Accurate Prediction of Non-Gravitational Aerodynamic Forces for the GOCE Satellite Using the Test Particle Monte Carlo Method

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Abstract

A fundamental prerequisite for satellites operating in a Low Earth Orbit (LEO) is the availability of fast and accurate prediction of non-gravitational aerodynamic forces, which is characterised by the free-molecular flow regime. However, conventional computational methods like the analytical integral method and direct simulation Monte Carlo technique are found failing to deal with flow shadowing and multiple reflections or computationally expensive. This work develops a general computer program for the accurate calculation of aerodynamic forces in free molecular flow regimes using the test particle Monte Carlo method, and non-gravitational aerodynamic forces for the GOCE satellite is calculated for different freestream conditions and gas-surface interaction models by the computer program.

1. Introduction

The Gravity field and steady-state Ocean Circulation Explorer (GOCE) satellite, successfully launched on March 17th, 2009 with the ROCKOT Launcher from Plesetsk Cosmodrome in northern Russia, is the first Earth Explorer Core Mission of ESA's Living Planet Program. The mission objectives of GOCE, as formulated in 1998 [1], are the determination of the global field of geoid heights with an accuracy of 1 cm and of gravity anomalies with an accuracy of $10^{-5}\ m/s^2$, as well as to achieve this with a spatial resolution of 100 km half wavelength. To accomplish the scientific objectives, GOCE flies in a Sun-synchronous Low Earth Orbit (LEO). The altitude has to be as low as possible, to maximize sensitivity to the gravity field. The design altitude range was 250-280 km [2]. However, at this altitude, non-gravitational aerodynamic forces, especially the drag effects are still significant due to the thermosphere, which is a major aerodynamic problem. Therefore, in order to guarantee the accuracy of gravity gradiometry and orbit determination by satellite-to-satellite tracking, the Drag Free and Attitude Control (DFAC) system has to be designed to compensate the along track non-gravitational force using an ion engine and a set of magnetic torques for attitude control, leaving the satellite to free fall subject only to gravity [3]. A fundamental prerequisite for the DFAC system is the availability of accurate prediction of non-gravitational aerodynamic forces for the GOCE Satellite. The highly rarefied atmosphere at the altitudes of interesting LEO (>200 km) is commonly characterized as free molecular flows. Computational methods like the analytical integral method and direct simulation Monte Carlo (DSMC) are important tools in accurately computing aerodynamic forces. However, the former can't deal with multiple reflections caused by complex concave geometries, and the latter is computationally expensive and cannot be employed real time.

Relatively speaking, the test particle Monte Carlo (TPMC) method can reduce a large amount of storage capacity and computer time and could simulate different gas surface interaction models for energy and momentum exchange. What's more, it can also model the effects of flow shadowing and multiple reflections caused by complex concave geometries. Therefore, the TPMC method is ideal for free molecular flows and more practical than the DSMC and analytical integral method for the predictions of non-gravitational aerodynamic forces of satellites in LEO.

This work develops a general computer program for the accurate calculation of aerodynamic forces in free molecular flow regimes using the TPMC method. The TPMC code developed is validated by comparing the aerodynamic coefficients computed here with those calculated by free molecular theoretical formulas [4] using a flat plate model. Then, the TPMC method is applied to the GOCE satellite with complex geometry and accurately known object characteristics (shape, size and attitude) for calculation of its aerodynamic forces and torques using different gas surface interaction models. More importantly, the effects of flow shadowing and multiple reflections caused by some surfaces of the GOCE satellite are quantitatively researched and analysed with heuristic arguments based on the molecular theory. The accurate prediction of aerodynamic forces for the GOCE satellite using the TPMC method could be used to not only design of the DFAC system but also derive accurate density estimates along the GOCE orbit. The atmospheric densities and winds are calculated by Koppenwallner [5] using aerodynamic data of satellites in LEO.

2. Test particle Monte Carlo method

The TPMC method was proposed by Davis [6] in 1960 for calculation of molecular flow rates through a cylindrical elbow and pipes of other shapes. The test particles, which represent real molecules, are sequentially fired into the computational domain and each test particle represents a large number of real molecules. The molecules are fired with velocities that are probabilistically determined. The velocity is composed of a constant freestream bulk velocity and the probabilistically determined thermal velocity. [7] The test particles interact with the surface but do not undergo intermolecular collisions. Therefore, The TPMC method is ideal for free-molecular flows, and can model hydrodynamic interactions, including the flow shadowing and multiple reflections caused by complex or concave geometries. Further, it can also simulate different GSI models for energy and momentum exchange. The distinguishing feature of the TPMC method is that the representative molecular trajectories are generated serially rather than simultaneously, and it has proved particularly useful for flows with complicated boundaries that lead to multiple surface reflections during a single trajectory [8]. The TPMC method has been successfully applied to calculation of free-molecular aerodynamics of spacecraft by Klinkrad et al. [9] and backscatter of outgassing molecules from simple spacecraft surfaces by Fan et al. [10].

2.1 General schemes of TPMC simulation

The TPMC method simulates flow by tracing the trajectories of test particles in a rectangular-prism computational domain which contains the GOCE geometric model. It is assumed that each test particle represents a large amount of real gas molecules, and then during the motion of a test particle, it can strike the surfaces of the GOCE and be reflected according to a given GSI model. When impinging on some surface, the test particles exchange a part of their momentum and energy with the GOCE, and the reflected particles either can collide on other surfaces of the GOCE again or leave the computational domain. After tracing a large quantity of test particles, the macroscopic aerodynamic characteristics of the GOCE are calculated statistically.

