# Numerical simulation of the effect of convection on a single aluminium droplet in combustion

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## Abstract

This paper studies the combustion of an aluminium droplet for a propellant gas like environment. Firstly, a numerical model is built based on a multispecies reacting system. A two-dimensional axisymmetric configuration is considered for a spherical droplet for which the temperature evolution is estimated. The 2D Navier-Stokes equations are solved for the gas phase with the in-house numerical tool CPS. This code considers a Arrhenius modeling for the kinetics. The transport coefficients are evaluated for each species on the basis of Lennard-Jones potential. The mixture laws derive from values of species dynamic viscosity, thermal conductivity and diffusion coefficients. The aluminium mass flow rate vaporized at the droplet surface is assumed to be controlled by the thermal flux resulting from the difference between the flame and the surface temperatures. In a simplified approach, the droplet temperature is uniform and its value is given by the thermodynamic equilibrium at the aluminium gas/liquid interface. The diffusion of gaseous species is also taken into account at the droplet surface. Hence, the aluminium mass flow rate is controlled by the convection and diffusion. The validation of this model is performed per the study of the combustion of a single droplet in air. Results agree with already published numerical and experimental data. The combustion time obtained for a 100µm droplet matches quite well into the Widener-Beckstead correlation.

Secondly, a specific attention is given for validation purposes on the regression rate modification by the convective stream. Estimates of the Ranz-Marshall correction law correspond well with the mass flow rate as it was formerly shown in the reference [1]. Furthermore, a series of calculations are performed for a propellant gas environment (AP/HTPB type) at different pressures in order to define a law corresponding to the Ranz-Marshall formulation and approximate the effect of the convective flow on the vaporization.

The result is that the vaporization rate is less sensitive to the convective stream in the propellant gas environment than in air, especially for high temperatures. The effect of pressure on combustion time is also investigated and illustrates little effect in the investigated configuration. Specific focus is given to the vaporization response of the droplet to an oxidizing flow disturbance. If the adopted method was considered to be limited to a quasi-steady behaviour, it is demonstrated that it could be extended to unsteady calculations with the restriction of low to moderate Schmidt numbers. A correct agreement is found between the numerical simulations results and the linearization of the new Ranz-Marshall law validating its use for two-phase flow simulations.

## 1. Introduction

Combustion of aluminium droplets has been studied for several decades due to the widely use of aluminium as a fuel component in solid propellant. In a standard AP-HTPB based composite propellant, the addition of a powder of thin aluminium particles (~40 µm diameter) induces a temperature increase in the combustion products but also in the formation of liquid alumina. Formation of large agglomerates (~100-200µm) is generally observed. The understanding of the physics involved in the droplet combustion, and especially the combustion time, appeared of prime importance for solid rocket motor performance estimation. A recent interest rose with SRM oscillations consideration. The effect of aluminium is suspected to strongly affect the motor stability. In that purpose, instabilities observed on the P80 SRM are explained by an effect of aluminium combustion. This highlights the need to have an accurate and efficient model to describe the combustion of aluminium droplets in solid rocket motors.

Earliest models tended to build a correlation between the initial particle diameter and the burning time. Several models have been proposed for calculating the burning time and flame temperature [2]. In the 70$^{ies}$, Law [3] suggested investigating the different behaviours of the combustion depending on the quantity of alumina coming back onto the droplet surface. This leads to an analytical model, but still considers a spherical symmetry for the droplet. The Ranz-Marshall correlation allows taking into account global effects of how a convective stream affects the aluminium vaporized mass rate [4]. However, in the case of a convective flow surrounding the particles, the symmetric assumption no longer holds. The distribution of physical quantities such as temperature, mass fractions or velocity cannot be predicted using this approach. This data is essential to correctly predict the aluminium vaporisation rate and the droplet burning time.
Liang and Beckstead first published results on numerical simulations on a single aluminium droplet combustion in air and in propellant combustion products [5], [6]. In their papers, aluminium vaporization, finite gaseous reactions and formation of an oxide cap are considered. At the same time, some experimental data was published by Bucher *et al*. [7]. In their work, PLIF was used to measure the radial profiles of AlO species and temperature during the quasi-steady burning stage of the droplet free fall. In addition with other experimental results [7], [8] giving the global burning time or the evolution of the droplet diameter, information on the temperature and AlO fields in the vicinity of the droplet surface is very helpful for the validation of numerical simulations.
The same approach is used in the study performed by Gallier et al. □[1]. The obtained results are much in line with the ones of Beckstead *et al.* which were used as validation for the model in the case of the steady combustion of an aluminium droplet in air. The great interest of this study is the extension of the model built for steady configurations to aluminium droplet unsteady combustion. The droplet vaporisation response is numerically evaluated thanks to an acoustic disturbance and the involved physical mechanism is explained. In an air environment, they also showed that for low frequencies, the quasi-steady combustion model (e.g. d² law) is still valid and the response is well predicted by linearization of the Ranz-Marshall correlation. The purpose of the present study is to enlarge the later results to a propellant combustion products environment (i.e. high temperature of ~2000K and oxidizers like $H_2O$ or $CO_2$). A specific focus is done on the unsteady combustion and a reviewed Ranz-Marshall correlation in this specific environment. Additional computations are dedicated to a better understanding of the combustion such as a continuous regression of the droplet diameter, the influence of the oxide cap on the combustion time or the variation of the oxidizer composition during the droplet burning.

