

Trajectory Optimisation for Mars - Earth Transfer.

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Abstract

Two-body (Mars to Earth) spacecraft trajectory relating Lambert Problem for interplanetary transfers. Keplerian parameters were incorporated in calculating the position and velocity vectors required for the programme. Optimisation of the transfer path for minimum cost function (ΔV) was undertaken once the initial transfer parameters and selected methodology were specified. The hyperbolic entry/exit was designed using patched-conics solution. The perturbation experienced by the spacecraft during the entire transfer was included and projected for one year based on the initial parameters (accounting both short and long term contributions). Major factors taken into consideration were solar pressures, atmospheric drag, and non-uniform mass distribution, so as to generate desired results within reasonable accuracy. The optimisation allowed a total transfer time of 108 days from 250 km Mars circular parking orbit to a predefined elliptical orbit around Earth. The propulsive cost computed for this was 8.69 km/s which included the plane change, hyperbolic exit and entry and the final orbit insertion. It was observed from the perturbation analysis that majority of orbital variation was arising from the uneven mass distribution of earth. The short period analysis caused the right ascension to drift by 30° while the long period caused 300° , the rest of the trends in Keplerian elements were tabulated and plotted.

1. Introduction

The primary objective of the project is to map an optimum transfer for a satellite from a circular orbit around Mars to an elliptical orbit around the Earth, followed by perturbation analysis for short and long term orbital periods. In the Mars SOI (sphere of influence), the exit conics and in the Earth's SOI, the entry conics are to be defined by sizing the hyperbolas in terms of Keplerian parameters.

Based on the entry into the final orbit around the Earth (the parameters are given in Table 1), and the flexibility to choose the remaining unknown variables, the patched conic section is to be designed. Using information from this table, the time period of the satellite around Earth is calculated incorporating Kepler's relation between semi-major axis, a , and Time period, T . $T=5888.62 \text{ s} = 1.64 \text{ hours}$. So in one solar year (365 days), the satellite orbits the earth $365/T = 5355.4$ times.

Table 1 – Defined orbital parameters

Height (Peri.)	Height (Apo.)	Inclination	Arg. of Perigee	Right Ascension
553 km	780 km	97.9°	0°	180°

One of the crucial factors driving this numerical analysis is the ΔV function. It is important to find a value which is minimum with respect to the onboard propulsion systems and hence the cost economics of the flight. The total ΔV is to be found which is derived by incorporating all manoeuvres during the course of the transfer.

The flexibility over choosing the most convenient start and end points, along with defining best suited hyperbolic elements gives a greater launch window for the mission. Only using this could it be feasible to meet the requirement of optimised cost function.

Initially this computation requires the input of position vectors for the start and end of the transfer, which, along with the total time of flight, is fed into the given Lambert solver for the corresponding initial and final velocity vectors. So, with this the trajectory of the path is obtained.

Table 2 – defined spacecraft parameters

S/c mass	Area	β	Reflectivity (ϵ)
100 kg	0.48 m ²	104.17 kg/m ²	0.98

It is required to calculate the hyperbolic transfer velocities for both exit and entry in order to be able to size the trajectory of the s/c in the heliocentric reference frame. It also helps with the knowledge of the velocity changes occurring at

various progressive stages of the transfer and the possibility of introducing an optimal value. Upon obtaining information about the exit and entry conics (in terms of the Keplerian parameters) and the transfer with sun as focus, it is crucial obtain the final hyperbola which allows the s/c to enter into its required orbit around earth. This entails the calculation of the entry angle, in terms of position and velocity vectors, in the geocentric frame, hence knowing the initial point of orbit insertion and its direction in the predefined retrograde orbit.

Next phase of the requirement for this project is the computation of perturbations. It is necessary to calculate both the short and long term perturbations. The contributions for this are derived from the solar pressure, atmospheric drag, uneven mass distribution, all analysis to run for a solar period.

During the time for the transfer, in either case (short and long), each perturbation is considered individually and its contribution to the overall anomaly is numerically assessed. The selected designs are discussed individually. Attention is given to the fact that in the case of any potential physical impact on Earth, the time of occurrence is noted and consequently the effects are to be discussed.

The trajectory is modelled as a patched conic approximation with three segments, reflecting the orbits around the three bodies: The escape from Mars, the transfer about the Sun (heliocentric reference), and the entry into Earth orbit. The ranges of each segment of the approximation is determined by the gravitational sphere of influence of each body in the system, with the assumption that the exit from and entrance to the sphere of influence is parallel to the velocity of the gravitational body. To calculate the changes in velocity a few necessary assumptions could be made. It may be assumed that the Earth and Mars are in a coplanar orbit around the Sun (saves the hassle of changing the inclination during transfer). This assumption made possible a transfer orbit via Hohmann Transfer. This interplanetary trajectory design changes as we research more into the orbital inclinations of the Earth and Mars. Also, it is assumed that the propulsive manoeuvres are instantaneous, also assuming that these propulsive manoeuvres are parallel to the current velocity vector, with no redirection, which reduces the problem by one dimension.

The problem requires that we find the position and velocity vectors in an inertial coordinate system. The approach is to find the representation of the position and velocity vectors in some convenient coordinate system and then find a transformation that allows us to find their representation in the desired inertial system. This idea of transferring the representation of vectors from one system to another is an extremely useful concept and is used in many other astromechanics problems as well as in many other disciplines. We relate these two representations in the two different coordinate systems to each other by use of what are called transformation matrices. The transformation matrix to take the simple representations of the position and velocity vectors, has the form (with a similar form for the velocity).

$$\vec{r}^I = \begin{bmatrix} -\Omega_z \\ -\Omega_z \\ -\Omega_z \end{bmatrix} \begin{bmatrix} -i_x \\ -i_x \\ -i_x \end{bmatrix} \begin{bmatrix} -(\omega + \nu)_z \\ -(\omega + \nu)_z \\ -(\omega + \nu)_z \end{bmatrix} \vec{r}^l$$

$$\begin{Bmatrix} X \\ Y \\ X \end{Bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos(\omega+\nu) & -\sin(\omega+\nu) & 0 \\ \sin(\omega+\nu) & \cos(\omega+\nu) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}$$

$$= T_{l2I} \vec{r}^l$$

where T_{l2I} is the transformation matrix given by:

$$t_{l2I} = \begin{bmatrix} \cos\Omega \cos(\omega+\nu) - \sin\Omega \sin(\omega+\nu) \cos i & -\cos\Omega \sin(\omega+\nu) - \sin\Omega \cos(\omega+\nu) \cos i & \sin\Omega \sin i \\ \sin\Omega \cos(\omega+\nu) + \cos\Omega \sin(\omega+\nu) \cos i & -\sin\Omega \sin(\omega+\nu) + \cos\Omega \cos(\omega+\nu) \cos i & -\cos\Omega \sin i \\ \sin(\omega+\nu) \sin i & \cos(\omega+\nu) \sin i & \cos i \end{bmatrix}$$

