Laminar-turbulent heat transfer on the surface of a hemisphere in the hypersonic air stream

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Abstract
The results predicted by two calculation methods (the effective length method and the Cebeci–Smith algebraic model of turbulence) are compared with experimental data on the heat flux distribution over the surface of a hemisphere at Mach number M_∞ = 5. A modification of the Cebeci–Smith turbulence model is proposed.

1. Introduction

Burning of hypersonic aircraft (HSA) thermal shielding observed under turbulent gas flow conditions has in many cases a noticeable effect on the aerodynamic characteristics of the vehicle. Therefore, the quality of the description of the heat transfer on the heat-stressed components in structures of this type arouses considerable interest. Such a description is usually based on either the effective length method by V.S. Avduevsky [1, 2] or the semiempirical algebraic or two-parameter differential turbulence models [2].
In this connection, it should be pointed out that the effective length method based on application of the experimental data on the convective heat transfer for a plate to the surface of an arbitrary curvature is one of the basic tools for designing of thermal shielding [2].
Moreover, in the literature, primary attention is paid to study of turbulent flows on the lateral surfaces of missiles and spacecraft, while hemispherical heat-stressed components of the HSA belong to the most common components used in practice.
All the above-mentioned conditions underlie the increased interest in analysis of errors inherent in basic techniques for computation of turbulent heat transfer on a hemisphere and in the search for ways of decreasing these errors. The aim of this work is to find a solution to these problems.

1.1 Mathematical formulation of the problem

The subject of this study is the experimental data obtained by Widhopf and Hall [3].
Location of the transitional region in the boundary layer was computed using the PANT method [4].
Algebraic Cebeci-Smith turbulence model [5] and the Avduevsky effective length method [1, 2] were considered as mathematical heat transfer models.
The analyzed experimental data were obtained at Mach number M_∞ in an oncoming air flow equal to 5 and a stagnation temperature on the order of 500 K. As a consequence, the computational study was carried out by solving numerically two-dimensional boundary layer equations written for a perfect gas in the Lie-Dorodnitsyn variables [6].
The transition region on the blunt elements of aircraft is localized in practice by method described in [4]. This method is a correlation technic, which based on numerous experimental data and allows to calculate transitional region according to wall roughness and free turbulence in the incident flow.
According to this method, Reynolds number that determines transition onset, can be calculated by formula:

$$\mathrm{Re}_{e,\vartheta,1} = \min\left(215/d^{0,7}, 300\right)$$
$$d = \Gamma_{Free} + \frac{\mathrm{Re}_{e,b}}{\mathrm{Re}_{e,\vartheta}} / \left[0,1G_w + \left(1+0,25G_w\right)\frac{\rho_e}{\rho_w}\right] \qquad (1)$$
$$\mathrm{Re}_{e,b} = \frac{\rho_e u_e b}{\mu_e}\ ;\ \ \mathrm{Re}_{e,\vartheta} = \frac{\rho_e u_e \vartheta}{\mu_e}$$

Here, Γ_{Free} is the free turbulence intensity in the incident gas flow, ρ_e, μ_e, u_e are the density, the dynamic viscosity coefficient, and the gas velocity on the outer boundary of the boundary layer, ρ_w is the gas density at the wall temperature, $\mathrm{Re}_{e,b}$ is the Reynolds number calculated by the gas flow parameters on the outer boundary of the boundary layer and height *b* of the wall roughness, $\mathrm{Re}_{e,\vartheta}$ is the Reynolds number calculated by the gas flow parameters on the outer boundary of the boundary layer and momentum thickness ϑ in the laminar boundary layer, and G_w is the mass gas injection rate in the boundary layer in terms of fractions of the heat transfer factor on an impermeable wall.

A necessary condition for occurrence of a turbulent gas flow mode in the boundary layer, the position of which on the surface of a body is determined by criterion (1), is satisfaction of the inequality:

$$\mathrm{Re}_{e,\vartheta,\max} > \frac{\max\left[215{,}255\left(1\text{-}\Gamma_{Free}/d\right)\right]}{d^{0{,}7}}$$

The Reynolds number for the beginning of the turbulent gas flow pattern in the boundary layer was calculated with using of the formula:

$$\mathrm{Re}_{e,\vartheta,2} = \sqrt{2}\,\mathrm{Re}_{e,\vartheta,1}\left\{1+\frac{\sqrt{2}\text{-}1}{\sqrt{ctg^2\left[\max\left(0,\alpha\right)\right]+1}}\right\}$$

Here, α is the effective angle of incidence under which the angle between the aircraft's velocity vector and the plane tangent to the body surface is understood.

Dynamic viscosity coefficient μ of the gas is calculated using intermittency factor Γ [2, 8] according to the formula:

$$\mu = \mu_L + \Gamma\mu_T$$

The magnitude of this factor is calculated, similarly to [7], using a cubic spline in the form:

$$\Gamma = \max\left\{0, \min\left[1, \xi^2\left(3-2\xi\right)\right]\right\},\ \xi = \left(\mathrm{Re}_{e,\vartheta} - \mathrm{Re}_{e,\vartheta,1}\right)/\left(\mathrm{Re}_{e,\vartheta,2} - \mathrm{Re}_{e,\vartheta,1}\right)$$

Here, μ_L and μ_T are the laminar and the turbulent dynamic viscosity coefficients.