## 2. Modelling

The proposed model is mainly based on gas phase reactions between gaseous aluminium and gaseous oxidizers to form alumina. A detailed description can be found in the reference [14]. The liquid phase of the aluminium droplet is not studied but characterized by a constant and uniform temperature.

### 2. 1 Equations in gas phase

It is assumed that the combustion physic is limited by diffusion phenomena. A kinetics scheme is defined in order to correctly estimate the flame temperature according to the species concentrations. For the reacting flow, the standard conservation equations for the mass, momentum and energy are written as bellow:

$$\frac{\partial}{\partial t}\rho Y_k + div(\rho Y_k \vec{v}) + div(\Im^k) = M_k \omega_k \quad \textbf{(1)}$$

$$\frac{\partial}{\partial t}\rho \vec{v} + div(\rho \vec{v} \otimes \vec{v}) + div(P\overline{\overline{I}} + \overline{\overline{\Pi}}) = 0 \quad \textbf{(2)}$$

$$\frac{\partial}{\partial t}\rho e_t + div(\rho e_t \vec{v}) + div(p\vec{v} + \overline{\overline{\Pi}}.\vec{v}) + div(\Im^q) = 0 \quad \textbf{(3)}$$

The diffusive fluxes of equations (1) to (3) can be expressed in a general formulation as follows:

$$\Pi = -\mu_{mix}\left(grad(\vec{v}) + grad(\vec{v})^t\right)$$

$$\Im^k = -\rho D_{k,mix}\, grad(Y_k) \qquad \textbf{(4)}$$

$$\Im^q = -\sum_k \rho h_k Y_k D_{k,mix}\, grad(Y_k) - \lambda_{mix}\, grad(T)$$

At the liquid / gas interface, it is considered that the energy needed to vaporize the liquid aluminium is supplied by the heat flux coming from the flame. Moreover, the droplet temperature is supposed to be uniform and equal to the vaporization temperature of aluminium for the calculation conditions. A Clausius-Clapeyron law is used to determine this internal temperature and it is explicitly assumed that the thermal conduction inside the droplet is neglected (the first term in the right member of eq. (9) is taken equal to zero). The gaseous aluminium mass rate is evaluated from the knowledge of the temperature gradient in the gas phase at the droplet surface and the heat of vaporization of aluminium taken at the value of 10.9MJ/kg.

$$\lambda_g \nabla T\big|_g = -\lambda_{Al,liq} \nabla T\big|_{Al,liq} + L_v \dot{m}_{Al,liq} \qquad \textbf{(9)}$$

In former calculations, (cf. Eucass paper [14]), pressure increase was a real issue and a upper limit of 30 bars was found to ensure a self-sustained combustion of the droplet. For higher pressure values, the vaporized mass flow rate tended toward a null value. This phenomenon was directly due to the vaporisation condition that links the vaporized mass flow to the heat flux according the following equation. At the droplet surface, the energy balance equation means that the conductive heat flux received from that gas phase is used to vaporize the aluminium quantity $\dot{m}_{Al}$, and equation (9) can be simplified as:

$$\dot{m}_{Al} = \lambda \frac{\overline{\nabla T}.\overline{n}}{Lv} \qquad \textbf{(10)}$$

Furthermore, the gradient estimation at the droplet surface needs to correctly estimate the surface temperature. In the previous model, the gases diffusion toward the droplet surface was neglected and the gradients were supposed to be zero at the droplet surface. In other words, the droplet was supposed to burn in aluminium vapour and the diffusion process to take place far from the surface. Hence, the droplet temperature was determined by considering an aluminium mass fraction of one. Accordingly, the pressure increase was linked with an increase of surface temperature and a decrease of thermal gradient and according equation 10, a decrease of aluminium vaporised mass flow rate.

To overcome this problem, the vaporisation condition was reviewed in order to take into account the diffusion of gaseous species at the droplet surface according the 1D eq. (11). The source term $\dot{\omega}$ only stands for aluminium production at the surface. For other gaseous species, its value is zero.

$$\dot{m}.\frac{\partial Y}{\partial x} = \rho D \frac{\partial^2 Y}{\partial x^2} + \dot{\omega} \qquad \textbf{(11)}$$

For aluminum, this relation means that the mass flow rate produced at the droplet surface is balance by the convective mass flow rate and the diffusive mass flow rate. For other species, as the source term is zero, the convection transfer exactly balances the diffusion.

## 2.2 Validation of the vaporization condition

This validation test deals with the modification of the boundary layer with respect to equation (11). One considers the simulation of an evaporating surface in a gas environment at 4000K in a 1D configuration as illustrated in Figure 2. The oxygen and aluminium concentrations are plotted along the X axis and the evaporating surface is located on the left side. No kinetics is considered here, and only aluminium and oxygen gases are taken into account. The main goal is to determine the vaporized mass flow rate for a pressure of 50 bar and to compare the numerical value with the analytical one. The results agree very well and both approaches give a value of 0,042 kg/m²/s.

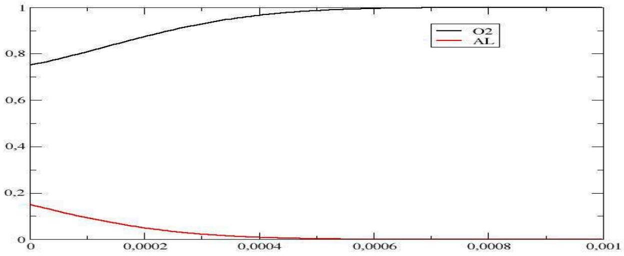


Figure 2 : $O_2$ and Al mass fraction for the vaporization validation

The set of equations (1) to (11) forms the unsteady compressible Navier-Stokes system which is solved by an explicit finite volume algorithm with the Herakles in-house developed code CPS. All calculations presented in this paper are second order accurate in space (approximate Roe-Toumi solver with appropriate slope limiter). The time accuracy is also second order thanks a two-step Runge-Kutta scheme.