2.2 Start of a test particle trajectory

The freestream molecules have a Maxwell-Boltzmann distribution of velocities with the mean velocity and temperature equal to those of gas at infinity, and the Maxwellian distribution function has the form:

$$f_0\left(\boldsymbol{c}'\right)=\pi^{-3/2}c_{\mathrm{m}}^{-3}\exp\left[-c_{\mathrm{m}}^{-2}\left(u'^2+v'^2+w'^2\right)\right], \tag{1}$$

where $c_{\mathrm{m}}=\sqrt{2kT/m}$ is the most probable molecular thermal speed, k the Boltzmann constant, T the temperature of gas molecules, m the gas molecular mass and u', v', w' are Cartesian components of $\boldsymbol{c}'$, the molecular thermal velocity vector. Each test particle starts from one of the sides of the rectangular-prism computational domain with the probability

$$p_l=Q_l\Big/\sum_{i=1}^{6}Q_i\,, \tag{2}$$

where Q_l is the particle number flux across the side numbered l with area A_l and the unit outward normal vector $\boldsymbol{n}_l$. The expression of Q_l is given by

$$Q_l=A_l\int_{-\boldsymbol{n}_l\cdot\boldsymbol{c}>0}n_{\mathrm{f}}\left(-\boldsymbol{n}_l\cdot\boldsymbol{c}\right)f_0(\boldsymbol{c}')\mathrm{d}\boldsymbol{c}, \tag{3}$$

where n_{f} is the freestream number density of gas molecules at infinity,

$$\boldsymbol{c}=\boldsymbol{c}_{\mathrm{f}}+\boldsymbol{c}' \tag{4}$$

is the velocity vector of gas molecules and $\boldsymbol{c}_{\mathrm{f}}$ freestream velocity at infinity. Having the probability determined, with the knowledge that the start points of test particles are uniformly distributed over the faces of the computational domain, their position coordinates in the wind axes system could be obtained straightforwardly.

Before proceeding further with the velocity of a test particle at start point, let us now define two dimensionless quantities as follows for later use:

$$\begin{cases} \boldsymbol{s} = \boldsymbol{c}_{\mathrm{f}} / c_{\mathrm{m}} \\ \boldsymbol{\xi} = \boldsymbol{c} / c_{\mathrm{m}} \end{cases}, \tag{5}$$

where $\boldsymbol{s}$ is the freestream speed ratio vector with s as its modulus and $\boldsymbol{\xi}$ the dimensionless velocity vector. Now we continue with the determination of the velocity at start point. According to the free-molecular theory, the density of distribution of the dimensionless velocity $\boldsymbol{\xi}$ could be written as

$$\begin{cases} p(\xi_{\mathrm{n}}) = \dfrac{2\xi_{\mathrm{n}} \exp\left[-(\xi_{\mathrm{n}} - s_{\mathrm{n}})^2\right]}{\exp(-s_{\mathrm{n}}^2) + \sqrt{\pi} s_{\mathrm{n}} \left[1 + \operatorname{erf}(s_{\mathrm{n}})\right]} \\ p(\xi_{\mathrm{t1}}) = \dfrac{1}{\sqrt{\pi}} \exp\left[-(\xi_{\mathrm{t1}} - s_{\mathrm{t1}})^2\right] \\ p(\xi_{\mathrm{t2}}) = \dfrac{1}{\sqrt{\pi}} \exp\left[-(\xi_{\mathrm{t2}} - s_{\mathrm{t2}})^2\right] \end{cases}, \tag{6}$$

where the subscripts n, t refer to the face normal and tangential components respectively, and hence $\mathrm{t1}, \mathrm{t2}$ represent arbitrary two orthogonal components in the tangent plane of the face. Starting from the density of distribution, we could readily gain access to the sampling of velocities of test particles at start points.

2.3 GSI model and reflection

The physics behind the exchange of energy and momentum is defined by the choice of energy accommodation coefficient and reflection model, respectively when gas molecules impinge on the plate surface. Energy accommodation coefficient, α_{E}, is formally defined as the fraction of the kinetic energy lost by the molecules incident on the plate surface upon reflection given by:

$$\alpha_{\mathrm{E}} = (E_{\mathrm{i}} - E_{\mathrm{r}}) / (E_{\mathrm{i}} - E_{\mathrm{w}}) \tag{7}$$

where E_{i} is the average energy of gas molecules imping on the surface, E_{r} the average energy of gas molecules leaving the surface, and E_{w} the average energy that gas molecules would carry away if they came into thermal equilibrium with the surface.

With respect to the momentum transfer, gas molecules are assumed to be reflected from the plate surface using the Maxwellian GSI model, which is constructed on the assumption that a fraction $1 - \sigma$ of molecules is specularly reflected from the surface, and a fraction of σ is reemitted diffusively. The fraction of incident molecules that are reflected diffusively is usually characterized by the surface accommodation coefficient also termed the Maxwell accommodation coefficient. Let the subscripts i, r denote the incident and reflected gas molecules, respectively, and then the velocity of test particles that are specularly reflected off has the form,

$$\boldsymbol{c}_{\mathrm{r}} = \boldsymbol{c}_{\mathrm{i}} - 2(\boldsymbol{n} \bullet \boldsymbol{c}_{\mathrm{i}})\boldsymbol{n}, \tag{8}$$

where $\boldsymbol{n}$ is the external normal of the surface. Diffusive reflection implies that the molecules reflected from a surface have a Maxwellian distribution of velocities with the mean velocity and temperature equal to those of surface, and hence the velocity of diffusively reflected test particles is sampled in a local spherical coordinate system, directed along the outward normal to the surface of the plate as follows:

$$\begin{cases} c_{\mathrm{r}} = c_{\mathrm{w}}\sqrt{-\ln\left(R_1R_2\right)} \\ \cos\theta = \sqrt{R_3} \\ \varphi = 2\pi R_4 \end{cases}, \tag{9}$$

where is c_{r} the modulus of $\boldsymbol{c}_{\mathrm{r}}$, $c_{\mathrm{w}} = \sqrt{2kT_{\mathrm{w}}/m}$ is the most probable molecular thermal speed corresponding to the temperature of the plate T_{w} and θ, φ are elevation and azimuth angles.

