Launcher/satellite interface optimization for payload comfort

Luc-Olivier Gonidou
CNES – Launchers Directorate
52 rue Jacques Hillairet, 75612 Paris, France – luc.gonidou@cnes.fr

Abstract
Satellites are submitted to various mechanical environments during launch preparation and then during flight. To status about the mechanical compatibility of a satellite with the loading environment of a launcher, all contributors have to be taken into account: quasi-static loads, low frequency environment, acoustic, shocks, thermal.
This paper intends to present an original method used on ARIANE 5 to guide the interface design to minimize stresses in the satellite primary structure. In our case the junction between the launcher and the satellite was hyperstatic. Starting from the various static and dynamic contributors, several combinations have been generated to cover all stress states in the satellite. The load cases inducing the smallest margins in the spacecraft were isolated. Using a Singular Value Decomposition approach, the interface deformations were maximized with respect to internal spacecraft loads leading to a set of natural interface deformations. The critical load cases were then projected on this basis in order to identify a single interface deformation to be limited through launcher dispenser modifications. The method will be presented in details and illustrated with results obtained on a real satellite.

1. Introduction

To ensure mechanical compatibility between a satellite and a launcher, limit loads and verification logic, usually based on tests, are defined by launcher operators [R1, 2]. A satellite designer has then to combine many contributors and to add load factors and factors of safety to verify its design. Typical load factors and factors of safety can be found in [R3]. The contributors to the stress state inside the primary structure of a spacecraft are mainly: Quasi-Static Loads; dynamic loads; line loads due to geometrical discontinuities, differences in local stiffness or non-uniform transmission of thrust; thermal loads and integration loads. Usually Quasi-Static Loads and dynamic loads are combined in a global QSL. When a satellite is matted to a launcher through a multi-point interface – this is usually the case for multiple payload dispensers – interface warping can occur and has to be properly taken into account. In such a case, limiting satellite warping through launcher or dispenser modifications can decrease the stress state within the satellite. The aim of this paper is to present a method to identify the worst warping to limit for satellite benefice.

2. Quasi-Static Loads and Warping specification

The general equations governing the behaviour of a payload coupled to a launcher through an interface is given by:

$$\begin{bmatrix} M_{jj} & M_{ji} \\ M_{ij} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{u}_j \\ \ddot{u}_i \end{Bmatrix} + \begin{bmatrix} C_{jj} & C_{ji} \\ C_{ij} & C_{ii} \end{bmatrix} \begin{Bmatrix} \dot{u}_j \\ \dot{u}_i \end{Bmatrix} + \begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix} \begin{Bmatrix} u_j \\ u_i \end{Bmatrix} = \begin{Bmatrix} F_j \\ 0 \end{Bmatrix} \quad \textbf{(1)}$$

where the satellite degree of freedom (dof) have been partitioned between internal (i) and interface (j) degree of freedom.
Adopting a Craig&Bampton approach [R4] by writing that internal dof displacements are the sum of interface static modes and internal dynamic modes leads to the following equation:

$$\{u\}=\begin{Bmatrix} u_j \\ u_i \end{Bmatrix}=\begin{bmatrix} I & 0 \\ \Psi_{ij} & \Phi_{ip} \end{bmatrix}\begin{Bmatrix} u_j \\ q_p \end{Bmatrix} \quad \textbf{(2)}$$

with Ψ_{ij} corresponding to the interface static modes and Φ_{ip} corresponding to the internal dynamic modes with fixed interfaces. $\Psi_{ij} = -K_{ii}^{-1} K_{ij}$ and $(K_{ii} - \omega^2 M_{ii})\Phi_{ip} = 0$.
Using Eq. 2, Eq. 1 can be written as:

$$[M_c]\begin{Bmatrix} \ddot{u}_j \\ \ddot{q}_p \end{Bmatrix}+[C_c]\begin{Bmatrix} \dot{u}_j \\ \dot{q}_p \end{Bmatrix}+[K_c]\begin{Bmatrix} u_j \\ q_p \end{Bmatrix}=\begin{Bmatrix} F_j \\ 0 \end{Bmatrix} \quad \textbf{(3)}$$