This provides the complete transformation matrix for transferring vectors represented in the local system to the same vectors represented in the inertial system. In order to get our desired results, we must multiply the matrix by the position and velocity vectors. For the position vector, the result is just 'r' times the first column of the transformation matrix:

$$\vec{r}^I = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} r \left[\cos\Omega \cos(\omega+\nu) - \sin\Omega \sin(\omega+\nu) \cos i \right] \\ r \left[\sin\Omega \cos(\omega+\nu) + \cos\Omega \sin(\omega+\nu) \cos i \right] \\ r \left[\sin(\omega+\nu) \sin i \right] \end{Bmatrix}$$

The vertical velocity in classical inertial coordinates is:

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \begin{Bmatrix} \dot{r} \left[\cos\Omega \cos(\omega+\nu) - \sin\Omega \sin(\omega+\nu) \cos i \right] + r \dot{\nu} \left[-\cos\Omega \sin(\omega+\nu) - \sin\Omega \cos(\omega+\nu) \cos i \right] \\ \dot{r} \left[\sin\Omega \cos(\omega+\nu) + \cos\Omega \sin(\omega+\nu) \cos i \right] + r \dot{\nu} \left[-\sin\Omega \cos(\omega+\nu) + \cos\Omega \cos(\omega+\nu) \cos i \right] \\ \dot{r} \left[\sin(\omega+\nu) \sin i \right] + r \dot{\nu} \left[\cos(\omega+\nu) \sin i \right] \end{Bmatrix}$$

Using these vectors help with the calculation of exit velocity from Mars, determining the exit hyperbola. As the transfer is not Hohmann's transfer (ideally suited for minimum ΔV in case of circular and co-planar orbits), the plane change angle, total transfer time and transfer angle are necessary inputs to compute the size of hyperbolas. There is a possibility of incorporating the additional velocity impulses due to plane change either at the start of the transfer, i.e. in the circular parking orbit, or at the end of the transfer, i.e. in the elliptical orbit around the earth. A combination of the two options could be selected as well, in which case the one that is least expensive is chosen.

The effects of different perturbations cause the orbit to change its configuration, from the original and stable one to an undesired one. Depending on the extent of these perturbations, the satellite may drift away from its main orbit into the attractor and cause collision. The main effects of these are seen in the change in Keplerian parameters which define an orbit. So in order to quantify them, the rate of change is measured for each and plotted to show their variations against time.

Two forms of perturbations are to be analysed when considering interplanetary transfers, namely short and long. As the names suggest, short involves only one complete rotation of the satellite around its planet, whereas the long form involves the total time period of the planet around the sun, which in turn leads to the satellite completing its orbit several times during this.

The major perturbations that can deviate the satellite orbit are non-uniform mass distribution of the attractor planet (due to oblateness effects), solar radiation pressures, atmospheric drag and presence of additional bodies (n-body problem). Oblateness mainly affects by imparting an uneven pull, but the extent of this greatly depends of the position of orbital plane with respect to the equatorial plane. For instance, if the two planes coincide then the polar attraction will cancel out resulting in no or negligible mass distribution effects.

For the case of solar radiations, there will be a significant variation in the short term analysis as the satellite remains in the light side of the planet (hence facing the sun) for about half its orbital time and remains in the dark side for the other half (hence having no direct link to the sun). Also, in the case of long term analysis, the amount of incident radiation changes per unit area due to the elliptical nature of earth's orbit around the sun.

When considering atmospheric drag into the analysis, it is important to note the minimum point on the orbit (periapse) will be most vulnerable to the effect as compared to any other point on the orbit. But again, as the atmosphere of planets only reaches out to a certain height, beyond it, empty space exist where the effect is nullified. So again for this effect, the size of the Keplerian parameters defines the degree of perturbations that can be taken into account for short and long term analysis.

There exist a special case when due to either one of the effects, the satellite orbit could change drastically that in fact the other effects come into significant consideration and the entire analysis goes into a loop wherein each factor causes an increasing amount of perturbed orbital deviation. Ultimately for this particular analysis kind, the final state vectors of the satellite will be depending on the end time of the programme being terminated. Or otherwise, in an event of the satellite collision with the attractor, the programme would be terminated, indicating the state vectors along with the time of collision.

2. State Vector and Orbit Determination

First and foremost, it is essential to define the initial and final orbits for the s/c to operate on. Along with that, using the provided Lambert solver, the state vectors were formulated. Due to the alignment of the chosen planets at certain recurring intervals of time, it is possible to opt for a satellite Hohmann transfer, which, in the case of Mars and Earth happens every 2.8 years and the launching date can be precisely determined. The time of flight for a Hohmann either way is 260 days. Setting the launch date as and inputting the flight time as, it is approximated to cost an equivalent of 5.6km/s delta-V. Now, as this is the ideal condition with the assumption of coplanar orbits, it is understandable that the total velocity change however will be higher. So, other feasible options were ventured that may involve not only a lower total delta-V, but also a shorter flight time.

Taking a trial initial value for the state vector and running it into a Matlab loop, a range of values for the transfer orbit delta-V was obtained. As the minimum possible delta-V for Earth-Mars transfer is 5.6 km/s (MIT), this was chosen as a reference point for subsequent calculations.

Table 3 – state and orbit vectors

	Position [km]	Velocity [km/s]
Mars Orbit	1.0e+08*[0.0094 0.6864 -1.9859]	[21.7741 -13.1389 -5.8533]
Earth Orbit	1.0e+08*[-0.8301 1.2175 0]	[-24.8539 -17.2160 0]

Normalising the vectors gives 2.1012e+08 km and 26.0960 km/s as initial position and velocity and 1.4736e+08 km and 30.2342 km/s for final position and velocities. As for the transfer, it is evident that the s/c is required to travel from an outer planet orbit to an inner one. According to this, the velocity in the heliocentric frame must be reduced while going from mars orbit to the transfer. Again, at the arrival point, the velocity must be reduced further such as to match the earth orbit.