2.4 Calculation of normalized aerodynamic characteristics

If a test particle is reflected, the next possible point of intersection with plate surfaces is evaluated, and in case of a hit, the total incident momentum is adjusted by adding the momentum of this secondary reflection. Correspondingly, if a third or even more reflection occurs, the aforementioned process should be repeated to taking into account the momentum and energy transfer in the process of reflection. Interaction properties such as momentum and energy received from each test particle colliding on and reflecting off the plate surface are collected in corresponding counters. On the basis of collected information, the aerodynamic coefficients are calculated in the following manner:

$$\begin{cases} \boldsymbol{C}_{\mathrm{F}} = A\sum_{j=1}^{N} m\left(\boldsymbol{c}_{\mathrm{r}} - \boldsymbol{c}_{\mathrm{i}}\right) \\ C_{\mathrm{h}} = \dfrac{A}{c_{\mathrm{f}}}\sum_{j=1}^{N}\left(E_{\mathrm{i}} - E_{\mathrm{r}}\right) \end{cases}, \tag{10}$$

where

$$A = 2\sum_{l=1}^{6} Q_l \Big/ \left(Nc_{\infty}^2 A_{\mathrm{ref}}\right) \tag{11}$$

is used for normalization of aerodynamic coefficients, A_{ref} is the reference area, and N is the total number of test particles.

3. Validation of computation method

In this part, numerical calculations have been carried out for flows past a single circular flat plate (Figure 1), and a comparison between TPMC results and free-molecular flow theoretical formulas are plotted in Figure 2. The computational conditions are given in Table 1. Variations of drag coefficient, lift coefficient, lift-to-drag ratio and heat transfer coefficient over a range of freestream angles of attack are shown in the figure. Note that all the four aerodynamic coefficients for three different freestream speed ratios ($s = 1, 5, 10$) are shown. As freestream speed ratio s rises, all aerodynamic coefficients decrease. With the increment of freestream angle of attack α, drag and heat transfer coefficient are increased monotonically, while the qualitative behavior of lift coefficient and lift-to-drag ratio are not monotonic, taking on both increasing at small α and decreasing at large values of α. Obviously, all TPMC results and analytical expressions match very well, proving that the method used here is valid.

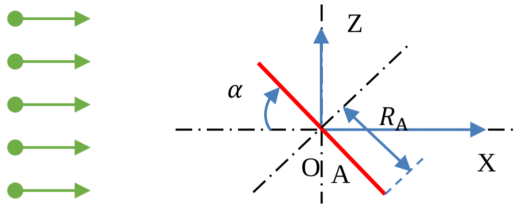
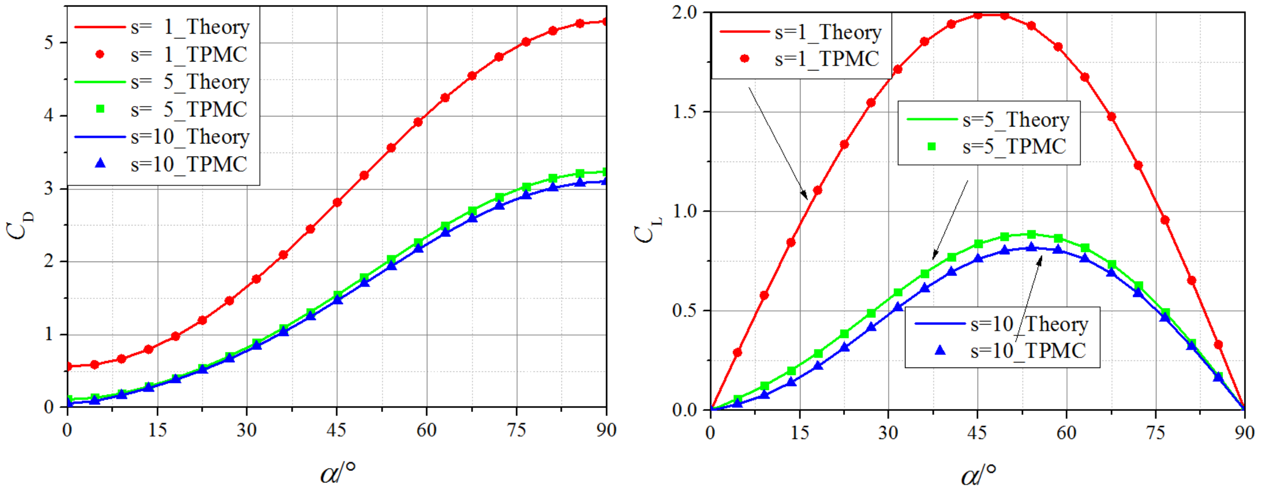


Figure 1: This is the schematic sketch of a free-molecular flow past a circular flat plate.