where:

$$[M_c]=\begin{bmatrix} M_{jj}+M_{ji}\Psi_{ij}+\Psi_{ji}M_{ij}+\Psi_{ji}M_{ii}\Psi_{ij} & M_{ji}\Phi_{ip}+\Psi_{ji}M_{ii}\Phi_{ip} \\ \Phi_{pi}M_{ij}+\Phi_{pi}M_{ii}\Psi_{ij} & \Phi_{pi}M_{ii}\Phi_{ip} \end{bmatrix}$$

$$[C_c]=\begin{bmatrix} C_{jj}+C_{ji}\Psi_{ij}+\Psi_{ji}C_{ij}+\Psi_{ji}C_{ii}\Psi_{ij} & C_{ji}\Phi_{ip}+\Psi_{ji}C_{ii}\Phi_{ip} \\ \Phi_{pi}C_{ij}+\Phi_{pi}C_{ii}\Psi_{ij} & \Phi_{pi}C_{ii}\Phi_{ip} \end{bmatrix}$$

$$[K_c]=\begin{bmatrix} K_{jj}+K_{ji}\Psi_{ij}+\Psi_{ji}K_{ij}+\Psi_{ji}K_{ii}\Psi_{ij} & K_{ji}\Phi_{ip}+\Psi_{ji}K_{ii}\Phi_{ip} \\ \Phi_{pi}K_{ij}+\Phi_{pi}K_{ii}\Psi_{ij} & \Phi_{pi}K_{ii}\Phi_{ip} \end{bmatrix}$$

Introducing the Guyan mass matrix: $M_G = M_{jj} + M_{ji}\Psi_{ij} + \Psi_{ji}M_{ij} + \Psi_{ji}M_{ii}\Psi_{ij}$ and writing: $L = M_{ji}\Phi_{ip} + \Psi_{ji}M_{ii}\Phi_{ip}$ and $\Phi_{pi}M_{ii}\Phi_{ip} = \lfloor \mu_p \rfloor$, $\Phi_{pi}K_{ii}\Phi_{ip} = \lfloor \mu_p \omega_p^2 \rfloor$, Eq. 3 becomes:

$$\begin{bmatrix} M_G & L \\ L^t & \lfloor \mu_p \rfloor \end{bmatrix}\begin{Bmatrix} \ddot{u}_j \\ \ddot{q}_p \end{Bmatrix}+\begin{bmatrix} C_{jj}+C_{ji}\Psi_{ij}+\Psi_{ji}C_{ij}+\Psi_{ji}C_{ii}\Psi_{ij} & C_{ji}\Phi_{ip}+\Psi_{ji}C_{ii}\Phi_{ip} \\ \Phi_{pi}C_{ij}+\Phi_{pi}C_{ii}\Psi_{ij} & \Phi_{pi}C_{ii}\Phi_{ip} \end{bmatrix}\begin{Bmatrix} \dot{u}_j \\ \dot{q}_p \end{Bmatrix}+$$

$$\begin{bmatrix} K_{jj}+K_{ji}\Psi_{ij} & 0 \\ 0 & \lfloor \mu_p \omega_p^2 \rfloor \end{bmatrix}\begin{Bmatrix} u_j \\ q_p \end{Bmatrix}=\begin{Bmatrix} F_j \\ 0 \end{Bmatrix} \quad \textbf{(4)}$$

Interface forces are then function of interface nodes acceleration, velocity and displacement and internal modes acceleration and velocity.