The date of departure falls within 6 months from present day, and results in one of the shortest transfer paths possible between Mars and Earth, them being the main reasons for its selection. The Mars orbit is inclined by 1.85° with respect to that of the Earth. The parking orbit around Mars is circular with no restrictions on choosing the position of satellite in this orbit or the direction of revolution around Mars. This provides the flexible option of aligning the orbit closely matching to that of the final elliptical orbit around Earth, which is predefined.

The changes in inclinations for the entire flight that need to be accounted are mentioned in table 4. This was used to determine the total contribution from the inclinations and hence size the circular orbit with respect to the mars equatorial plane. So when considering the overall inclination (assuming the earth's orbit as reference i.e. 0°), we obtain $(-23.45+97.9-1.85) = 72.6^\circ$. This was now taken in the Mars frame of reference (hence including its equatorial inclination) to result in $(72.6-25.19) = 47.41^\circ$, i.e. the circular orbit inclination around mars, with respect to its equatorial plane.

Table 4 – Fixed Earth/ Mars parameters

Type	Inclination [°]
Earth equator	23.45
Mars orbit	1.85
Mars equator	25.19
Required final orbit	97.9

Figures 1 and 2 show the scaled plots of the Mars, Earth and transfer orbit, indicating the position of mars on its orbit at spacecraft departure and earth in its orbit upon spacecraft arrival, clearly mapping the trajectory being followed alongside the relative inclinations of all the orbits.