## 2.3 Kinetics

An important assumption is that aluminium droplet burns in a vapour phase diffusion flame. Then, it is strongly dependant on the nature of gaseous oxidizers and inert species. In the case of the combustion of aluminium in air, only gas phase reactions were considered by using a ninestep mechanism. This reduced mechanism for the combustion in air was proposed by CNRS/LCSR [9]. For a propellant gas environment, it is necessary to consider the reactions between Al and $CO_2$, $H_2O$ which are the main oxidizer species. Oxidizing reactions with $O_2$ have not been considered because $O_2$ appears in very low concentration (<1%) in the propellant combustion products. Considering $CO_2$, kinetic scheme, defined from data provided by the literature [9][11], [12], suggests the following 10 reactions scheme. A second scheme was also taken into account. It was proposed by Onera in the work they performed in the framework of R&T CNES ODP. It is composed of 20 chemical reactions and 23 species. A specific treatment is applied to small alumina particles (< 1µm). This species is considered as a gas with limited diffusion properties.

Table 1: Kinetic schemes used for calculations in propellant combustion products

| | Scheme n°1 | | Scheme n°2 |
|---|---|---|---|
| 1 | $Al_2O = Al + AlO$ | 1 | $AL + O_2 = ALO + O$ |
| 2 | $Al_2O_2 = AlO + AlO$ | 2 | $AL + O = ALO$ |
| 3 | $Al + CO_2 = AlO + CO$ | 3 | $ALO + O_2 = AL O_2 + O$ |
| 4 | $Al_2O + 2CO_2 = Al_2O_3 + 2CO$ | 4 | $ALO_2 = ALO + O$ |
| 5 | $2AlO + CO_2 = Al_2O_3 + CO$ | 5 | $AL_2O = AL + ALO$ |
| 6 | $Al + H_2O = AlO + H_2$ | 6 | $AL_2O_2 = ALO$ |
| 7 | $Al_2O + 2H_2O = Al_2O_3 + 2H_2$ | 7 | $AL_2O_2 = AL + ALO_2$ |
| 8 | $2AlO + H_2O = Al_2O_3 + H_2$ | 8 | $AL_2O_2 =$ AL2O $+ O$ |
| 9 | $H + OH + M = H_2O + M$ | 9 | $O_2 = 2 O$ |
| 10 | $H + H + M = H_2 + M$ | 10 | $AL + CO_2 = ALO + CO$ |
| | | 11 | $AL + H_2O = ALO + H_2$ |
| | | 12 | $H + OH = H_2O$ |
| | | 13 | H2 $= 2 H$ |
| | | 14 | $AL_2O_3 = AL_2O_2 + O$ |
| | | 15 | $AL + HCL = ALCL + H$ |
| | | 16 | $ALH + CL = ALCL + H$ |
| | | 17 | $ALCL + HCL = ALCL_2 + H$ |

| | |
|---|---|
| 18 | ALCL + HCL = $ALCL_2$ + H |
| 19 | $AL_2O_3$ = ALO + $ALO_2$ |
| 20 | ALO + $CO_2$ = $ALO_2$ + CO |

## 2.4 Burning time estimation

A critical point for the definition of the burning time is the estimation of the average vaporised mass flow according to the local mass flux $\dot{m}(\theta)$ at the droplet surface (θ is the polar angle). Integration over the entire surface (approximated on *Npt* cells) is made to determine the instantaneous mass flux *M'* following the relation:

$$\overline{m} = \frac{M'}{4\pi R^2} = \frac{1}{2}\int_0^{\pi} \dot{m}(\theta)\sin(\theta)d\theta \approx \frac{\pi}{2Npt}\sum_i \sin(\theta_i)\dot{m}(\theta_i) \quad \textbf{(11)}$$

From this relation, the burning time is defined as:

$$t_b = 2.5\frac{4}{3}\frac{\pi\rho_{Al}}{\overline{m}}\left(\frac{d}{2}\right)^3 \quad \textbf{(12)}$$

The 2.5 corrective coefficient was first introduced by Widener and Beckstead [6]. The justification is simple: the simulations are performed for a fixed diameter, although the droplet diameter decreases during the combustion. The simulation geometry is then representative of an intermediate state of the droplet. This value is obtained by considering the remaining aluminium mass to the initial mass. If we now consider that the diameter is representative of the average surface during the droplet combustion, a value of *2.8* is found. A third way consists in integrating the standard $d^2$ law by replacing the combustion rate K by its expression: $K = -2r\frac{dr}{dt}$. In that case a value of 3.3 is found for the coefficient.

Whatever the chosen approach, the value of *2.5* is considered in the following for the determination of the combustion time according to the definition of $t_b$.
For comparison purpose, the combustion times are compared to the ones obtained with the Widener-Beckstead correlation. A parameter *Xeff* is defined according the oxidizers concentrations according to $Xeff = XO_2 + 0.66\, XH_2O + 0.22\, XCO_2$.

$$t_b^{WB} = \frac{1}{6.27\times 10^{-8}}\frac{d^{1.8}}{X_{eff}^{1}T_{\infty}^{0.2}P^{0.1}} \quad \textbf{(13)}$$

## 2.5 Mesh generation

All the simulations hereafter consider the combustion of a spherical droplet. An axisymmetric mesh is used where a 100µm droplet is located in the centre of the domain. The boundaries of the domain are placed at around 20 radii of the droplet in order to limit the effects on boundary conditions. A specific refinement is imposed to the nearest cells of the droplet surface. The selected size is 3µm which is small enough to ensure a correct estimation of the temperature gradient at the surface. The mesh is composed of nearly 3430 cells.