The heat conductivity factor is calculated by a similar formula. The turbulent Prandtl number is assumed to be equal to 1 in this case.

1.2 Algebraic double-layer Cebeci-Smith model of turbulence

Within the framework of the Cebeci-Smith turbulence model [5], in the internal (near-wall) and the external flow regions different formulas are applied to calculate the dynamic viscosity factor component determined by the turbulent gas pulsations. In the former of the above regions, the Prandtl formula $\mu_T = \rho l_T^2 \left|u_y\right|$ is used in which turbulence scale l_T is assumed to be proportional to the distance of the computational node from the wall, i.e., to coordinate *y*, and for a smooth transition of the gas flow in the near-wall region to the flow in the laminar sublayer, the van Driest damping function *D(y)* is used.
Thus, in this gas flow region:

$$\mu_T = \mu_{T,in} = \rho\left[\kappa y D\left(y\right)\right]^2 \left|u_y\right| \qquad (2)$$

$$D(y)=1\text{-}\exp\left(-\frac{y/y^*}{26}\right),\ y^*=\frac{\mu_w}{\rho_w v^*},\ v^*=\sqrt{\frac{\tau_w}{\rho_w}}$$

Here, ρ is the gas density, u is the tangential projection of the gas velocity vector, y is the coordinate counted from the wall in the direction of its external normal, κ is the von Karman constant equal to 0.41, y^*, v^* are the characteristic length and velocity values used as scales, and τ_w is the friction stress on the wall. Subscript *w* refers to the parameters of the gas on the wall.

To calculate μ_T in the second, external region, it was used the formula:

$$\mu_T=\mu_{T,out}=0{,}0168\rho u_e\int_0^{y^{**}}\gamma\left(1-\frac{u}{u_e}\right)dy \tag{3}$$

$$\gamma=\left[1+\beta\left(\frac{y}{y^{**}}\right)^{\alpha}\right]^{-1},\ \alpha=6,\ \beta=5.5$$

The limit of integration in Eq. (3) is defined from the condition:

$$\frac{u\left(y^{**}\right)}{u_e}=0{,}995$$

2. Experimental data on convective heat transfer

The results of numerous experimental studies of the laminar-turbulent heat transfer in [3] were carried out at three different flow conditions. Radius *R* of the model's spherical rounding was equal to 0.0635 *m* in all experiments. The roughness of the model's surface was close to 8×10^{-6} *m*. The information about the other experimental conditions is presented in Table 1, where *N* is the number of the test mode, $\mathrm{Re}_{\infty,R}$ is the Reynolds number calculated by the parameters of the air in the incident flow and the radius of the spherical blunting of the model, q_{w0} is the specific heat flux in the critical point of the model calculated by the Fay-Riddell equation [9], and $\overline{T}_w$ is ratio of the wall temperature to the air flow stagnation temperature.

Table 1. Experimental conditions reported in [3]

N	M_∞	$\mathrm{Re}_{\infty,R}$	q_{w0}, W/m^2	$\overline{T}_w$
I	5	10^7	0.41×10^6	$0.24-0.27$
II	5	4×10^6	0.25×10^6	0.26
III	5	2.5×10^6	0.19×10^6	0.23

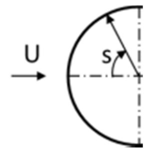
Apparently, the data presented in Table 1 do not suffice for computational studies. However, the missing information can be obtained by constructing an iteration process on the basis of the temperature in the oncoming air flow that ensures a preset specific heat flux value in the critical point of the spherical nose type. The results obtained by solving this problem are presented in Table 2.

Table 2. Established experimental conditions

N	T_∞, ^{0}K	ρ_∞, kg/m^3	V_∞, m/s	T_0, ^{0}K	p_0, MPa	T_w, ^{0}K
I	82.3	0.9895	909.3	493.8	0.762	123.45
II	81.4	0.3872	904.3	488.4	0.296	126.984
III	79.9	0.2310	896	479.4	0.173	110.26

3. Computational study and results

It was assumed in all computations that the gas was perfect and the conditions at the outer boundary of the boundary layer were formed on the basis of the numerical solutions of the Euler equations using a spline approximation procedure [10] afterwards.
Comparison of the experimental data computation results is shown in the figure 1.



On the all figures *S* is the coordinate of the point on the hemisphere measured in radians and it is counted from the stagnation point; $q_w = q_w(s)/q_{w0}$ is the specific heat flux in terms of fractions of its value at the critical point of the model's spherical blunting calculated by the Fay-Riddell equation [9].

