow controlling underactuated robots and manipulators [4, 5], there are still many open problems and room for new research in this area, since controls are dedicated to certain missions and space robots.

1.3 Contribution

The paper presents a space robot control oriented 3D dynamics, a theoretical control development for approaching, capturing and acquisition of an object. The assumptions are that the space robot is supposed to track and capture an object of a relatively small size with respect to that of the robot, and which does not spin. The theoretic development is illustrated by the simulation results.

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The research contribution is two - folded. Its results may provide the better insight into control design for space systems and may constitute a control theoretic basis for future applications in space or even for ground manipulators when one of their actuators fails.

2. A free – floating robot's dynamics development

Motivations for taking this research are the potential significance of its results in the future, in the face of constantly growing interest in conquest and exploration of space. The growing space exploration by a man results in generation of more space debris and requires sophisticated services [3]. The latter ones are often delivered by astronauts, e.g. *Extra Vehicular Activities* (EVA). The latter may be harmful for the astronaut performing the tasks. Debris moving in space may cause danger for satellites and need to be removed successively. Therefore, both, services in space and debris removal need to be performed by specialized space robots in order to minimize or eliminate potential dangers. Also, this research, through a development of new control strategies for space robots may provide a new insight into nonlinear control methods for missions in space.

2.1 Physical and mathematical model of the *free – floating* space manipulator

A free – floating operation mode requires the following conditions to be fulfilled: spacecraft thrusters need to be turned off and the system's linear and angular momenta are to be conserved. The condition of linear momentum conservation generates the holonomic constraint on a robot. However, due to the angular momentum conservation space robots are non – holonomic control systems. The free – floating mode implicates that a robot is underactuated. In control setting, the underactuation is treated as a second order non – holonomic constraint, so the free – floating space robot is a multi-constraint control system [1, 2].
The vehicle analysed in this paper is space manipulator consisting of base and two arms. Moreover, specific assumptions have been made for this research. Although the vehicle is three dimensional, a certain limitation to its motion is applied: the joints can rotate only around the z – axis. Detailed description of the physical properties is provided in the Table 1 below.

Table 2.1: Physical parameters of the system

Body No.	$\mathbf{a}_i$ [m]	$\mathbf{l}_i$ [m]	$\mathbf{m}_i$ [m]	$\mathbf{I}_i$ [kg m^2]
0	1	0.5	40	6.667
1	1	0.5	4	0.333
2	1	0.5	3	0.250

Physical model of the robot is presented in the Figure 2.1.

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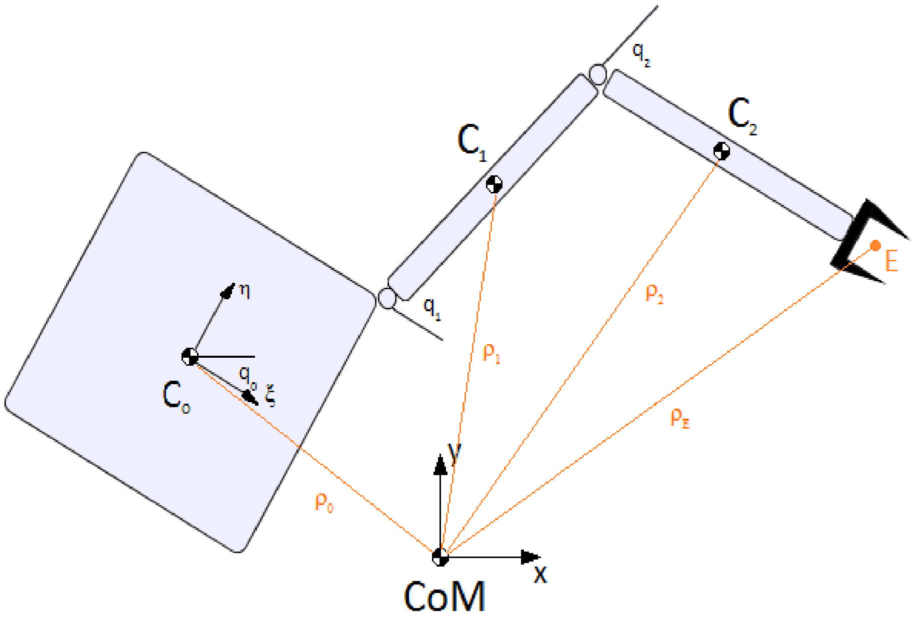

Figure 2.1 Two – dimensional representation of the three – dimensional robot

The analysed system is described by the joint coordinates (bold font means vector or matrix values): $\boldsymbol{q} = [q_0, q_1, q_2]^T$. The following variables define the centers of mass of each body in the system: C_0, C_1, C_2. Vectors ρ_0, ρ_1, ρ_2 define the leading vectors of the centers of masses, where $\rho_i = [\xi, \eta, \zeta]^T$. Also ρ_E defines the end effector of the manipulator.