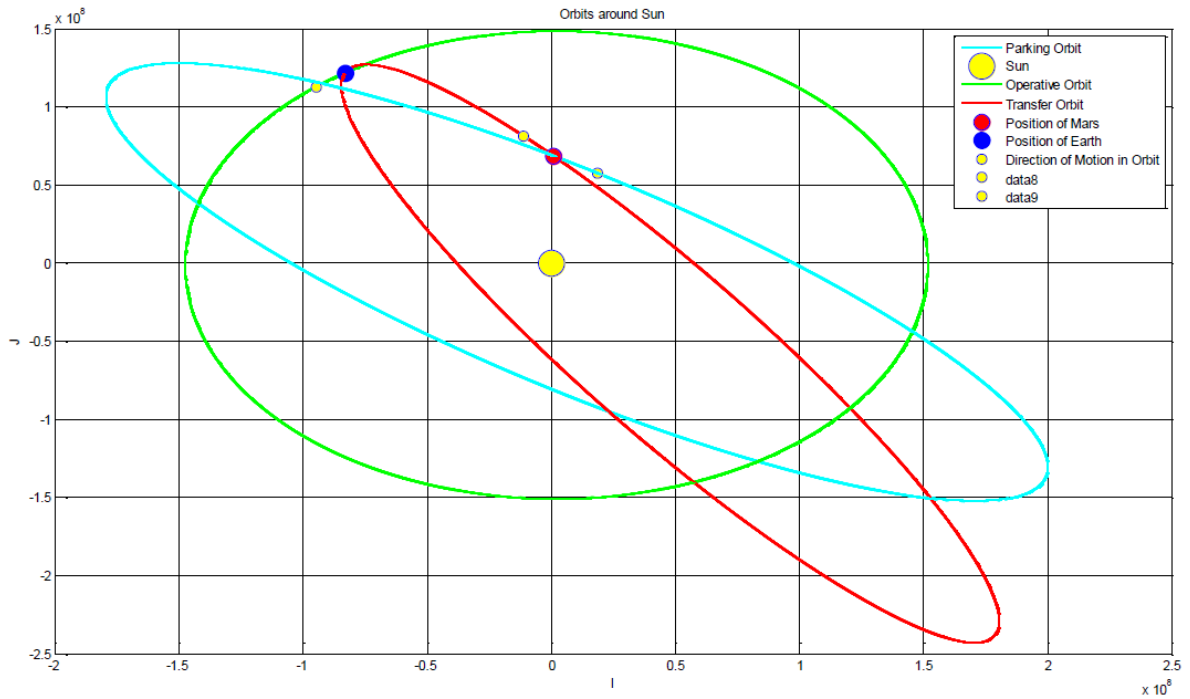


Figure 1 – sun-centric orbit in 2D

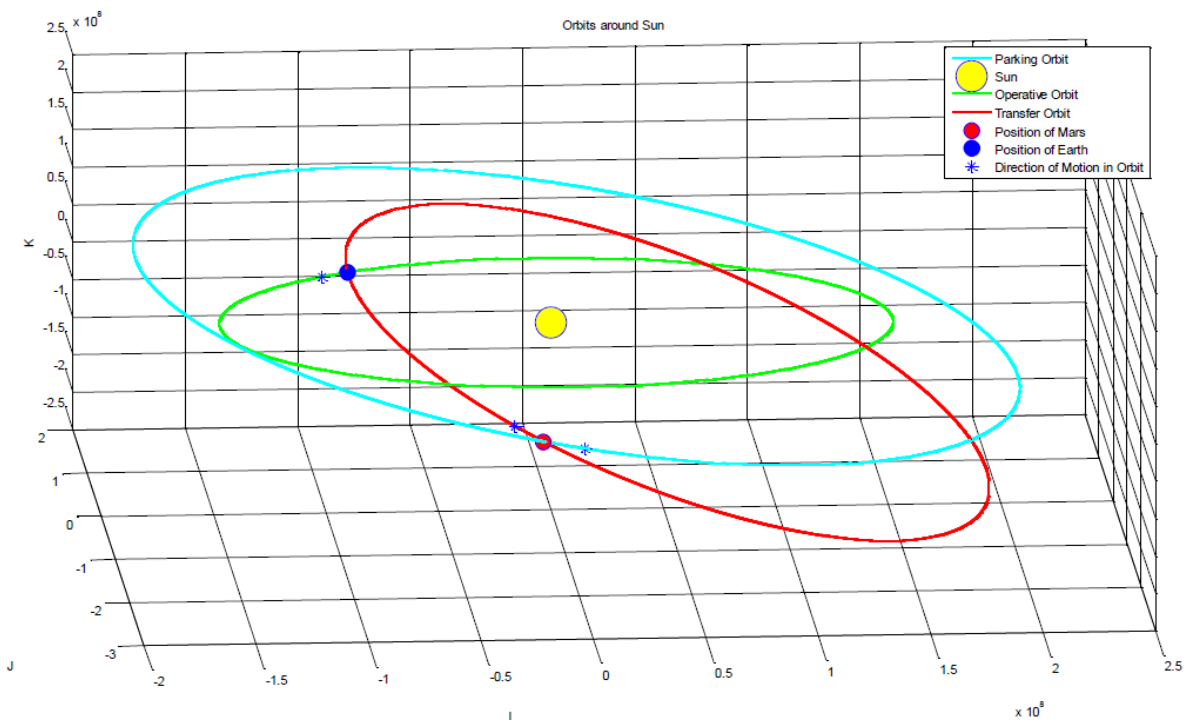


Figure 2 – sun-centric orbit in 3D

Once the basic parameters are determined, it is convenient to use the results to design the conics, which is presented in the following section.

3. Exit and Entry Conics

The spacecraft for a transfer from Mars to Earth means it must initiate from the back side of the sphere of influence during departure and travel in the direction opposite to the velocity vector of Mars in the heliocentric frame. For the case of exit hyperbola, we know the circular orbit velocity is 3.4273 km/s and the transfer orbit velocity is 26.0342 Km/s, which implies the hyperbolic exit velocity at departure is 4.8473 km/s, leading to a v_{∞} of 0.016 km/s at the Mars sphere of influence. The eccentricity, $e = 1.0003$ and $\beta = 1.4627^\circ$.

The transfer orbit inclination is 1.67° with respect to Earth. On doing the entry manoeuvre, there is another degree of freedom - whether to take the direction of v_{∞} in the transfer from above the Earth equatorial plane or below. When choosing the positive angle (i.e. entering Earth SOI from above its equatorial plane), due to the orientation of the final elliptical orbit ($\Omega=180^\circ$), the approach hyperbola would require a delta-V to change the plane configuration. On the other hand, when the approach is towards below the equatorial plane (i.e. with a negative reference angle), then the spacecraft's direction of approach would also require delta-V of another magnitude. Now, bearing in mind the direction spacecraft revolves in, adopting the former case will incur a change of 44° (approximate calculation assuming change in Earth's position on its orbit is 1° per day), whereas when using the latter, that value will be $136^\circ (=180-44)$.

So, the spacecraft, originating at 250km above Mars, follows a trajectory around the sun before entering a fixed point into the Earth's SOI, the specifications for which are mentioned in the following section.

3.1 Size of entry hyperbola-

According to the transfer orbit obtained previously, the spacecraft was yet supposed to change its inclination (i) and orientation (ω) to achieve the final requirement. So, upon insertion into the SOI, a capture orbit was designed, which would take care of these while also maintaining a low delta-V.

Two different approaches were designed to incorporate this solution. A balance was struck between these variable parameters and the best possible approach was selected that would fulfil the necessary changes in the spacecraft trajectory. These are discussed below.

Approach 1 -The perigee for the capture orbit was selected to be the same as the apogee of the final orbit, meaning that the hyperbolic entry is maintained at the final elliptical orbit's apogee. A manoeuvre was made at this instant to put the spacecraft from the hyperbola into a 'capture' ellipse, with a semimajor axis of 306768 km and eccentricity of 0.9767. This ellipse was chosen to minimise the delta-V for the remaining part of the transfer. A propulsive manoeuvre takes place at the apogee of the capture orbit, a distance of 600000 km from Earth. The time for this flight (perigee to apogee of capture) was calculated as 9.79 days. Once at apogee, the inclination change manoeuvre is made in order to align the spacecraft with the final orbit in the geocentric frame. The spacecraft travels back to the perigee now (also the apogee of the final orbit), making another impulsive change to get itself into the final orbit (a change of ω here). The total cost of this approach added up to 9.6 km/s. On sensing it to be a high value, led to a slightly different way of approaching, described as follows.

Approach 2 -This time the hyperbolic entry was chosen to be drifted away from the original point mentioned previously (i.e. apogee of the final orbit), which upon a simple iteration, yielded a semimajor axis of 310000 km and eccentricity of 0.9166 for the capture elliptical orbit. The time to reach the capture orbit apogee in this case is 10.24 days, which was kept fixed at 600000 km. The rest of the steps were similar to the previous approach- inclination change at apogee of the capture orbit following by final insertion into the required orbit after travelling back from the apogee of the capture to the apogee of the final orbit. The time for this was 9.79 days. This approach was preferred as it costs 8.7 km/s to the overall mission, almost 1 km/s reduction from the previous one.

The v_{∞} for entry is 4.63 km/s, with $\beta=65.60^\circ$ and $e=2.4210$. As this entry is 26378 km from Earth's centre, the aiming radius could be obtained, i.e. $\Delta = 40928$ km. Figure 7 shows the use of different elliptical orbits, where the delta-V changes are performed and allows a minimal change from the hyperbolic entry to required elliptic.

The point of entry is in the same axial line as the apogee for the final orbit (it is the perigee of a bigger elliptic around Earth). It is done in order to first use this outer orbit to put the spacecraft from a hyperbola into an elliptical orbit around Earth. After having reached the apogee of this orbit (where we know the velocity is minimum), a delta-v manoeuvre allows to change the inclination to the final required one (97.9° with respect to Earth equatorial plane). With this, the spacecraft is now approaching the perigee once again, but in an orbit that is co-planar with the final orbit.

The last of the manoeuvres is done at the apogee to enable the spacecraft to revolve in the specific required orbit, with the right inclination and direction of motion. Figure 3 shows the trajectories taken by the spacecraft after entry from the hyperbolic path to the capture orbit.