2.1 Isostatic interface

For an isostatic interface, displacements of interface dof do not create any deformation inside the satellite. In that case, interface static modes are rigid body modes $\Psi_{ij} = \Psi_{Rij}$. Those modes do not create elastic or viscous forces. We have: $[K]\{u_R\}=\{0\}$ and $\{u_R\}^t[K]\{u_R\}=\{0\}$ or

$$\begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix}\begin{Bmatrix} I \\ \Psi_{Rij} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} I \\ \Psi_{Rij} \end{Bmatrix}^t\begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix}\begin{Bmatrix} I \\ \Psi_{Rij} \end{Bmatrix}=0$$

This is equivalent to the following relations (same relations are obtained with $[C]$ matrix):

$$K_{jj} + K_{ji}\Psi_{Rij} + \Psi_{Rji}K_{ij} + \Psi_{Rji}K_{ii}\Psi_{Rij} = 0$$

$$K_{jj} + K_{ji}\Psi_{Rij} = 0$$

$$K_{ij} + K_{ii}\Psi_{Rij} = 0$$

Using all those relations, Eq. 4 can be simplified as follow:

$$\begin{bmatrix} M_G & L \\ L^t & [\mu_p] \end{bmatrix}\begin{Bmatrix} \ddot{u}_j \\ \ddot{q}_p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \Phi_{pi} C_{ii} \Phi_{ip} \end{bmatrix}\begin{Bmatrix} \dot{u}_j \\ \dot{q}_p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [\mu_p \omega_p^2] \end{bmatrix}\begin{Bmatrix} u_j \\ q_p \end{Bmatrix} = \begin{Bmatrix} F_j \\ 0 \end{Bmatrix} \quad \textbf{(5)}$$

Interface forces are then function of interface nodes and internal modes accelerations only:

$$M_G \ddot{u}_j + L \ddot{q}_p = F_j \quad \textbf{(6)}$$

When internal local satellite modes have a minor contribution to interface forces, they can be neglected and a global QSL can be derived directly from interface loads for dimensioning purpose:

$$M_G \ddot{u}_j^{'} \approx F_j \quad \text{or} \quad M_{CDG} \ddot{u}_{QSLj} = \psi_{R\_CDGj} F_j \quad \textbf{(7)}$$

where M_{CDG} is the condensed mass matrix at CoG and ψ_{R_CDGj} are the satellite rigid modes with respect to CoG (the $\psi_{R\_CDGj} F_j$ force vector equilibrates interface force F_j).

2.2 Hyperstatic interface

For an hyperstatic interface it is clear that displacements of interface dof create a deformation inside the satellite.

Starting with Eq. 1 and neglecting loads due to viscous forces, the second line can be written as:

$$M_{ij}\ddot{u}_j + M_{ii}\ddot{u}_i + K_{ij}u_j + K_{ii}u_i = 0 \quad \text{or} \quad u_i = -K_{ii}^{-1}\left(M_{ij}\ddot{u}_j + M_{ii}\ddot{u}_i\right) - K_{ii}^{-1}K_{ij}u_j \quad \textbf{(8)}$$

Reinjecting the expression of internal displacement in the first line leads to:

$$\left(M_{jj} - K_{ji}K_{ii}^{-1}M_{ij}\right)\ddot{u}_j + \left(M_{ji} - K_{ji}K_{ii}^{-1}M_{ii}\right)\ddot{u}_i + \left(K_{jj} - K_{ii}^{-1}K_{ij}\right)u_j = F_j \quad \textbf{(9)}$$

The contribution of interface displacement to interface loads can be modified by removing the isostatic part for which there is no contribution.

To do so the interface dof can be splitted between 6 isostatic dof u_S and the remaining dof u_H :

$$\{u_j\} = \begin{Bmatrix} u_S \\ u_H \end{Bmatrix} \quad \textbf{(10)}$$

From Eq. 9, interface loads due to interface displacement are then equal to:

$$\left(K_{jj} - K_{ii}^{-1}K_{ij}\right)u_j = \begin{bmatrix} K_{SS} & K_{SH} \\ K_{HS} & K_{HH} \end{bmatrix}\begin{Bmatrix} u_S \\ u_H \end{Bmatrix} = \begin{Bmatrix} F_S \\ F_H \end{Bmatrix} \quad \textbf{(11)}$$

What we call warping are the displacements $\breve{u}_H$ creating the same interface loads without any isostatic displacements.