2.2 Angular momentum conservation

The conservation of the angular momentum results in the non – holonomic constraint. This condition requires the sum of angular momenta of all the bodies in the system to be equal zero. Taking into account the applied notation, angular momentum is angular position (q) and angular velocities (dq/dt) dependent. Thus, the formula for angular momentum conservation can be written as follows:

$$K = \sum_{i=0}^{2} K_i = \Phi(q_i, \dot{q}_i) = 0 \qquad (1)$$

In this equation Φ is the known function of joint coordinates and their velocities. Subsequently, the angular momentum of a single body can be written as follows:

$$K_i = \rho_i \times m_i \dot{\rho}_i + I_i \omega_i \qquad (2)$$

Where m_i is the mass, I_i is the inertia tensor and ω_i is the angular velocity vector of the i – th body. Although, the model of the robot is three – dimensional, the motion of the robot is limited. Thus, rotation occurs only around the z – axis. Therefore, equation (2) can be written in a simplified form:

$$K_i = \rho_i \times m_i \dot{\rho}_i + I_{i\zeta\zeta} \omega_{iZ} \qquad (3)$$

Where $I_{i\zeta\zeta}$ is the inertia tensor of the i-th body and ω_{iZ} is the angular velocity component of the i-th body around the axis perpendicular to the plane in which the motion occurs. Angular velocity of each body can be written as the sum of time derivatives of the rotational angles:

$$\omega_0 = \dot{q}_0$$
$$\omega_1 = \dot{q}_0 + \dot{q}_1$$
$$\omega_2 = \dot{q}_0 + \dot{q}_1 + \dot{q}_2$$

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2.3 3D robot dynamics

In order to proceed with the computations, kinetic energy dependent on joint coordinates and angular velocities is required. Total kinetic energy of the system, *T*, is devised as the sum of kinetic energy of each body. Kinetic energy of a single body is the sum of kinetic energy of linear motion of the centre of mass and the kinetic energy of the circular motion of the body around the centre of mass of the system. This can be noted as follows:

$$T_i = \frac{1}{2} m_i (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) + \frac{1}{2} I_{iCM} \omega_i^2 \quad (3)$$

In this equation, x_i, y_i, z_i are the linear velocities of the centre of mass of the body. I_{iCM} is the moment of inertia around the centre of mass of the system. It is derived from the Steiner's theorem as follows:

$$I_{iCM} = I_i + m_i d_i^2 = I_i + m_i (x_i^2 + y_i^2 + z_i^2) \quad (4)$$

Total kinetic energy of the system can be then computed as follows:

$$T(q_i, \dot{q}_i) = \sum_{i=0}^{2} T_i(q_i, \dot{q}_i) \quad (5)$$

3. Methods

Dynamics of the space robot is modelled by using the methods of analytical mechanics appropriate for the multibody mechanical systems. From the mathematical point of view, dynamical model is a set of non – linear ordinary differential equations (ODE). Further on, dynamical model is transformed into the symbolical model in order to conduct the numerical computations.
The motion of the system is described by the Lagrange equations of motion:

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{q}_i}\right) - \frac{\delta T}{\delta q_i} = Q_i + \lambda \frac{\delta \Phi}{\delta \dot{q}_i} \quad (6)$$

After appropriate operations these equations can be transformed into matrix form.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau + J_0^T \lambda \quad (7)$$

Matrices ***M***, ***C*** and ***J*** are derived directly from equation (6). Furthermore, the unknown Lagrange multipliers are decoupled from the control momenta which results in the reduced form of the equations. This procedure is applied to non – holonomic systems in order to obtain dynamical control model.