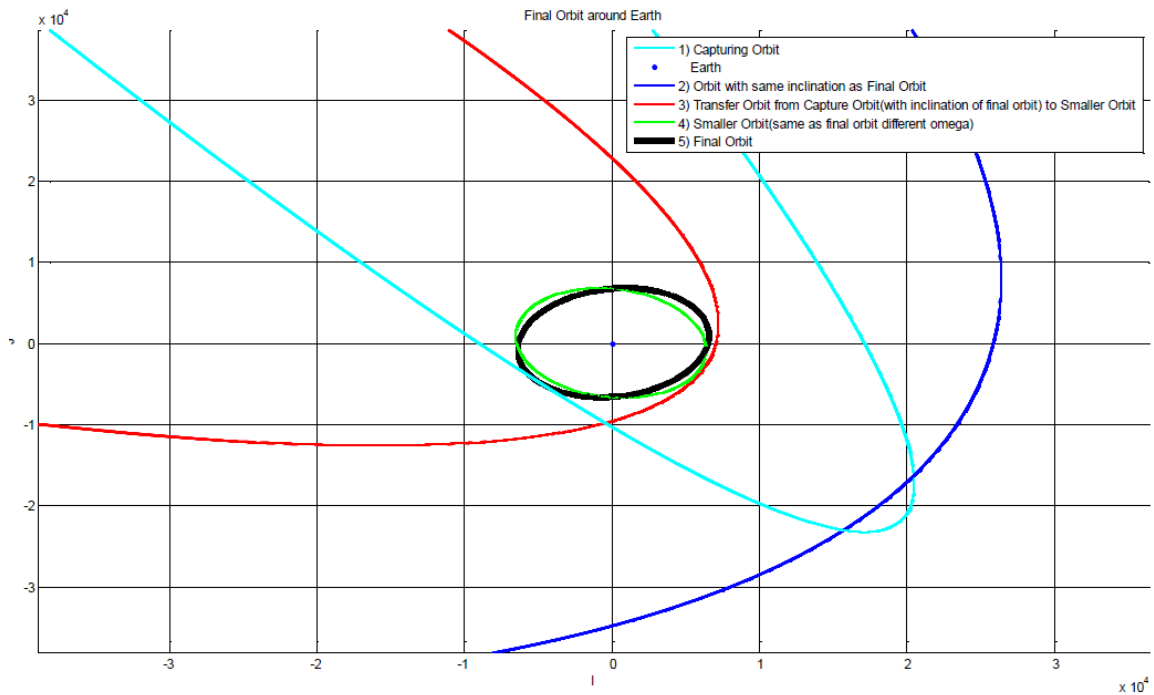


Figure 3 – Spacecraft earth entry

3.2 Delta-V calculations-

The main contributions to the overall mission delta-V come from heliocentric transfer, hyperbolic exit and entry, capture orbit around earth and final orbit insertion. The initial and final positions for mars and earth over the duration of this transfer give their heliocentric velocities as 26.096 km/s and 30.234 km/s respectively. The Δv obtained for each step of the transfer has been mentioned in table 5, with their values and cumulative expense of the entire mission computed at the end.

Table 5 – total delta-V caculations

DESCRIPTION	VALUE [KM/S]	REFERENCE
Mars velocity	26.0960	
Mars transfer orbit velocity	26.0342	
$\Delta v(1)$	0.0619	
Earth velocity	30.2342	
Earth transfer orbit velocity	34.8681	
$\Delta v(2)$	4.6339	
$\Delta v(1)+\Delta v(2)$	4.6958	
Mars circular orbit velocity	3.4273	
Hyperbolic exit velocity	4.8474	
Δv to exit Mars	1.4200	A
Capture orbit perigee velocity	5.3817	
Hyperbolic entry velocity	7.1899	
Δv to enter into earth capture orbit	1.8083	B
Δv for inclination alignment ($\Delta i=73.11^\circ$)	2.3079	C
Δv for final orbit insertion	3.0360	D
Δv for argument of perigee alignment ($\Delta \omega=44^\circ$)	0.1212	E

The total time from initial position of spacecraft to its final destined orbit is **108.0344 days**.

This constitutes 88 days for Mars – Earth heliocentric transfer, 10.25 days in capture orbit for inclination change, 9.79 days for reaching final orbit.

4. Perturbation analysis

The analysis was conducted individually for four factors (solar radiation, 3rd body effect, atmospheric drag, non-uniform mass distribution on central body) and their effects, appropriately identifying the dominant contributions. Under each classification, the first five plots (a, e, i, Ω , ω) indicate the analysis results run for short period, i.e. the satellite orbit around Earth once, and the following five plots indicate results of the same parameters for long term analysis, i.e. one solar period. Last three plots are 3D illustration of the actual perturbations in the geocentric frame of reference derived from these plots.

4.1 Atmospheric Drag

Table 6 lists the range of values obtained from the analysis for short and long term orbital periods. The last column indicates the original keplerian parameters.

Table 6 – atmospheric drag perturbation effects

	Short Period Min.	Short Period Max.	Long Period Min.	Long Period Max.	Earth Orbit
a[km]	7044.5	7044.50	7044.5	7044.5	7044.5
e[-]	0.0161	0.0161	0.0161	0.0161	0.0161
i[rad]	1.7087	1.7087	1.7087	1.7087	1.7087
Ω [rad]	3.1416	3.1416	3.1416	3.1416	3.1416
ω [rad]	-0.000	0.000	-0.000	0.000	0
θ [rad]	3.1415	9.4294	2.7925	33700.12	3.1415