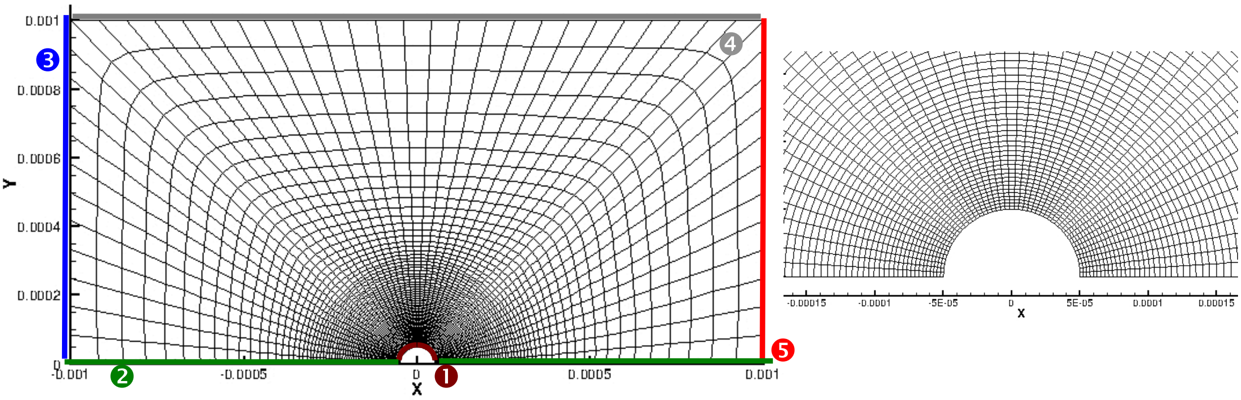



Figure 2: Mesh definition with placement of boundary layers and detail on the droplet surface

The boundary conditions are defined as:

- ❶ : Aluminium vaporization controlled by eq. (11)
- ❷ : Symmetry
- ❸ : Inlet with imposed mass flow rate at constant temperature
- ❹ : Slip condition
- ❺ : Outlet with constant pressure

## 3. Results and discussions

### 3.1 Preliminary calculation

The first simulation aims at validating the model by considering the steady combustion of a 100µm aluminium droplet in air and to compare the new results with the previous ones obtained without the diffusion at the propellant surface. The ambient temperature is 300K and the pressure is 1bar. The kinetic scheme is taken from the reference [13]. The carried out simulation considers an oxidizer mass flow rates at $5kg/s/m^2$, which corresponds to a flame Reynolds numbers $Re_f$ of $0.4$ ($Re_f$ are calculated at flame conditions for the viscosity and density and the gas velocity is taken at oxidizer injection).

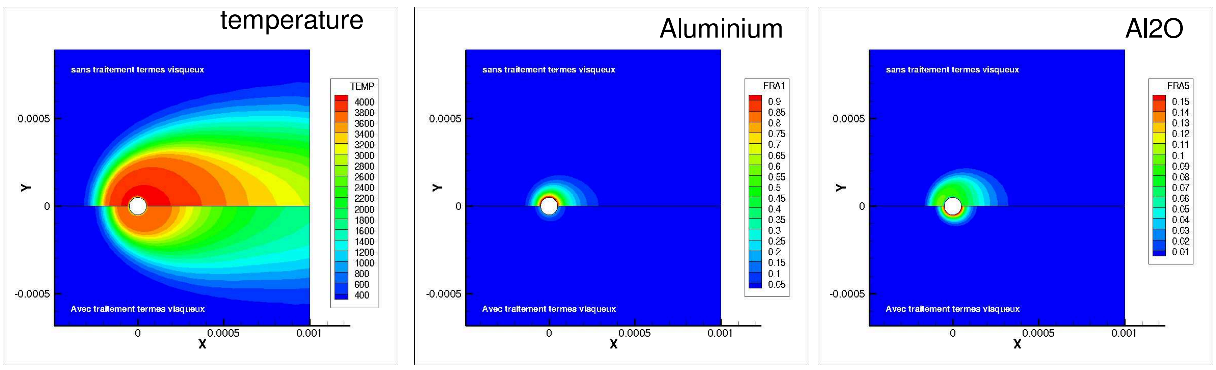



Figure 3 : Temperature field and Al, Al2O mass fractions around the droplet for Re=5
Upper part: with no diffusion at the droplet surface
Lower part: with diffusive terms at the surface

Several remarks can be done. Firstly, the maximum temperature of 3600K is reached with the new condition. This value is lower than the one obtained in previous calculations in the same conditions (>4000K). It can be seen that the flame location is nearer to the droplet surface and there is a drastic decrease of the aluminium vapour surrounding the droplet. The effect is that the burning time is reduced to 22ms instead of 55ms for the old calculation. The increase in aluminium vaporization is due to the much lower droplet temperature associated with the low concentration of aluminium. Considering the Beckstead correlation, the corresponding burning time is ~40ms.

### 3.2 Combustion in propellant gas environment

The chemical composition of the propellant products is as follows. It corresponds to fresh gases that oxidize the aluminium droplet and is obtained by a thermodynamic equilibrium calculation. The temperature is taken at 2400K.