$$M_{12}(q)\ddot{q}_2 + C_{12}(q, \dot{q}_2)\dot{q}_2 = \tau + J_1^T \lambda \quad (8)$$

$$M_{22}(q)\ddot{q}_2 + C_{22}(q, \dot{q}_2)\dot{q}_2 = \tau \quad (9)$$

$$\dot{q}_1 = D(q)\dot{q}_2 \quad (10)$$

Where $\boldsymbol{q_1} = [q_0]$ and $\boldsymbol{q_2} = [q_1, q_2]^T$. Equations (9) and (10) state the system's dynamical control model. They are used during the computation. Calculations are conducted in MatLAB 2008a. To summarize the variables used in the equations : K_i are the angular momenta of each body, $\Phi(q_1, dqi)$ is the the known function of joint coordinates and their derrivatives with respect to time; *T* is the system's kinetic energy; Q_i is the vector of generalized forces and λ is the unknown Lagrange multiplier. Generalized forces are equal to the the control torques τ_i. In this research only the angles q_1 and q_2 are controlled. Orientation of the base, q_0, is not controlled. Thus, $\tau_0 = 0$ and the only acting torques are τ_1 and τ_2.
Moreover, equation (1) can be written in the form of A(q) dq/dt = 0 and therefore, can be used as non – holonomic constraint.

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4. Simulation results

This section of the article presents the results obtained in the research process. Two algorithms have been implemented in this research: computed torque control algorithm and Wen – Bayard algorithm.

4.1 Exact linearisation algorithm

This algorithm requires the knowledge of complete configuration of the analysed system. It uses the linearisation of the dynamics model with statical feedback loop. The model can be written using the following equations:

$$M_{22}(q)\ddot{q}_2 + C_{22}(q,\dot{q}_2)\dot{q}_2 = \tau \quad (9)$$

$$\dot{q}_1 = D(q)\dot{q}_2 \quad (10)$$

According to this algorithm, the input consists of control torques, as follows:

$$\tau = M_{22}(q)\nu + C_{22}(q,\dot{q}_2)\dot{q}_2 \quad (11)$$

In this equation υ is the new vector of control inputs.

Control of the linearised system is basically feeding the input with the signal from PD regulator:

$$\nu = \ddot{q}_{2d} - K_d\dot{e} - K_p e \quad (12)$$

Where K_d and K_p are the amplifications of the PD controller and e is the error function defined as follows ($q_2(t)$ is the trajectory and $q_{2d}(t)$ is the desired trajectory):

$$e(t) = q_2(t) - q_{2d}(t) \quad (13)$$

The described above algorithm was used to track the circular trajectory defined by the following equations:

$$X = \begin{bmatrix} x_d \\ y_d \end{bmatrix} = \begin{bmatrix} x_s - 0.3\cos\left(\frac{2\pi t}{15}\right) \\ y_s + 0.3\sin\left(\frac{2\pi t}{15}\right) \end{bmatrix} \quad (14)$$

It is a circle of radius r = 0.3 m centered in (x_s, y_s) and it takes 15 seconds to make the full circle. For the control algorithm, first and second derivative with respect to time is required.

$$\dot{X} = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} = \begin{bmatrix} 0.3\left(\frac{2\pi}{15}\right)\sin\left(\frac{2\pi t}{15}\right) \\ 0.3\left(\frac{2\pi}{15}\right)\cos\left(\frac{2\pi t}{15}\right) \end{bmatrix} \quad (15)$$

$$\ddot{X} = \begin{bmatrix} \ddot{x}_d \\ \ddot{y}_d \end{bmatrix} = \begin{bmatrix} 0.3\left(\frac{2\pi}{15}\right)^2\cos\left(\frac{2\pi t}{15}\right) \\ -0.3\left(\frac{2\pi}{15}\right)^2\sin\left(\frac{2\pi t}{15}\right) \end{bmatrix} \quad (16)$$

Simulation time was set to $t_t = 30\ s$ so the end effector can make two full circles. The results for this motion are presented below in Figure 4.1.