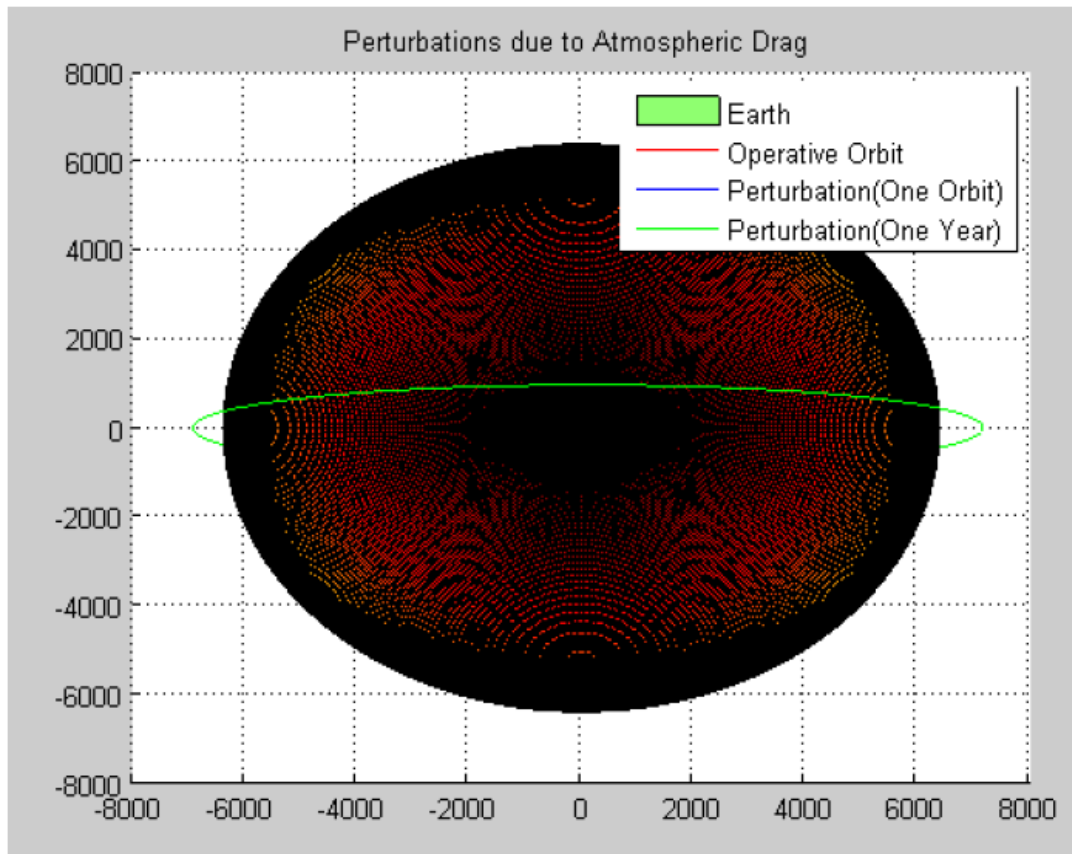


Figure 4 – Atmospheric drag for long and short term

4.2 Non-uniform Mass distribution

Table 7 summarises the important variations between long and short analyses as a consequence of earth's mass distribution only.

Table 7 – Non uniform mass distribution perturbation effects

	Short Period Min.	Short Period Max.	Long Period Min.	Long Period Max.	Earth Orbit
a[km]	7026.39	7045.12	7039.0184	7099.1328	7044.5
e[-]	0.0156	0.01746	0.01312	0.01636	0.0161
i[rad]	1.70867	1.70885	1.7081	1.7087	1.7087
Ω [rad]	3.1416	3.1427	3.1416	9.2029	3.1416
ω [rad]	-0.107	0.103	-20.843	0.000	0
θ [rad]	3.1416	9.427	3.1416	33447.959	3.1416

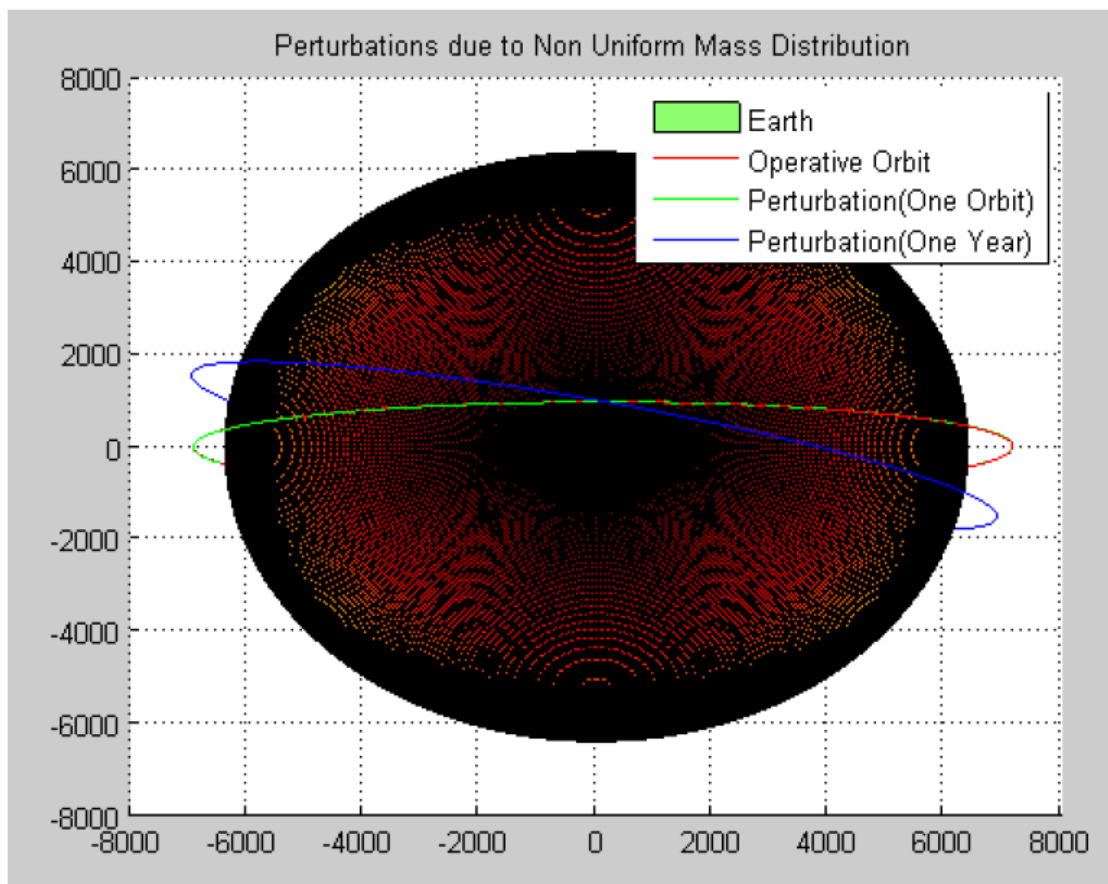


Figure 5 – Non uniform mass distribution for long and short term

4.3 Solar Radiation Pressures

Table 8 – solar radiation pressures perturbation effects

	Short Period Min.	Short Period Max.	Long Period Min.	Long Period Max.	Earth Orbit
a[km]	7044.5	7044.50	7044.5	7044.5	7044.5
e[-]	0.0161	0.0161	0.0161	0.0161	0.0161
i[rad]	1.7087	1.7087	1.7087	1.7087	1.7087
Ω [rad]	3.1416	3.1416	3.1416	3.1416	3.1416
ω [rad]	-0.000	0.000	-0.000	0.000	0
θ [rad]	3.1415	9.4248	3.1415	33674.5622	3.1415