- CO : 0.19
- CO2 : 0.083
- H2 : 0.143
- H2O : 0.321
- N2 : 0.262

A first series of calculations deal with the chemical mechanism with 10 reactions. The pressure varies from 1 bar to 50 bar for the same particulate Reynolds number of 5. As for the combustion in air, we notice a decrease of the temperature with the pressure as it can be seen on figure 4. The AlO production is also less active. The decrease of the gas velocity due to the pressure increase yields to a more symmetric shape of the flame.

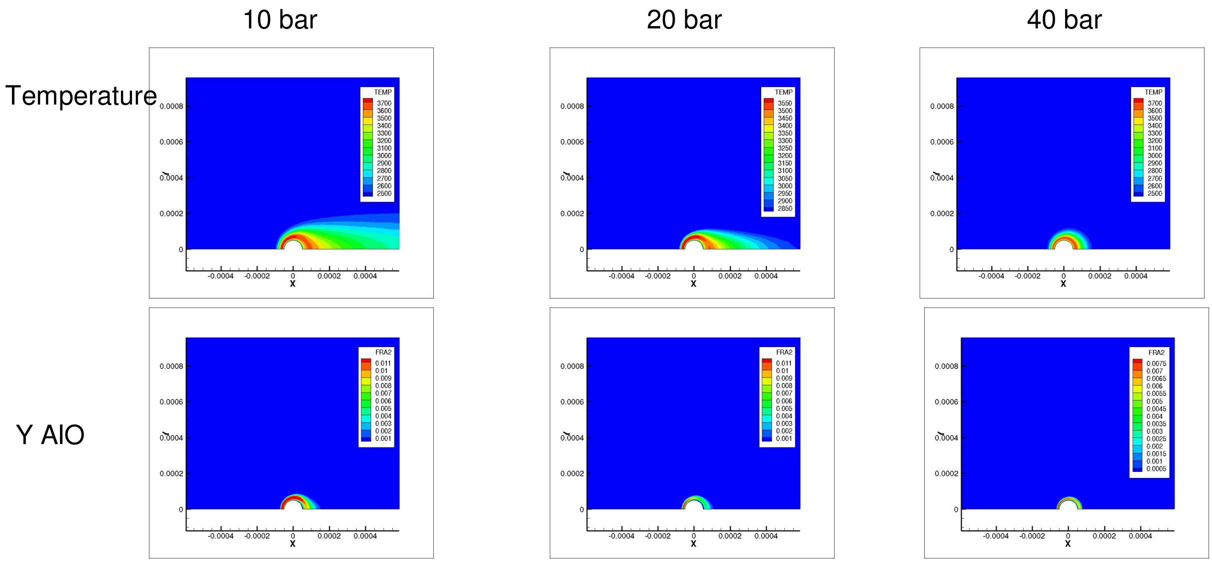



Figure 4 : Temperature field and AlO mass fraction

The next figure presents the evolution of the temperature with the angular location. The considered point is determined according to the angle θ.

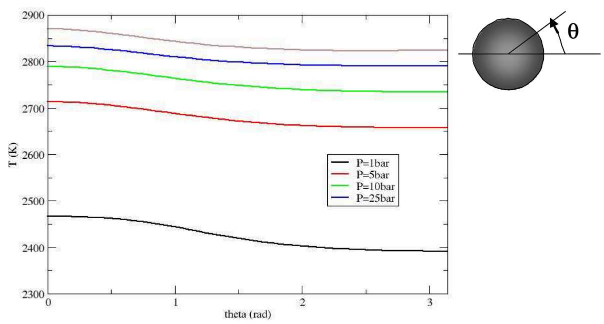



Figure 5 : Temperature evolution at the droplet surface
for $Re_p$=5 (pressure effect)

The pressure effect is associated with an increase of the temperature into the droplet. The forward part of the droplet is characterised by the highest temperature because the stagnation point tends to concentrate the aluminium vapour. It can be noticed that the temperature difference between the forward and the backward parts of the droplet is limited to 50K. This is a good indication that the assumption of a isothermal temperature in the droplet is valid in first approximation.

The aluminium mass fraction is plotted in figure 6. With the pressure, it tends to low values and the difference between the forward and the rear parts is less important. This is a consequence of the oxidizer velocity decrease.

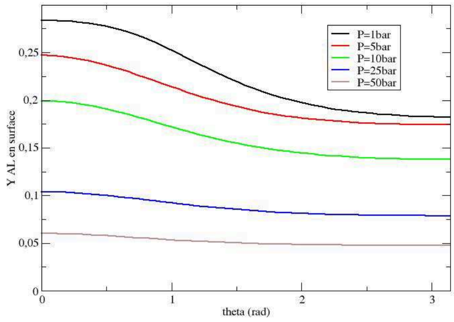



Figure 6 : Evolution of aluminium mass fraction at the droplet surface for $Re_p$=5 (pressure effect)

The same study was performed considering the 21 reactions chemical scheme. The oxidizer mass flow rate is increased to a particulate Reynolds number of 20. The results in terms of burning time are presented on figure 7. The effect of convection is clearly seen. A factor of 4 on the oxidizer velocity induces an increase of 100% of the burning time. Surprisingly, the pressure effect is less sensitive as it was expected. For both series, the same tendency is observed and a small increase in the burning time is obtained. However, this is in opposition with the burning times provided by the application of the Widener-Beckstead law (eq. 13) for which a decrease is observed with the pressure. However, the variation is not so important and between 10bar and 50bar, the burning time range is coherent with the calculations at Re=20.