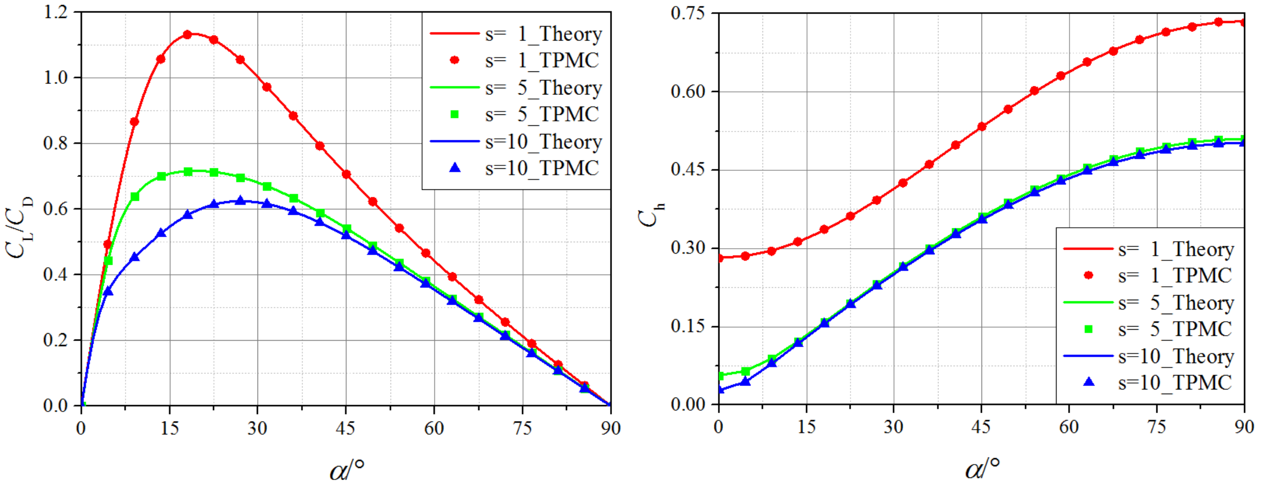
Table 1: This is the computational conditions of free-molecular flows past a flat plate.

Quantities	**Values**	**Quantities**	**Values**
Freestream molecular weight, M_f	29.0	The radius of plate A, R_A	1.0 m
Freestream temperature, T_f	300 K	The plate surface temperature, T_w	300 K
Freestream specific heat ratio γ	1.4	Maxwell accommodation coefficient, σ	0.5
Freestream molecular speed ratio, s	5.0	Energy accommodation coefficient, α_E	1.0



(a) Drag Coefficient

(b) Lift Coefficient



(c) Lift-to-Drag ratio

(d) Heat Transfer Coefficient

Figure 2: Validations of the aerodynamics coefficients of the flat plate.

4. An analysis of statistical fluctuations

As a class of Monte Carlo methods, the TPMC method suffers from statistical fluctuations intrinsically, and there are computational procedures that can cause the fluctuations to be enhanced or reduced as a result of numerical effects. According to the statistical theory, an effective measure to reduce the fluctuations is to increase the sample size. In other words, the more the total number of test particles, N_tp becomes, the less the statistical fluctuations keep. However, it is recommended that N_tp should not be set to an excessively larger value, because extremely large number of test particles could lead to the fact that computational procedures become unduly inefficient. It is therefore desirable to have a quantitative description of the fluctuations that occur in the TPMC simulation of aerodynamic force acting on the GOCE satellite to obtain a recommended value of N_{tp}.

Several test cases were run with the variable N_{tp} increased by a factor of ten, so that the total number of test particles N_{tp} has been set to 10^8. In order to check that the procedures produce satisfactory results and the statistical fluctuations remain negligible, twenty computations has been implemented with N_{tp} set to 10^8 with computational conditions tabulated in Table 2 and results presented in Table 3. Besides, a statistical analysis has been made in Table 4, which gives an average axial force of 6.384 mN and an average axial force coefficients of 4.84788, with a standard deviation of 0.004 and 0.003, respectively, and a coefficient of variation of 0.0006344 and 0.0006333. Note that each run takes about 30 minutes, which is acceptable in most cases. These results indicate that statistical fluctuations keep negligible and the computational efficiency is relatively high when the total number of test particles N_{tp} is set to 10^8.

Table 2: This is the computational conditions for the calculation of aerodynamic forces for the GOCE Satellite.

Quantities	**Values**	**Quantities**	**Values**
Freestream molecular weight, M_{f}	19.19	Temperature of surfaces of GOCE, T_{s}	941.33 K
Freestream speed, c_{f}	$7760\ \mathrm{m/s}$	Maxwell accommodation coefficient, σ	1.0
Freestream temperature, T_{f}	941.33 K	Energy accommodation coefficient, α_{E}	1.0
Freestream molecular speed ratio, α	0°	Freestream specific heat ratio, γ	1.4
Freestream mass density, ρ_{f}	$6.073 \times 10^{-11}\ \mathrm{kg/m^3}$	Total number of test particles, N_{tp}	10^8

Table 3: TPMC results of twenty computations for $N_{\mathrm{tp}} = 10^8$

Run	$\boldsymbol{F_{\mathrm{X}}}$ **(mN)**	$\boldsymbol{C_{\mathrm{X}}}$	**Run**	$\boldsymbol{F_{\mathrm{X}}}$ **(mN)**	$\boldsymbol{C_{\mathrm{X}}}$	**Run**	$\boldsymbol{F_{\mathrm{X}}}$ **(mN)**	$\boldsymbol{C_{\mathrm{X}}}$
1	6.38465	4.84835	**8**	6.38634	4.84964	**15**	6.38191	4.84627
2	6.38498	4.84860	**9**	6.39234	4.85419	**16**	6.38183	4.84621
3	6.38300	4.84710	**10**	6.37603	4.84181	**17**	6.38816	4.85102
4	6.37902	4.84408	**11**	6.38881	4.85152	**18**	6.38693	4.85009
5	6.38654	4.84979	**12**	6.37893	4.84401	**19**	6.38247	4.84670
6	6.38308	4.84716	**13**	6.37970	4.84460	**20**	6.38089	4.84549
7	6.38703	4.85016	**14**	6.38783	4.85077			

Table 4: A descriptive statistical analysis of TPMC results of twenty computations for $N_{\mathrm{tp}} = 10^8$

	Mean value (mN)	**Standard deviation (mN)**	**Coefficient of variation**
$\boldsymbol{F_{\mathrm{X}}}$	6.38402	0.00405	0.000634397
$\boldsymbol{C_{\mathrm{X}}}$	4.84788	0.00307	0.000633266