$$\begin{bmatrix} K_{SS} & K_{SH} \\ K_{HS} & K_{HH} \end{bmatrix}\begin{Bmatrix} 0 \\ \breve{u}_H \end{Bmatrix} = \begin{Bmatrix} F_S \\ F_H \end{Bmatrix} \quad \textbf{(12)}$$

From the second line of Eq. 12, $K_{HH}\breve{u}_H = F_H$ or $\breve{u}_H = K_{HH}^{-1} F_H$

$$\breve{u}_H = \begin{bmatrix} 0 & K_{HH}^{-1} \end{bmatrix} \begin{Bmatrix} F_S \\ F_H \end{Bmatrix} \quad \textbf{(13)}$$

Using Eq.11, one can express the warping as a function of the complete displacements:

$$\breve{u}_H = \begin{bmatrix} 0 & K_{HH}^{-1} \end{bmatrix} \begin{bmatrix} K_{SS} & K_{SH} \\ K_{HS} & K_{HH} \end{bmatrix} \begin{Bmatrix} u_S \\ u_H \end{Bmatrix} = \begin{bmatrix} K_{HH}^{-1} K_{HS} & 1 \end{bmatrix} \begin{Bmatrix} u_S \\ u_H \end{Bmatrix} \quad \textbf{(14)}$$

Eq. 9 can then be rewritten as:

$$\left(M_{jj} - K_{ji} K_{ii}^{-1} M_{ij}\right)\ddot{u}_j + \left(M_{ji} - K_{ji} K_{ii}^{-1} M_{ii}\right)\ddot{u}_i + \left(K_{jj} - K_{ii}^{-1} K_{ij}\right)\breve{u}_j = F_j \quad \textbf{(15)}$$

We can also write that interface nodes displacements are the sum of a rigid body motion and a warping:

$$\begin{Bmatrix} u_S \\ u_H \end{Bmatrix} = \begin{bmatrix} I & 0 \\ \varphi_{HR} & I \end{bmatrix} \begin{Bmatrix} u_S \\ \breve{u}_H \end{Bmatrix} \quad \textbf{(16)}$$

As rigid body motion does not create any elastic loads, we have:

$$\begin{bmatrix} K_{SS} & K_{SH} \\ K_{HS} & K_{HH} \end{bmatrix} \begin{Bmatrix} I \\ \varphi_{HR} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \textbf{(17)}$$

Which is equivalent to: $K_{SS} + K_{SH}\varphi_{HR} = 0$ and $K_{HS} + K_{HH}\varphi_{HR} = 0$

The isostatic resultant of forces due to a pure warping can be found by projecting those forces on rigid body modes:

$$\begin{bmatrix} I & \varphi_{RH} \end{bmatrix} \begin{bmatrix} K_{SS} & K_{SH} \\ K_{HS} & K_{HH} \end{bmatrix} \begin{Bmatrix} 0 \\ \breve{u}_H \end{Bmatrix} = (K_{SH} + \varphi_{RH} K_{HH})\breve{u}_H = (K_{HS} + K_{HH}\varphi_{HR})^t \breve{u}_H = 0 \quad \textbf{(18)}$$

Warping contribution to QSL is therefore equals to zero but warping has nevertheless to be mastered as it contributes to the stress state inside the satellite.

For a satellite hyperstatically matted to a launcher, a dimensioning specification should therefore associate a QSL ($\ddot{u}_{QSLj}$) and a warping ($\breve{u}_H$).

In the next paragraph we will present a method to optimize a launcher/dispenser design in order to limit the warping effect inside a satellite.

3. Singular Value Decomposition approach to limit warping effect

For satellite comfort it can be useful to limit warping at its interface. However, if we consider a 4 point interface, warping is a combination of 18 dof and it is clearly not straightforward to determine which type of warping should be limited through a launcher/dispenser modification. Testing many reinforcements to determine which combination offers the best efficiency to mass ratio can be quite long. Indeed this requires to model and to mesh several types of reinforcements. We have developed a method based on Singular Value Decomposition to identify critical warping for a given satellite. Knowing the warping to limit will enlighten the design work. The underlying hypothesis is that a great part of the loads inside the satellite is due to warping and that a limited number of warpings can be isolated.