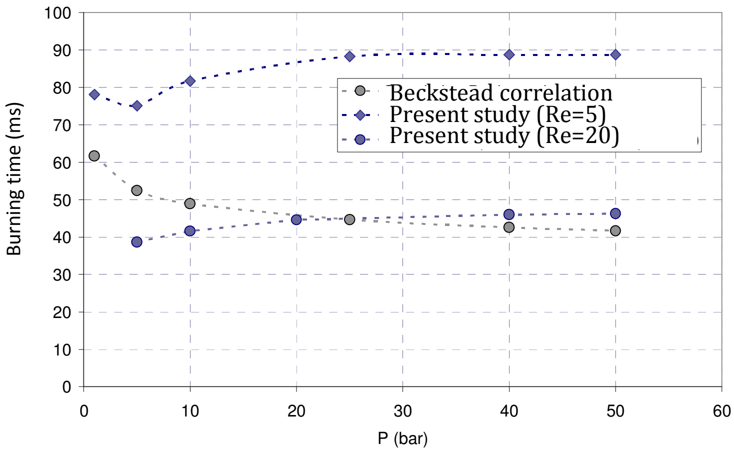



Figure 7 : Evolution of burning time with pressure

### 3.3 Effect of gas composition on the burning time

Until now, no evolution of the oxidizer environment was considered. During the droplet lifetime, the compositions of gases and temperature vary from the propellant surface (oxidizer rich gases referred as *Xeff=0.18* at table 2 and 18% of unburnt aluminium in propellant) to a zone where the aluminium combustion is completed (combustion products where *Xeff=0.07* due the complete combustion of aluminium). However, the decrease of oxidizer concentration is balanced by an increase of the temperature and one may wonder which effect has the strongest influence on the burning rate. To investigate this aspect, several gas compositions are studied for a pressure of 50bar and a Reynolds number of 5.

Table 2: Environments compositions

| composition | mol frac. | mol frac. | mol frac. | mol frac. | mol frac. |
|---|---|---|---|---|---|
| O2 | 0 | 0 | 0 | 0 | 0 |
| CO2 | 0.01 | 0.014 | 0.021 | 0.038 | 0.066 |
| H2O | 0.103 | 0.133 | 0.166 | 0.227 | 0.275 |
| Al2O3 | 0.08 | 0.069 | 0.055 | 0.027 | 0 |
| Xeff | 0.070 | 0.089 | 0.110 | 0.149 | 0.180 |
| Température | 3382 | 3166 | 2959 | 2726 | 2361 |

The combustion times derived from calculations are reported on figure 8. No significant modification is observed with the oxidizer composition changes. The burning times are ~120ms for all simulations although the Beckstead correlation exhibits a decreasing tendency.

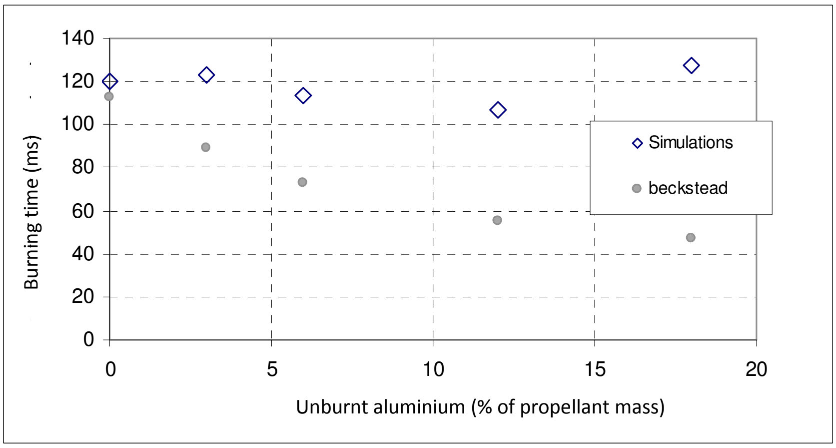
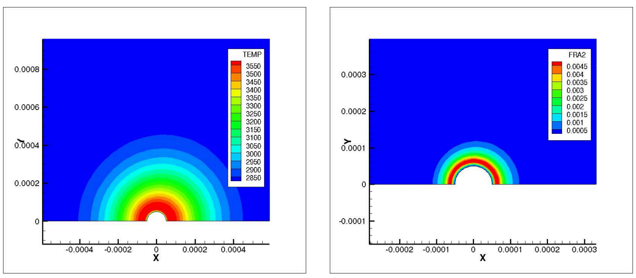



Figure 8 : Effect of oxidizer on burning time (P=50bar, Re=5)

### 3.4 Effect of pressure

In order to accurately investigate the effect of pressure on the burning time, some additional calculations are performed in an environment without convection. A quiescent atmosphere is considered for the oxidizer. As the oxidizer is not renewed, it is continuously consumed by the aluminium vapour. So, after the setting up of the flame a stationary phase of the combustion takes place. The presented results are taken at that moment. A small tendency seems to be revealed more in line with the Widener-Beckstead correlation. However, its interpolation with an exponential law provides an exponent value of -0.033, which is slightly different from the value of -0.1 proposed by Beackstead. This result indicates that the pressure has a moderate effect on burning time and the observed variations are more due to modification of the convection around the droplet.