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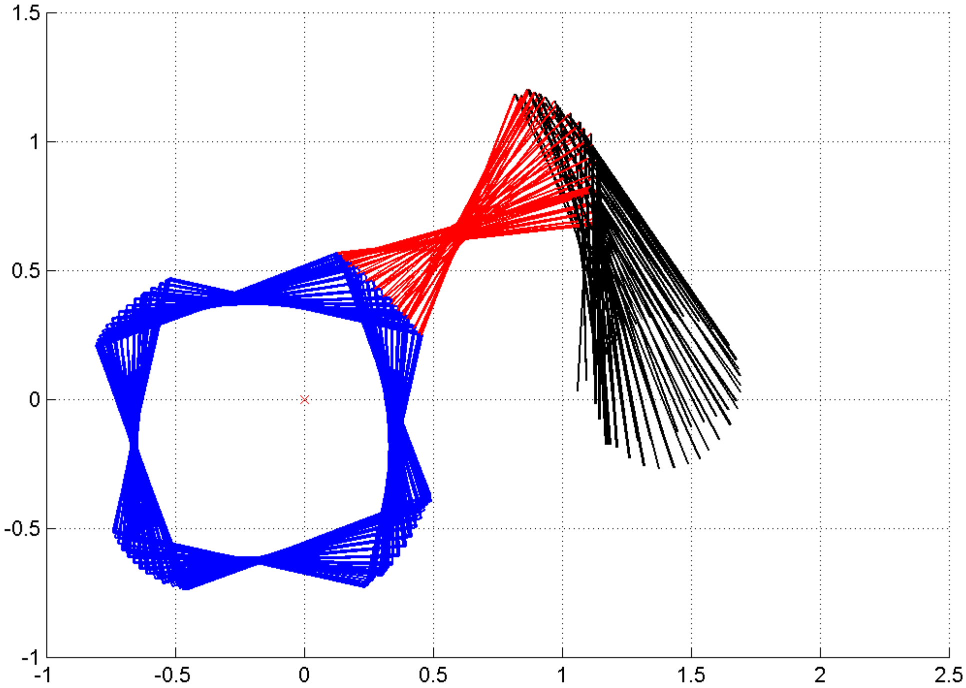

Figure 4.1. 2D representation of robot's motion computed with the exact linearisation algorithm

4.2 Wen – Bayard control algorithm

Wen – Bayard algorithm is a computed torque algorithm. It also requires the full knowledge of the system. However, it has a certain advantage over the exact linearisation algorithm: there is no requirement for the mass matrix to be non – singular. Control torques stating the input are computed using the following formula:

$$\tau = M_{22}(q)\ddot{q}_{2d} + C_{22}(q_d, \dot{q}_{2d})\dot{q}_{2d} - K_d \dot{e} - K_p e \quad (17)$$

The error function, $e = e(t)$, is defined earlier.

Simulation results of the Wen – Bayard algorithm implementation are shown in the following figure (**Figure** 4.2).

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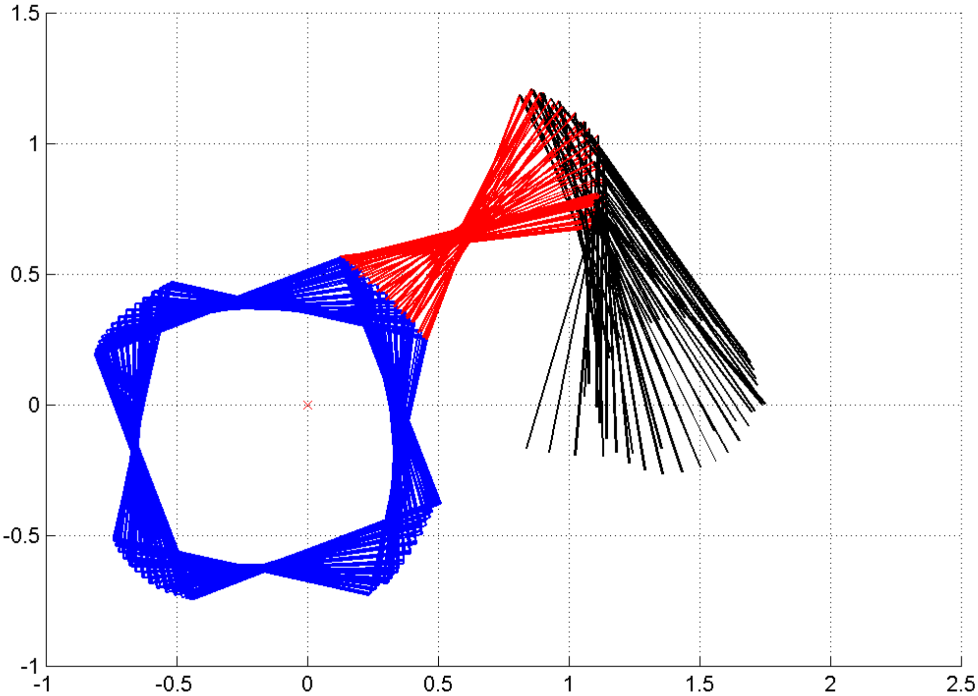