Starting from a satellite dynamic model from which "N" internal critical loads are accessible – directly or through restitution matrices – a (Nx24) matrix is built linking interface displacements and static internal loads. This matrix can be generated thanks to 24 static resolutions:

$$\{F_i\} = [\sigma_{ij}]\{u_j\} \quad \textbf{(19)}$$

To quantify the global criticality of a warping with respect to internal loads, a scalar criterion is defined as follow:

$$C_0 = \{F_i\}^t\{F_i\} \quad \textbf{(20)}$$

Using Eq; 19, the criterion can be written as:

$$C_0 = \{u_k\}^t[\sigma_{ik}]^t[\sigma_{ij}]\{u_j\} \quad \textbf{(21)}$$

This criterion being dependant on the warping shape but also on its amplitude, a second criterion is defined removing the amplitude dependence:

$$C = \frac{\{u_k\}^t[\sigma_{ik}]^t[\sigma_{ij}]\{u_j\}}{\{u_l\}^t\{u_l\}} = \frac{\{u_k\}^t[\Gamma_{kj}]\{u_j\}}{\{u_l\}^t\{u_l\}} \quad \textbf{(22)}$$

This new criterion is a Rayleigh quotient from which an optimisation process leads to a SVD problem. Indeed, derivating Eq. 22 with respect to $\{u_k\}^t$ to find maximal values leads to:

$$2[\Gamma_{kj}]\{u_j\}\left(\{u_l\}^t\{u_l\}\right) - 2\left(\{u_k\}^t[\Gamma_{kj}]\{u_j\}\right)\{u_k\} = 0 \quad \textbf{(23)}$$

equivalent to: $[\Gamma_{kj}]\{u_j\} = \dfrac{\{u_k\}^t[\Gamma_{kj}]\{u_j\}}{\left(\{u_l\}^t\{u_l\}\right)}\{u_k\}$

or:

$$[\Gamma_{kj}]\{u_j\} = C\{u_k\} \quad \textbf{(24)}$$

Solutions of Eq. 24 are interface displacements $\{\delta_j\}$ such that: $[\Gamma_{kj}]\{\delta_j\} = C_k\{\delta_k\}$

Once the initial (Nx24) matrix $[\sigma_{ij}]$ is built, the matrix $[\Gamma_{kj}]$ is obtained as $[\sigma_{ik}]^t[\sigma_{ij}]$.
An SVD decomposition of $[\Gamma_{kj}]$ leads to:

$$[\Gamma_{kj}] = [U][S][U]^t \quad \textbf{(25)}$$

where $[S_{kj}] = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & C_{24} \end{bmatrix}$ and $[U_{jk}] = [\delta_1 \quad \dots \quad \delta_{24}]$.

The eigenvectors $\{\delta_k\}$ are the natural interface displacements maximizing the criterion and the eigenvalues C_k are the criterion values corresponding to each natural displacement.

For a real interface displacement $\{\delta_{LCi}\}$ due to a launcher load case, one can calculate the participation of each natural displacement as:

$$\{\delta_{LCi}\} = \lfloor \delta_{ij} \rfloor \{\alpha_j\} \quad \textbf{(26)}$$

where: $\{\alpha_j\} = \dfrac{\lfloor \delta_{ij} \rfloor^t \{\delta_{LCi}\}}{\lfloor \delta_{ij} \rfloor^t \lfloor \delta_{ij} \rfloor}$

The criterion value for this real interface displacement is:

$$C_{0\_LCi} = \{\delta_{LCi}\}^t \lfloor \Gamma_{kj} \rfloor \{\delta_{LCi}\} \quad \textbf{(27)}$$