5. Contours of aerodynamic coefficients on each surface

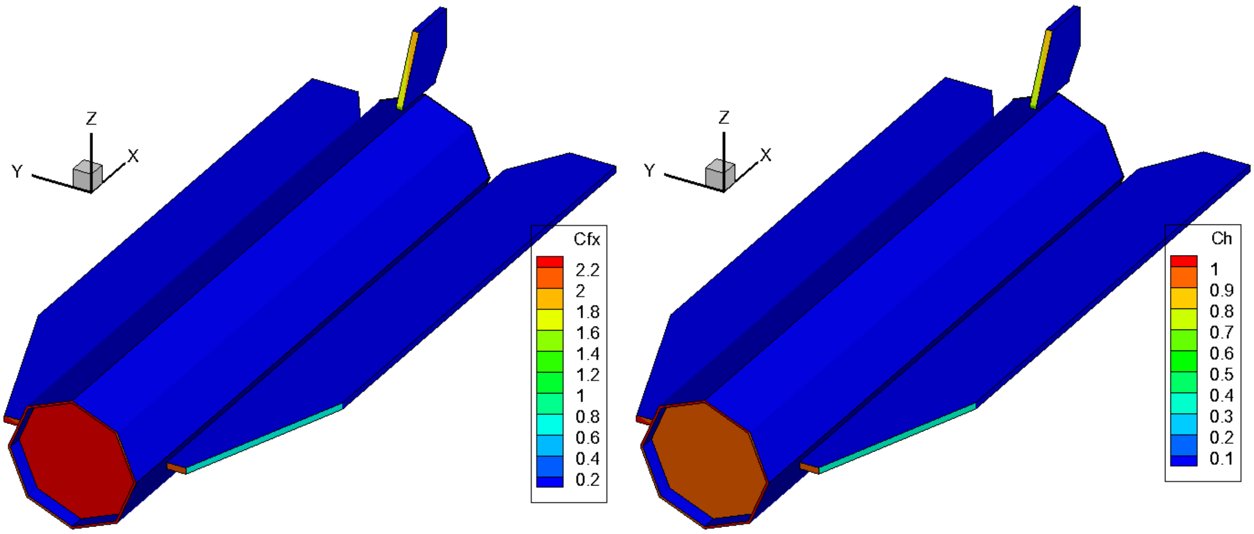
Having the computational method validated and statistical fluctuations analysed, we now focus on the contours of aerodynamic coefficients, including axial forces, slide forces, normal forces and heat transfer coefficients on each surface of the GOCE, aiming at gaining insight of aerodynamic force and heat flux distribution on each surface. Contours of aerodynamic coefficients resolved into the vehicle body axes system on each surface of the GOCE satellite are given in Figure 3.

As is shown in Figure 3(a) and (b), it is clear that the axial force and heat transfer are mainly distributed on the front surface at the head of the GOCE satellite because the angle made by the surface with freestream is 90°. Those surfaces which are parallel to the freestream have a much smaller axial force and heat flux in comparison with the front surfaces at the head of the GOCE. However, with regard to the surfaces which are shaded from the freestream, the axial force and heat flux acting on them keep extremely, even almost zero.

In terms of slide forces, as is illustrated in Figure 3(c), (d), slide forces of most surfaces of the GOCE stay zero as a result of the case that the freestream is parallel to the axis of symmetry. However, the eight inner surfaces at the heat of the GOCE have a noticeable slide force. Moreover, the contours of slide force coefficients on the eight surfaces are symmetrical about the plane $y = 0$. More specifically, some surfaces whose components of unit outward normal vectors in the y direction keep positive receive a negative axial force, while a positive axial force exerts on other surfaces whose outward normal vector components in the y direction remain negative. Furthermore, the more normal vectors of the surfaces become parallel to the y direction, the more magnitudes of their slide force coefficients are.

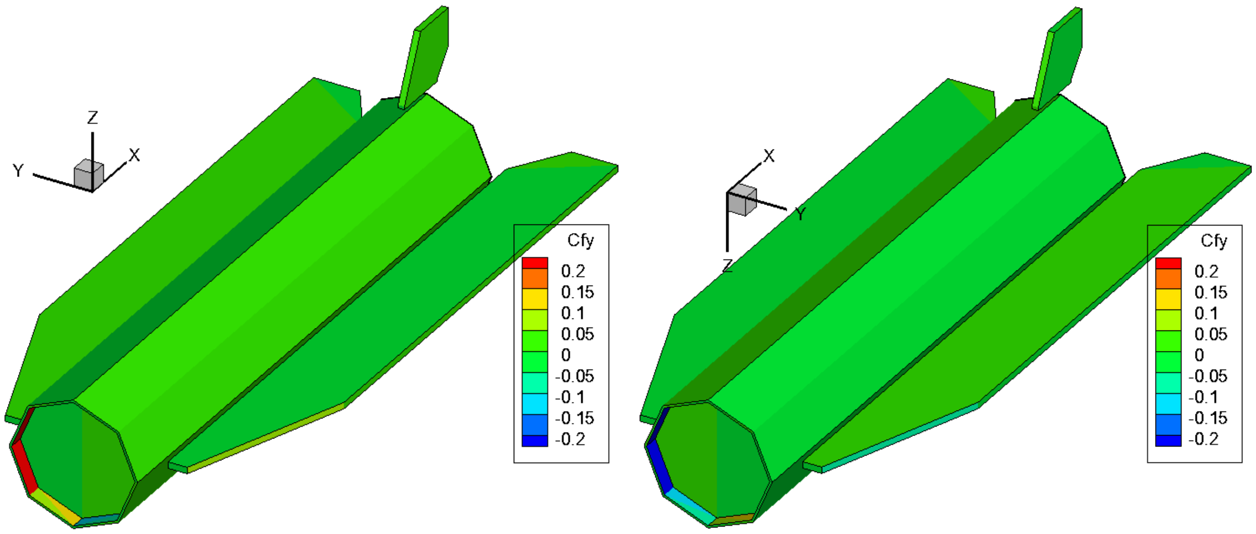
Similarly, with respect to normal forces, as is plotted in Figure 3(e), (f), normal forces of most surfaces of the GOCE stay zero. However, the eight inner surface at the heat have a considerable normal force. Moreover, the contours of normal force coefficients on the eight surfaces are symmetrical about the plane $z = 0$. Furthermore, the more normal vectors of the surfaces become parallel to the z direction, the more magnitudes of their normal force coefficients are.