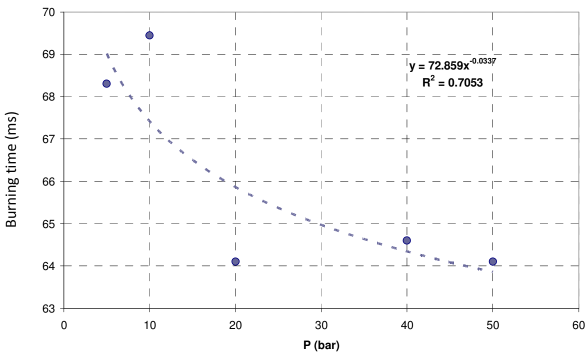



Figure 8 : Temperature and AlO mass fraction
for a quiescent atmosphere

## 3.3 Convective correction or application a Ranz-Marshall law

Calculations of a solid rocket motor operating may require a modelling of aluminium combustion to take into account the two phase flow and provide more accurate results. The available aluminium models are not as complex as the model presented in this study. They are simplified to be included in CFD codes. An example is based on the well know $d^2$ law: $d^2 = d_0^2 - Kt$ . However, the effect of convection is modelled by a term such as the correction of Ranz-Marshall where the modified mass flow is expressed in function of Reynolds and Schmidt numbers.

$$\frac{\dot{m}}{\dot{m}_0} = 1 + \delta \mathrm{Re}_{flam}^{\alpha} Sc^{\beta} \tag{14}$$

with $\alpha = 0.5$, $\beta = 0.33$ and $\delta = 0.3$. $\dot{m}_0$ represents the vaporization mass flow without convection $Sc$ the Schmidt number.

The exploitation of all results in the propellant environment allows us proposing new values for parameters *α, β* and *δ*. On the following figure, the combustion in air at 300K is replaced in comparison with the standard Ranz-Marshall correlation (eq. 14). These results are used as reference for validating purposes. It is clearly seen that the standard correction for air cannot be directly applied to propellant environment. The extrapolation of results obtained in a propellant gas environment leads to different values of parameters *α, β* and *δ* and the previous ones no longer hold. The proposed values are:

$$\alpha = 0.65, \quad \beta = 0.5, \quad \delta = 0.07 \tag{15}$$

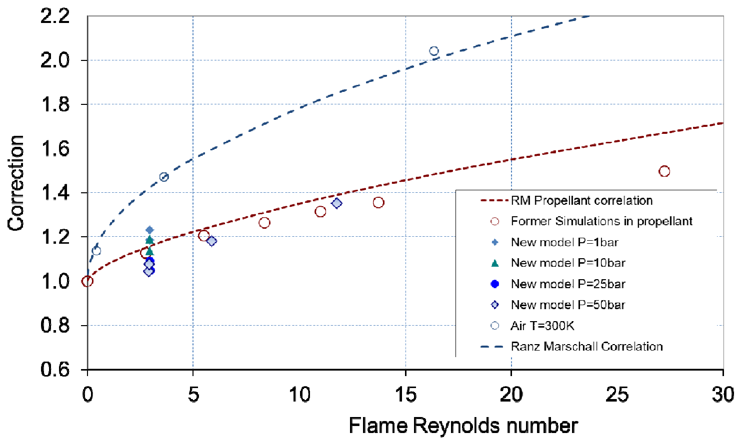



Figure 9 : Ranz-Marshall correction

### 3.4 Unsteady application

The proposed law apply to steady calculations. Its validity was stadied in the hypothesis of its use for an unsteady aluminium droplet burning. A periodic disturbance of the oxidizer flow rate is considered in the range of 5% of the initial mass flow rate. This low value of the disturbance allows a development of equation 14 in order to write the response function $R_u$ defined as :

$$\frac{\Delta \dot{m}}{\bar{\dot{m}}} = R_u \frac{\Delta V}{\bar{V}} \quad \textbf{(16)}$$

This expression leads to the analytical form: $R_u = \frac{\delta}{2} \frac{\mathrm{Re}_{flam}^{\alpha}\, Sc^{\beta}}{1+\delta \mathrm{Re}_{flam}^{\alpha}\, Sc^{\delta}}$

On the next figure, several simulations with various disturbance frequencies are presented. The pressure is constant at 50 bar and the oxidizer Reynolds number is 20. For lower values of Reynolds numbers (i.e. Re=5), the precision is not sufficient to be compared to results from eq. 16 (typically, the obtained results are of the same order of magnitude than the precision of the numerical simulations). It is noticed that for frequencies lower than 5000hz, a trend is observed which values tend to the theoretical Ru of 0.2. At a larger frequency, the numerical results no longer hold because of the existence of a limit frequency defined as $\Omega = 2\pi f \tau_{diff}$ of the order of unity and even less (i.e. $\Omega \approx 0.2$). Depending on the characteristic diffusion time $\tau_{diff}$, $\Omega$ can be larger than one, and the proposed approach is no longer valid due to a resonance between the diffusion time and the characteristic time of the disturbance. Typical values of $\tau_{diff}$ ranges from 5 $10^{-6}$s to 2.5 $10^{-4}$s

A second series of calculations were performed for different pressure values ranked from 5 to 50 bar at the given frequency of 1000Hz. As it can be seen on next figure, the results are scattered but the order of magnitude is obtained.
Hence, for moderate frequencies, as those encountered in large solid rocket motors, the proposed model can be applied with the modification of the Ranz-Marshall corrective term.