Or using Eq.25 and 26 $$C_{0\_LCi} = \{\alpha_j\}^t \lfloor \delta_{ij} \rfloor^t \lfloor \delta_{ij} \rfloor \lfloor S_{kj} \rfloor \lfloor \delta_{ij} \rfloor^t \lfloor \delta_{ij} \rfloor \{\alpha_j\} \quad \textbf{(28)}$$

With normalized natural deformation ($\lfloor \delta_{ij} \rfloor^t \lfloor \delta_{ij} \rfloor = \lfloor I \rfloor$), we can get the participation of each natural deformation to the criterion:

$$C_{0\_LCi} = \{\alpha_j\}^t \lfloor S_{kj} \rfloor \{\alpha_j\} = \sum_{j=1}^{24} \alpha_j^2 C_j \quad \textbf{(29)}$$

We can also build a global criterion summing the contribution of 'M' real interface displacements by summing each criterion:

$$C_{0\_G} = \sum_{i=1}^{M} C_{0\_LCi} \quad \textbf{(30)}$$

4. Application to a real satellite

The methodology developed in previous paragraph has been applied to a real satellite connected through a 4 point interface to a dispenser. 144 internal forces were kept to define the criterion. First, 24 static resolutions were performed to build the 144x24 $\lfloor \sigma_{ij} \rfloor$ matrix.

The SVD decomposition of $\lfloor \Gamma_{kj} \rfloor$ lead to the following eigenvalues:

Table 1: Eigenvalues of $\lfloor \Gamma_{kj} \rfloor$

Mode #	**C**	**Mode #**	C	**Mode #**	C
1	2.5E14	9	3.5E12	17	8.4E11
2	1.9E14	10	2.0E12	18	4.3E11
3	1.3E14	11	1.5E12	19	1.5
4	1.1E14	12	1.4E12	20	0.9
5	9.2E13	13	1.1e12	21	0.7
6	3.8E13	14	1.1e12	22	0.3
7	5.3E12	15	1.0E12	23	0.2
8	3.6E12	16	9.1E11	24	0.1

The eigenvalues are quite scattered demonstrating that some interface displacements are more stressing for the 144 forces locations inside the spacecraft. The eigenvalues and associated eigenmodes can be divided into 3 groups.

a) Modes 1 to 6 are associated with interface displacements for which relative translations between the 4 attachment points are present. They are the more stressing for the satellite.

b) Modes 7 to 18 are associated with interface displacements for which rotations can be observed at attachment points.
c) Modes 19 to 24 correspond to the 6 rigid body motions. Those motions do not stress the satellite.

The 2 next figures illustrate those types of eigenmodes. To be able to visualize the interface displacements and especially the rotations, the 4 interface points are plotted as cubes. The central cube corresponds to the mean displacement of the 4 interface nodes.