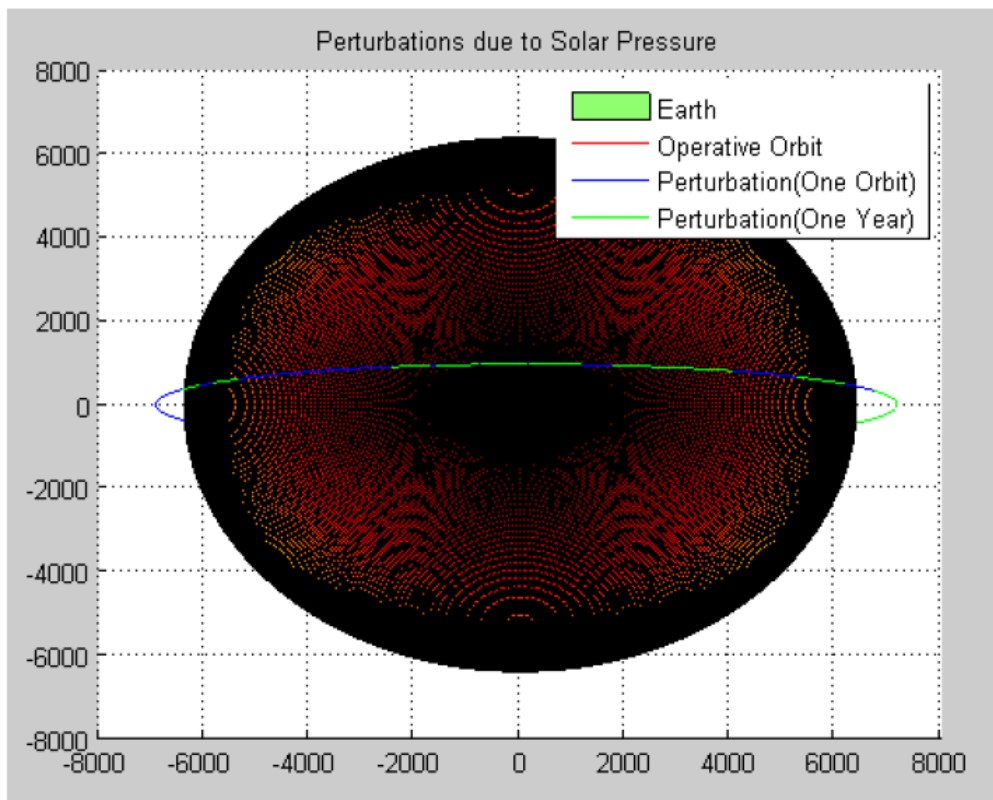


Figure 6 – solar radiation for long and short term

4.4 Three Body

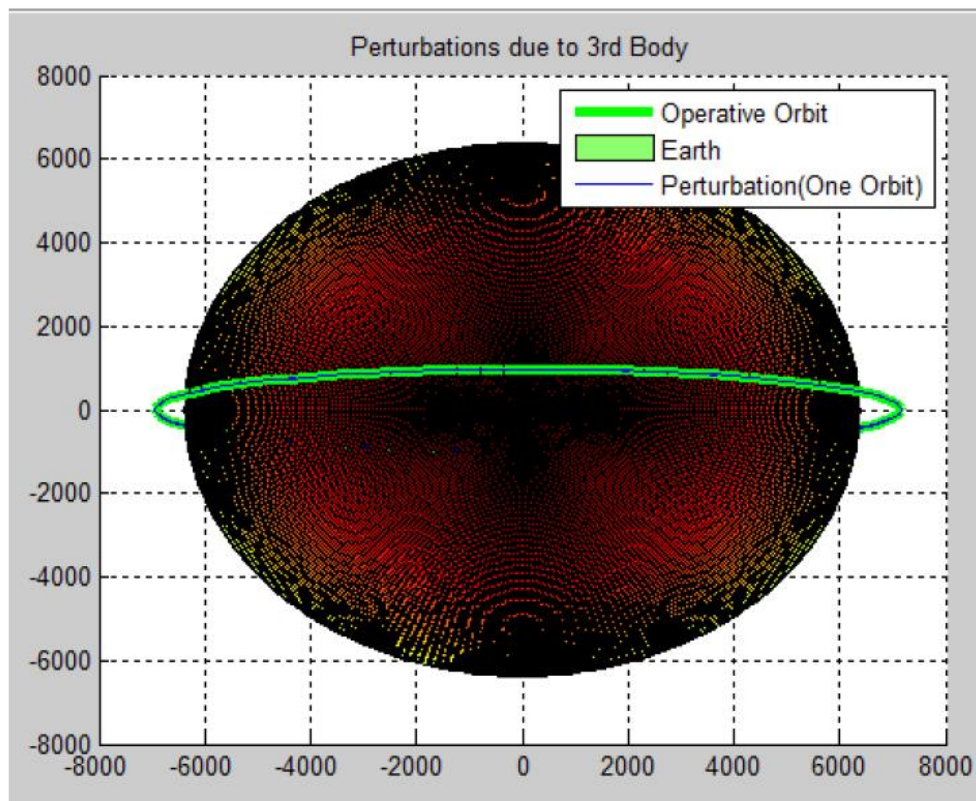


Figure 7 – atmospheric drag for long and short term

4.5 Total Effects

The combined effects of all perturbations considered are in fact comparable to non-uniform gravity case, in the sense that the results and plots show are striking similarity. This is the significant perturbation that affects the satellite in orbit.

The change in the right ascension over the entire secular period is 9.3425, which is nearly 1.5 rotations of the apseline. However in the diurnal analysis, it is seen that there are some observable changes in the Keplerian parameters, while in the short period the analysis is the same as for the non-uniform gravity.

Due to the presence of all the perturbation effects, the semi major axis changes by about 30 km and 20 km for long and short period respectively. The eccentricity of the orbit shows a deviation from its original value, changing by 0.002 in short and 0.045 in long term analysis.

For the variation of the inclination, same results are obtained for non-uniform gravity for the short period analysis while for the long period, the inclination increases more, although the variation range of 0.01° is not very significant.

In this case neither the short nor long period analyses are affected by other perturbations other than non uniform gravity. The reasons for the individual perturbation effects are already discussed in their respective sections.