According to the free molecular aerodynamics theory, both slide and normal force coefficients on the inner side surfaces at the head of the GOCE satellite should be equal to 0. Therefore, it is concluded that the fact that some of the surfaces have a positive slide or normal force while others have a negative slide or normal force results from the effects of multiple reflections between the front surface and the inner side surfaces. Besides, the TPMC method is capable of simulating the effects of multiple reflections quantitatively that the analytical integral method can't deal with.

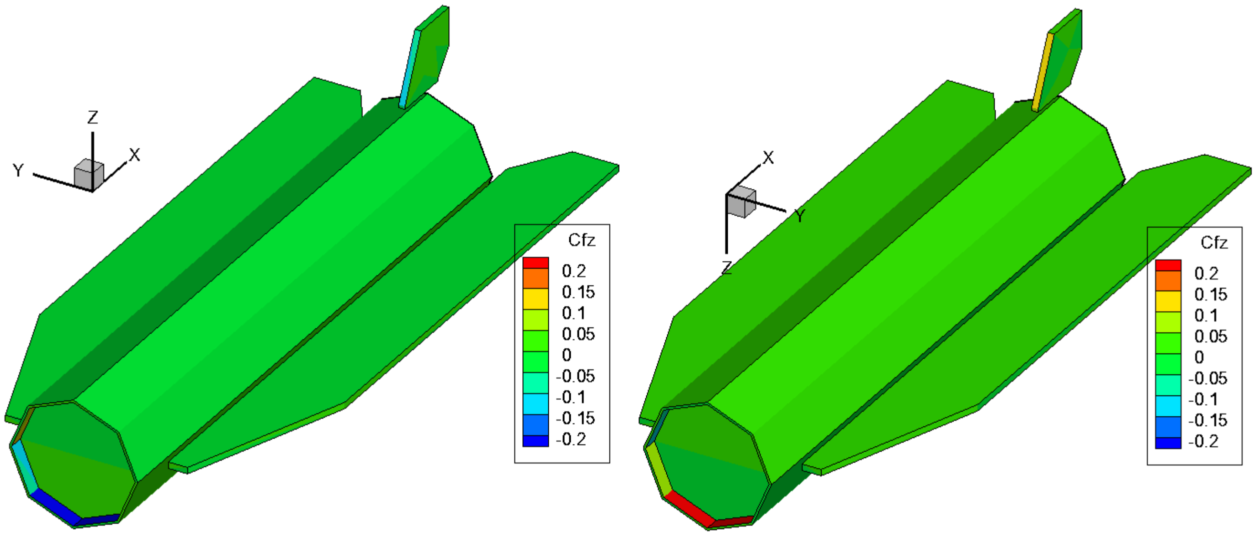


(a) Axial force coefficient

(b) Heat transfer coefficient



(c) Slide force coefficient (d) Slide force coefficient (seen in other orientation)



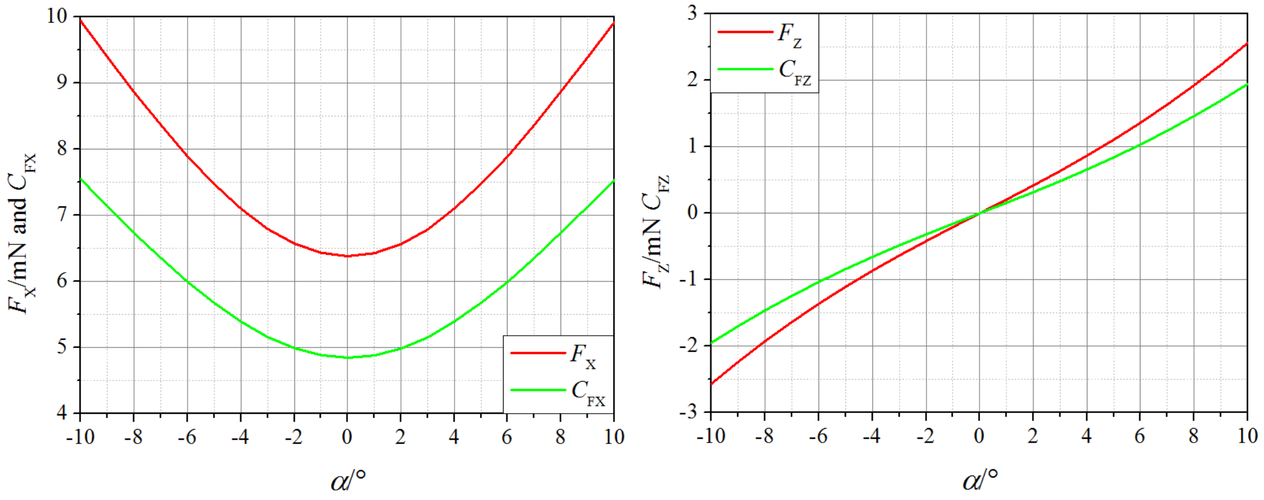
(e) Normal force coefficient (f) Normal force coefficient (seen in other orientation)

Figure 3: Contours of aerodynamic coefficients on each surface of the GOCE satellite.