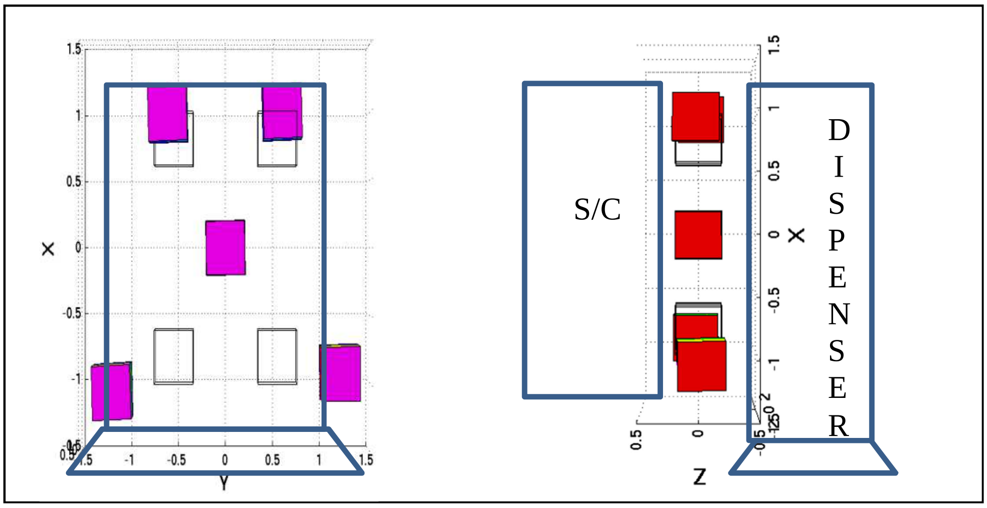

Figure 1: Eigenmode N°1

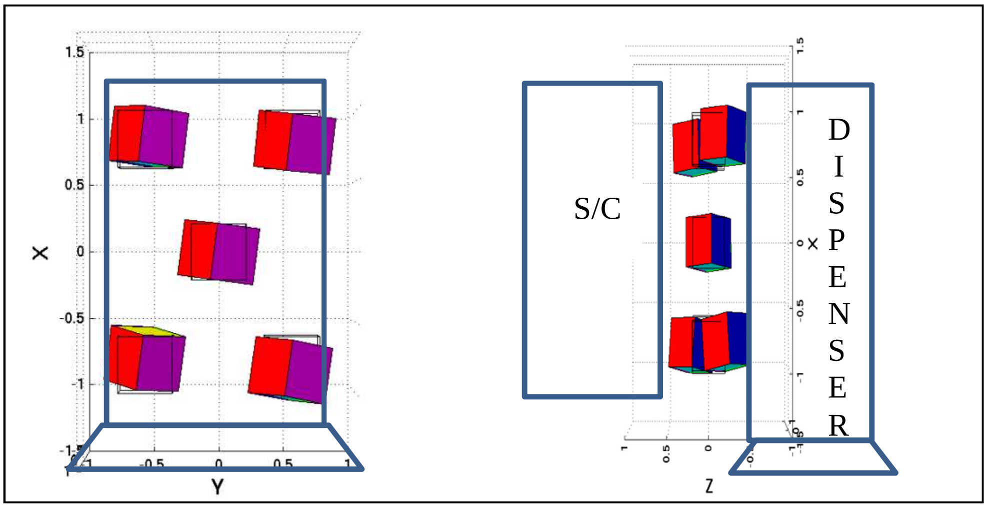

Figure 2: Eigenmode N°7

12 load cases were considered as the most stressful for the satellite. The participation of the 24 eigenmodes to the global criterion summing those 12 load cases was considered (Eq. 30).

The participation is given in table 2.

Table 2: Participation factor of the 24 eigenmodes to the global criterion C

Mode #	Participation (%)	Mode #	Participation (%)	Mode #	Participation (%)
1	4.0	9	**58.2**	17	0.02
2	3.7	10	4.0	18	0.007
3	2.4	11	1.1	19	ε
4	2.6	12	6.0	20	ε
5	0.9	13	2.1	21	ε
6	0.8	14	0.2	22	ε
7	1.4	15	0.1	23	ε
8	**12.3**	16	0.2	24	ε

One can observe that the dispenser initial design is quite optimal since the first six modes contribute very little to the global criterion. Two eigenmodes are the most participating (N°8 and N°9). They are represented on figures 3 and 4. One can realize that these two modes are very close in nature. Even so they are orthogonal, their main characteristic is a local rotation at each interface around the Y axis.
From this analysis it is clear that modifying the launcher/dispenser design in order to limit local rotation around Y axis at each interface point would significantly decrease the global loads inside the spacecraft.