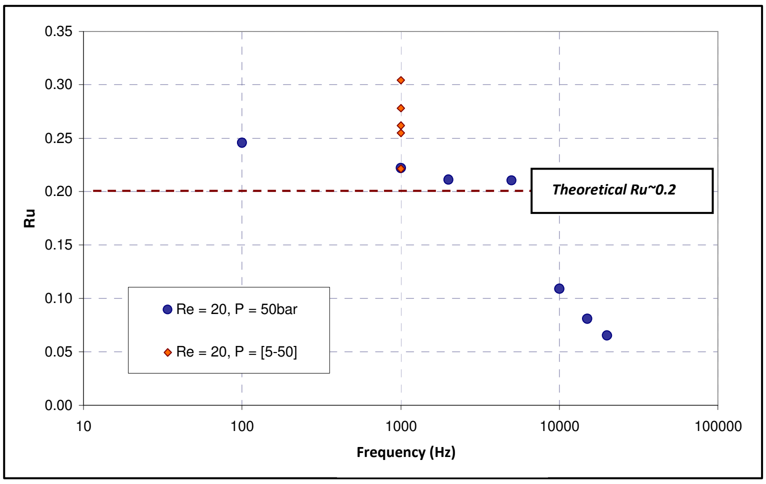



Figure 10 : Response of the vaporized mass flow rate with frequency

### Conclusion

The studied model involves a modification of the aluminium vaporization condition by considering the viscous diffusion of gaseous species at the droplet surface. A simulation of the burning of a droplet in air was used to numerically validate the model. Hence, its application could be extended to a larger pressure range, typical of those encountered in solid propulsion and allowing a study of the pressure effect.
For the conditions considered in this study, it was found that pressure has a low effect on combustion time. It is also the case for a variation of oxidizer concentration. This is a surprising result because the widely used correlation of

Beckstead predicts moderate effects with a decrease in the burning time especially for a pressure of between 30 and 50bar. The effects of convection on burning time are taken into account by application of a corrective term modelled by the Ranz-Marshall correlation. If the entered coefficients are found to work correctly for application in the air at 300K, they need to be modified for the case of propellant environment at high temperature. New simulations are performed in order to correct the values. Its application for the study of unsteady combustion appears to be valid for a frequency lower than 2000hz and moderate pressure (i.e. <50bar).

The proposed model offers a powerful tool to more accurately determine the combustion time of aluminium droplets. Furthermore, it provides interesting information on the physical phenomena involved in the combustion process. However, the deposit of alumina and its creation at the surface is not taken into account but research study has been engaged to overcome this.

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## References

[1] S. Gallier, F. Sibe et O. Orlandi, Combustion response of an aluminium droplet burning in air, *Proceedings of the Combustion Institute*, vol. 33, The Combustion Institute, Pittsburgh, PA, ed. Elsevier, 2011, pp.1948-1956.

[2] K. P. Brooks, M. W. Beckstead, 1995, Dynamics of aluminium combustion, *Journal of Propulsion and Power*, vol. 11, n° 4, pp. 769-780.

[3] C. K. Law, 1973, A Simplified theoretical model for the vapor phase combustion of metal particles, *Combustion Science and Technology*, Vol. 7, pp. 197-212.

[4] D. Scherrer, 1986, Etude des instabilités de combustion : loi de combustion des gouttes N2O4 et UDMH ; description des modèles numériques ; résultats de l'exploitation du modèle bidimentionnel, ONERA internal report 30/6112 YE.

[5] Y. Liang and M. W. Beckstead, 1998, Numerical Simulation of Unsteady, Single Aluminum Particle Combustion in Air, *AIAA Paper 98-3825*.

[6] J. F. Widener, Y. Liang, M. W. Beckstead, 1999, Aluminum combustion modeling in solid propellant environments, *AIAA paper 99-2629*.

[7] P. Bucher, R. A. Yetter, F. A. Dryer, T. P. Parr, D. M. Hanson-Parr, 1998, Plif Species and Radiometric Temperature Measurements of Aluminum Combustion in O2, CO2, and N2O Oxidizers, and comparison with Model Calculations, *Proceedings of the Combustion Institute*, vol. 27, The Combustion Institute, Pittsburgh, PA, ed. Elsevier, pp. 2421-2429.

[8] E. L. Dreizin, 1996, Experimental Study of Stages in Aluminum Particle Combustion in Air, *Combustion and Flame*, 105, 541-556.

[9] I. Gokalp, B. Legrand, C. Chauveau, L. Catoire, 1999, Etude des mécanismes réactionnels de l'aluminium. Dispositifs, d 'étude de la combustion de particules d'aluminium, LCSR Report, *ASSM program*.

[10] A. Fontijn, W. Felder, W., HTFFR, 1977, kinetics studies of $Al+CO_2 \rightarrow AlO+CO$ from 300 to 1900 K, a non-Arrhenius reaction, *The Journal of Chemical Physics*, Vol. 67, N° 4, pp. 1561-1569.

[11] J. C. Melcher, J. T. Brzozowski, H. Krier, R. L. Burton, 2000, Combustion of Aluminum in Solid Rocket Motor Flows, *AIAA paper 2000-3333*.

[12] S. Yuasa, S. Sogo, 1992, Ignition and combustion of aluminum in carbon dioxide streams , *Proceedings of the Combustion Institute*, vol. 24, The Combustion Institute, Pittsburgh, PA, ed. Elsevier, pp. 1917-1825.

[13] S. Gallier, F. Godfroy, 2009, Aluminum combustion driven instabilities in solid rocket motors. *Journal of Propulsion and Power*, vol. 25, n°2, pp.509-521.

[14] O. Orlandi, S. Gallier, Y. Moonsamy, N. Cesco 2013, Numerical simulation of a single aluminium droplet burning in a propellant environment. 5th *Eucass conference*, Saint-Petersburg.