Figure 4.2. 2D representation of robot's motion computed with the Wen – Bayard algorithm implementation

Moreover, the results of both algorithms' implementations have been compared. In both cases, manipulator was supposed to follow the desired trajectory (in this case it was circle with radius $r = 0.3$ m). Effects of this comparison are presented in Figure 4.3. Red colour represents the desired trajectory, blue colour represents the results obtained by implementation of the *exact linearisation algorithm* and black colour shows results obtained with the Wen – Bayard algorithm.

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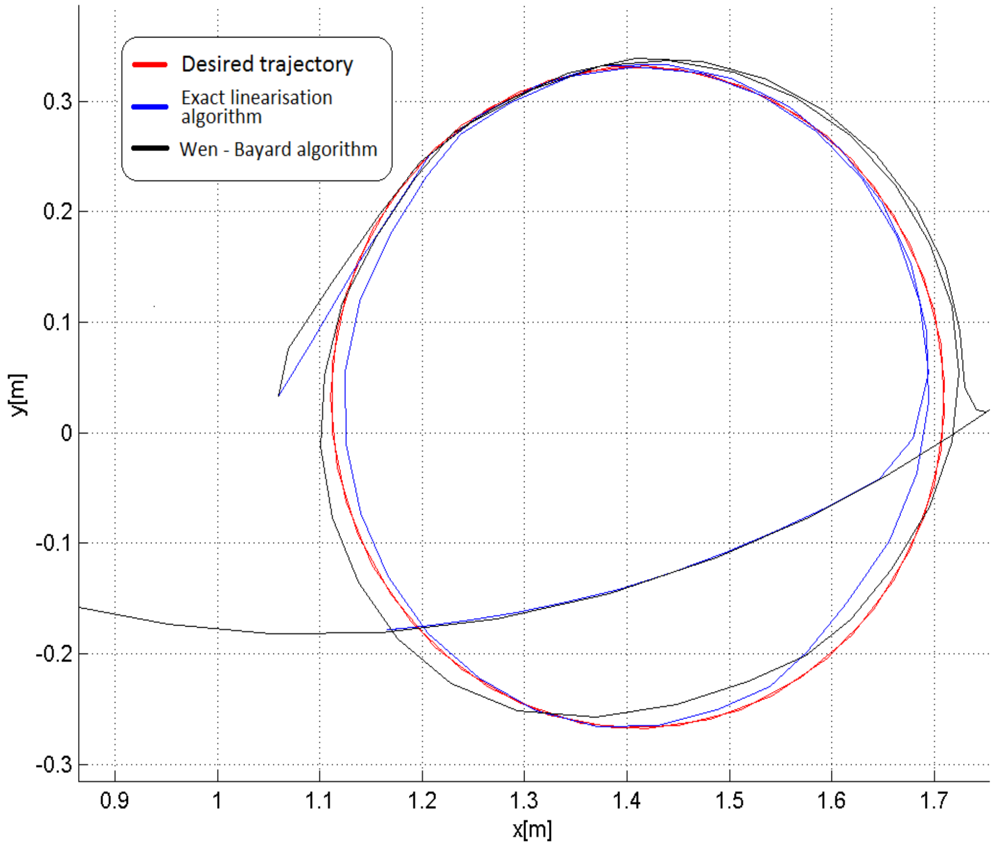

Figure 4.3. Comparison of the results obtained by two algorithms (*exact linearisation algorithm* – in blue, *Wen – Bayard algorithm*, in black) with reference to the desired trajectory (in red).

5. Conclusions

Control design for space robots is a challenging task. Presented research shows the simulation results of controlling the three – dimensional space vehicle in two – dimensional motion. Although, the computations were time consuming, the results are promising. Therefore, they present a good starting point for further actions and control strategy development for on – line computations. Next step to be undertaken is to design control strategies for three – dimensional motion.

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