6. Analyses of aerodynamic properties of the GOCE

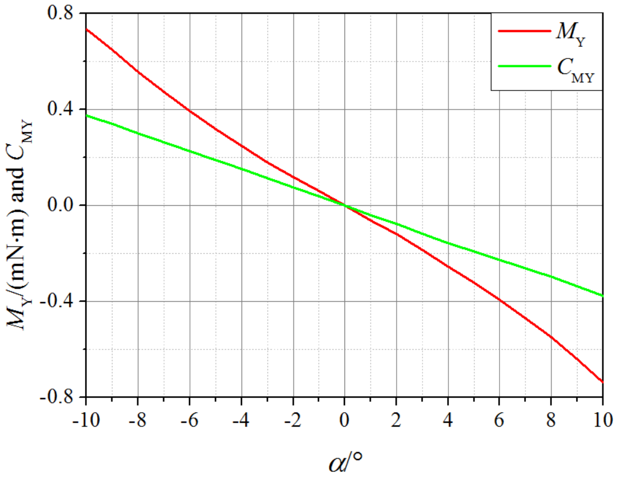
Let us proceed with the analyses of aerodynamic properties of the GOCE for a variety of freestream conditions and GSI models, in order to provide some prerequisites when the GOCE cruises in a LEO or alters its altitude. At first, variations of the axial force, normal force, pitching moment and their corresponding coefficients over a range of the freestream angles of attack are shown in Figure 4. Note that the computational conditions are tabulated in Table 2.

As is shown in Figure 4(a), the axial force (coefficient) keeps positive and symmetrical about line $\alpha = 0°$ in the whole range of α. More detailedly, the axial force (coefficient) beomes decreasing with the growth of α from $-10°$ to $0°$ while keeps increasing when α is increased from $0°$ to $10°$, and it takes its maximus value at the angle of $0°$. Figure 4(b) illustrates that the normal force (coefficient) stays negative in the range of α from $-10°$ to $0°$ while remains positive when α ranges from $0°$ to $10°$. Resulting from the geometrical symmetry, the normal force equals zero when α has a value of $0°$. In terms of the pitching moment (coefficient), as is plotted in Figure 4(c). It is clear that the quantitative behaviour is exactly opposite to that of the normal force. Similarly, the pitching moment goes to zero at the value of $\alpha = 0°$.



(a) Axial force (coefficient)

(b) Normal force (coefficient)



(c) Pitching moment (coefficient)

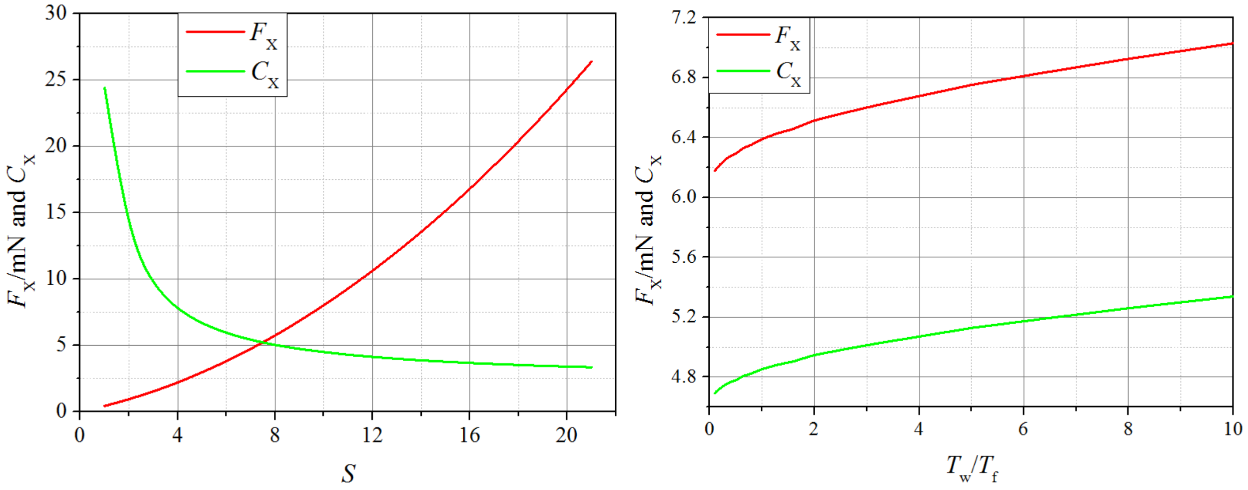
Figure 4: Aerodynamic properties of the GOCE satellite in terms of freestream angle of attack α.

We continue with a detailed observation insight the aerodynamic properties of the GOCE by providing some results of the axial force and its corresponding coefficient as a function of freestream conditions and GSI model parameters, including the freestream molecular speed ratio s, the ratio of freestream temperature to the surface temperature $T_\mathrm{w}/T_\mathrm{f}$, Maxwell accommodation coefficient σ and energy accommodation coefficient α_E. The corresponding plots are shown in Figure 5.

Variations of the axial force and its corresponding coefficient over a range of freestream molecular speed ratio s are shown in Figure 5(a). Observing the two results in this figure, it is seen that there are major qualitative differences. With the increase of s, the axial force is increased monotonically while the axial force coefficient has an exponential decrease resulting from the denominator keeps a second-order growth.