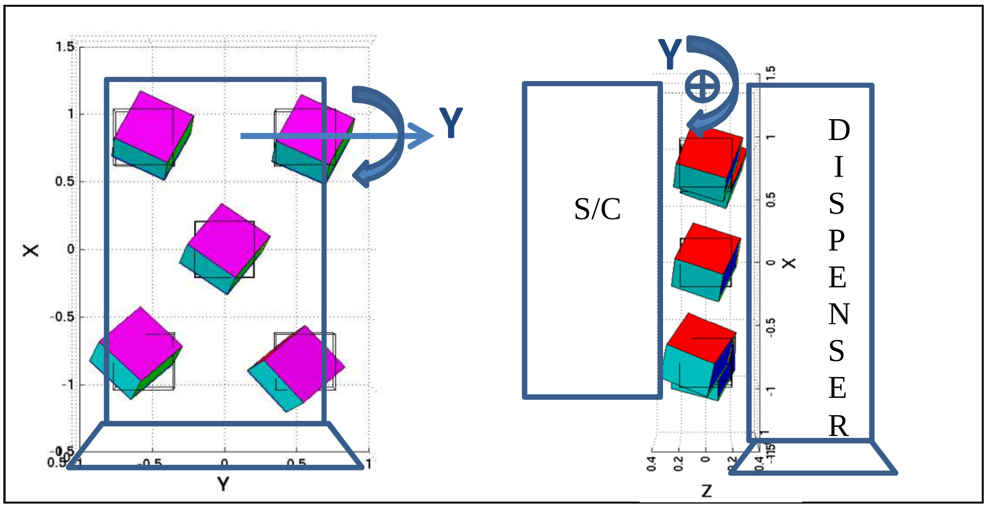

Figure 3: Eigenmode N°8

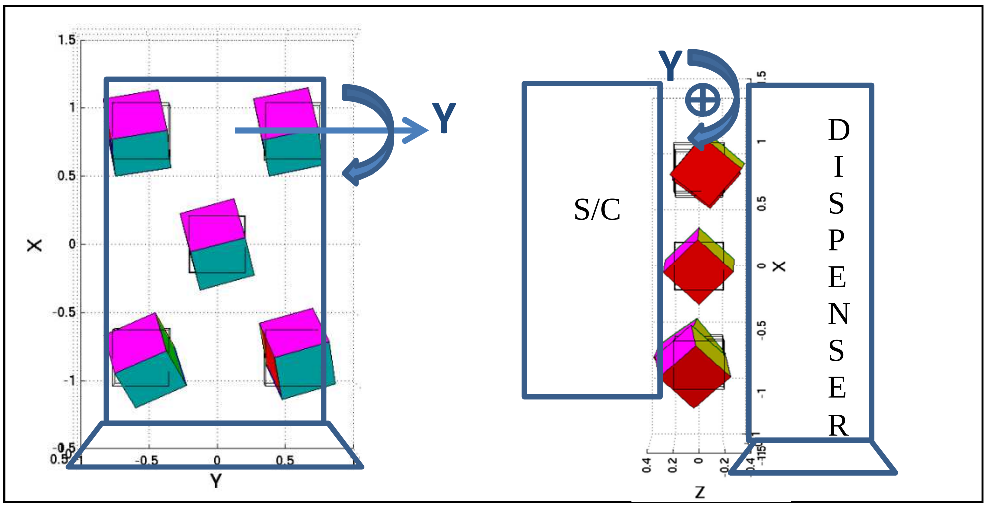

Figure 4: Eigenmode N°9

3. Conclusions

A design approach has been presented to optimize a launcher/satellite interface for payload comfort. The method is dedicated to satellites whose interface with launcher is hyperstatic and for which warping is an important contributor to internal stress state. Using available data coming from initial design model and identified critical load cases, a SVD method is proposed to determine which warping should be limited through design modifications. This intermediate step can save a lot of energy when trying to find the best reinforcement solution. The obtained results on a real configuration have shown that this method was efficient to identify a precise warping to limit through launcher/dispenser modifications for satellite benefit. It is clear that this method can be useful to optimize any mechanical system for which warping between subcomponents is an issue.

References

[1] Ariane 5 User's Manual 5/1, July 2011.
[2] Vega User's Manual 4/0, April 2014.
[3] Structural factors of safety for spaceflight hardware, ECCSS-E-ST-32-10-C Rev.1 March 2009.
[4] R. R. Craig Jr, M.C.C. Bampton. 1968. A hybrid method of component mode synthesis. *Computers and Structures.* Vol. 1(4), pp. 581-601.