As far as the perigee anomaly is concerned, it is evident that the short period analysis gives identical results as compared to non-uniform gravity analysis whereas the long period analysis gives slightly different plots. In particular, the minimum value that ω finally obtains is smaller than the previous one (both diurnal and secular), so the overall perturbation effects amplifies the variation of this anomaly.

Table 9 – total perturbation effects

	Short Period Min.	Short Period Max.	Long Period Min.	Long Period Max.	Earth Orbit
a[km]	7044.5	7063.23	7015.041	7047.262	7044.5
e[-]	0.0152	0.01721	0.01133	0.01618	0.0161
i[rad]	1.70849	1.70868	1.7086	1.70896	1.7087
Ω [rad]	3.1416	3.1427	3.1416	9.3425	3.1416
ω [rad]	-0.00	0.2085	-19.4	0	0
θ [rad]	1.5708	7.8316	1.571	33735.421	3.1416

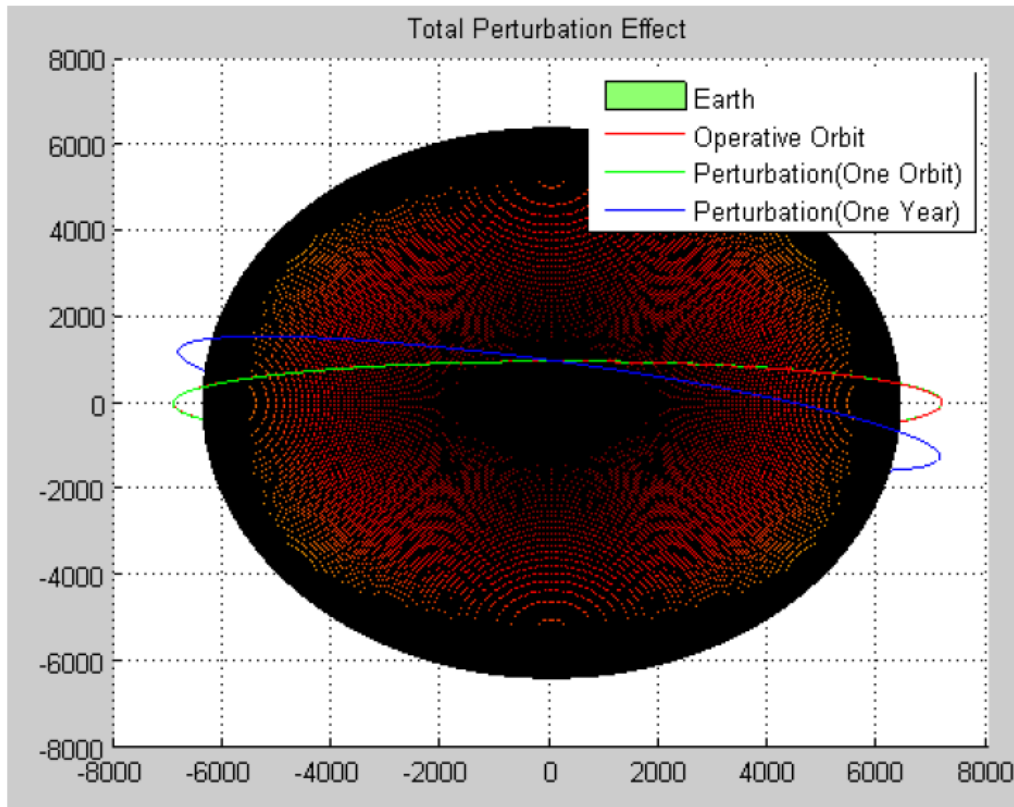


Figure 8 – total perturbation for long and short term

5. Discussions

A transfer of satellite from circular Mars orbit (250 km) around the sun and into a specified final orbit around Earth was conducted. With the flexibility of selecting the initial Keplerian parameters, it was convenient to opt for the most desirable solution by choosing the one which is most cost effective. After several combinations of start dates and nature of transfer orbit, the total time for the entire mission right from the time of departure to the required orbit insertion at end, was 108 days, of which 88 was just for the heliocentric transfer.

The exit occurs from the dark side of Mars with a hyperbolic velocity of 4.8 km/s and achieving 0.016 km/s at the Mars Sphere of Influence. On approach to the Earth's sphere of Influence, the velocity is 4.63 km/s and aiming radius is 40928 km, entering into a hyperbola with eccentricity of 2.421.

On striking a balance for the optimal θ value (for the first point of orbit insertion), two main approaches were designed (entering directly into the final orbit from the hyperbolic entry and changing the plane and inclination thereafter, or initially entering into a larger elliptic and performing the necessary manoeuvres prior to final orbit insertion) and consequently their delta-V figures were obtained, showing clear indication of the best path to opt.

Ultimately the cost function of the entire mission was minimised to 8.7 km/s. Ideally for a minimum transfer from Mars to Earth, a Hohmann trajectory results in 5.6 km/s delta-V. As this case involved non-circular orbits, different plane transfers and finally an orbit around Earth inclined at 97.9° , the extra impulsive costs were due to them.

The perturbation analysis work involved two major types- Diurnal and secular, accounting for the short (single orbit) and long (one solar year) terms.

The major contributor towards the overall perturbations was non-uniform mass distribution of the Earth. The effects resulting from atmospheric drag, solar radiation pressures and 3rd body presence was minimal, maybe even negligible only for the diurnal case. The orbit around Earth was really close to its surface to be affected by its oblateness but not close enough to be affected by presence of atmosphere. The latter had little influence for both diurnal and secular periods of consideration.

The orbit was inclined at 97.9° to the equatorial plane, hence more likely to be affected by the uneven mass distribution. It means that the satellite was revolving almost above the two poles in an orbit crossing the equator. The maximum difference arising in the non-uniformity of mass is at the poles, which are 21 km closer to the centre of earth than the equator.

The spacecraft was assumed to be a cube and having a direction normal to the velocity vector, which meant a uniform reception of solar pressure at all times, which was then divided by 6 (for single side of cube), to obtain the pressure at any point time. Errors could also arise due to the fact that solar action is not valid during the orbital time when the spacecraft is on the dark part of its orbit. Also, a uniform mass distribution of the spacecraft itself ignores any gravitational effects by producing torque as a result.

The most significant change due to the total perturbation effects was the shift in Right ascension of the orbit that caused the apseline to rotate by 0.07° during one orbit and as much as 350° over one solar year.

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