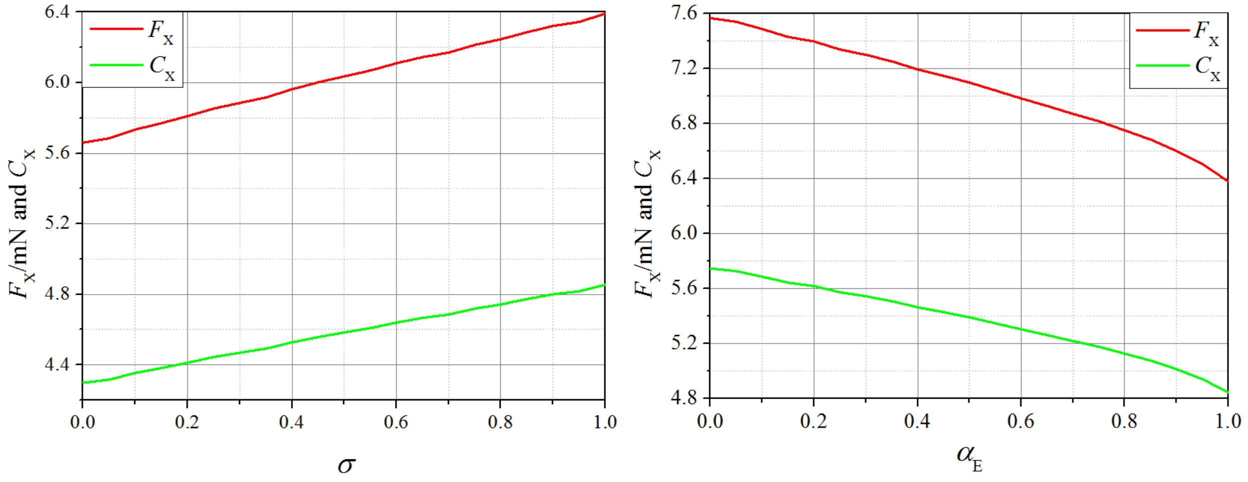
In reference to the influence of the surface temperature (related to the temperature ratio $T_\mathrm{w}/T_\mathrm{f}$) on axial forces, as is plotted in Figure 5(b), when surfaces of the GOCE turn a heat wall from a cold one, both the axial force and its coefficient have a slight rise, and the rise becomes nearly linear at large values of $T_\mathrm{w}/T_\mathrm{f}$.

Figure 5(c) and (d) illustrate variations of the axial force (coefficient) over a range of GSI model parameters. The trends in the data of Figure 5(c) indicate that, when the reflection model changes from a fully specular reflection to a purely diffusive one, the axial force (coefficient) has a linear increase. That is to say, the axial force goes to its minimum value when Maxwell accommodation coefficient σ equals 0, i.e. GSI model is a fully specular, while reach its maximum value when σ equals 1, i.e. GSI model is a purely diffusive. However, when energy accommodation coefficient α_E is increased from 0 to 1, the axial force (coefficient) has a slight drop. When all reflected molecules keep adaptive to the surface temperature, the axial force goes to its minimum value, and reaches its maximum value when all reflected molecules could not adjust to the surface temperature.



(a) Freestream molecular speed ratio, s

(b) Temperature ratio, T_w/T_f



(c) Maxwell accommodation coefficient, σ

(d) Energy accommodation coefficient, α_E

Figure 5: Variations of axial force (coefficient) in terms of different freestream conditions and GSI models.

7. Concluding remarks

In this work, a general computer program for the accurate calculation of aerodynamic forces in free molecular flow regimes using the TPMC method has been developed and non-gravitational aerodynamic forces for the GOCE satellite operating in a LEO have been calculated for different freestream conditions and GSI models by the computer program. Before the calculation, some primary procedures of the TPMC simulation technique were described, and a test case was performed using a flat plate model. At the same time, an analysis of statistical fluctuations of the TPMC results was made. We have obtained the following conclusions:

In the case of a free-molecular flow past a flat plate, the aerodynamic coefficients obtained by the TPMC method are in good agreement with the corresponding expressions derived from the free-molecular flow theory, illustrating the validity and reliability of the method used here.

Since larger number of test particles will give more accurate results but require more computation time, a compromise between operation time and data accuracy must be made. The analysis of statistical fluctuations made in the proceeding sections indicates that, when the total number of test particles has the value of 10^8, the computation time is approximately thirty minutes in a single computational state, and the standard deviation keeps an order of 10^{-3}. These results indicate that statistical fluctuations keep negligible and the computational efficiency is relatively high when the total number of test particles N_{tp} is set to 10^8.

From a knowledge of the contours of aerodynamic coefficients on each surface of the GOCE, it can be concluded that, at the pitch angle of 0°, the axial force and heat transfer are mainly distributed on the front surface at the head of the GOCE satellite. In addition, resulting from the effects of multiple reflections between the front surface and the inner side surfaces, some of the eight inner surfaces at the heat of the GOCE have a noticeable positive slide or normal force while others have a negative slide or normal force. Besides, the slide and normal force distributions on the eight surfaces are perfectly symmetrical about the plane $y = 0$ and $z = 0$, respectively.

When the GOCE needs to alter its altitude or orbit, the freestream speed and angle of attack become variable. Thus, it is necessary to let the variations of aerodynamic forces in terms of aforementioned two parameters be known. With the increase of s, the axial force keeps positive throughout, and is increased monotonically while the axial force coefficient has an exponential decrease. With the growth of α from $-10°$ to $10°$, the axial force becomes decreasing firstly while keeps increasing lastly. However, the normal force stays negative in the range of α from $-10°$ to 0° while remains positive when α ranges from 0° to 10°. Besides, the quantitative behaviour is exactly opposite to that of the normal force. Resulting from the geometrical symmetry, both the normal force and pitching moment equal zero when α has a value of 0°.

It is a fundamental prerequisite for the GOCE's effective work to have a knowledge of the influence of surface temperature and GSI model parameters on its aerodynamic forces. When surfaces of the GOCE turn a heat wall from a cold one, the axial force has a slight rise, and the rise becomes nearly linear at large values of $T_\mathrm{w}/T_\mathrm{f}$. The axial force goes to its minimum value when the GSI model is fully specular, while reach its maximum value with a purely diffusive model. Besides, when all reflected molecules could not be adaptive to the surface temperature, the axial force goes to its maximum value, while reaches its minimum value with all reflected molecules adjusting to the surface temperature.

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