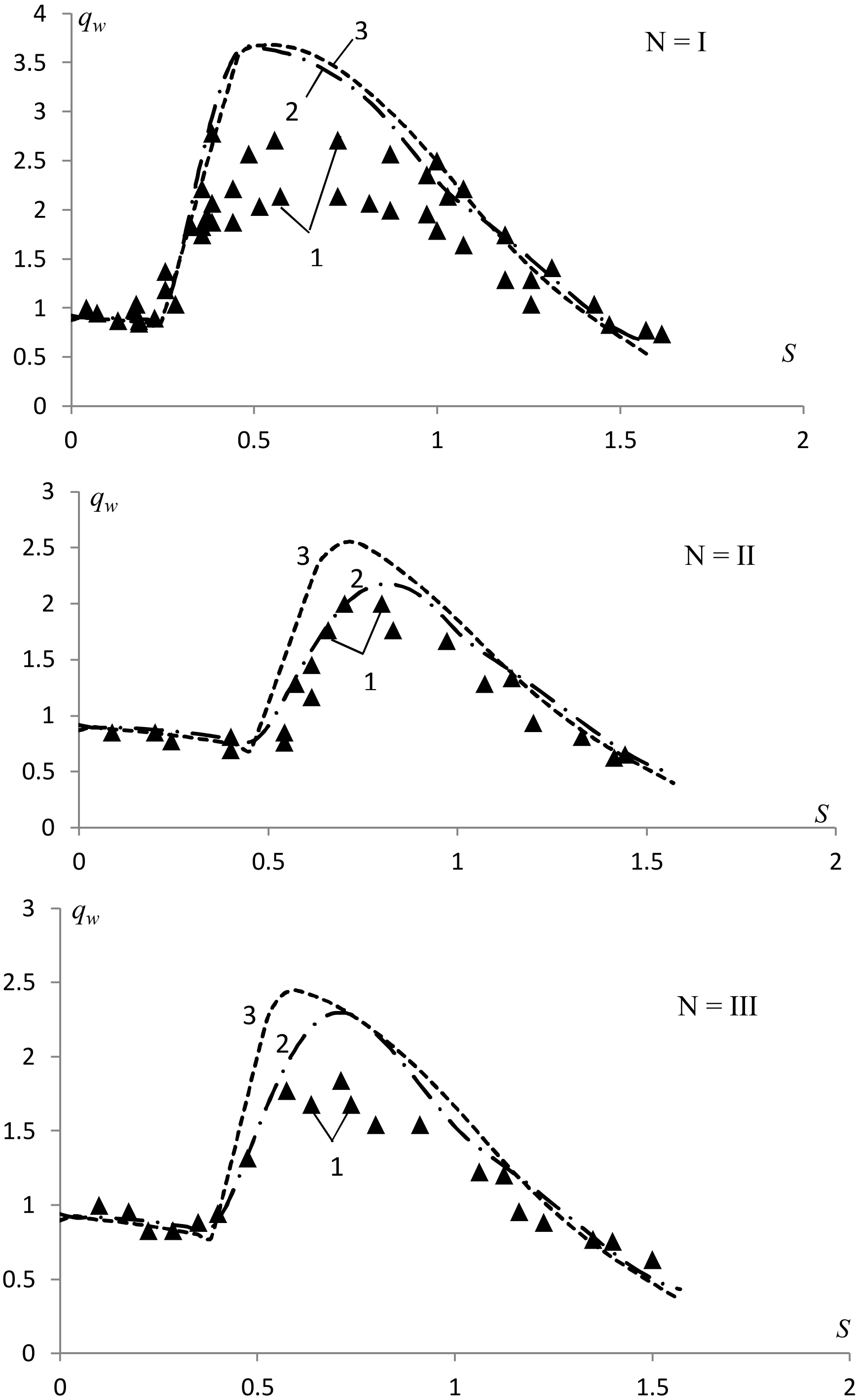


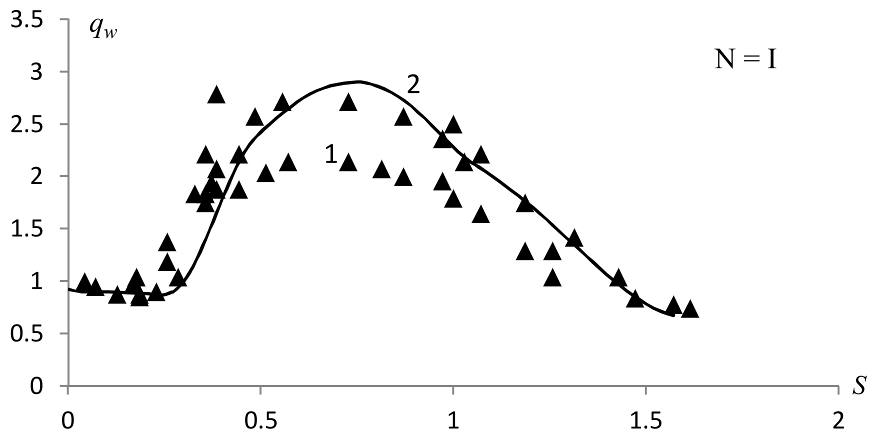
Figure 1. Distribution of the specific heat flux over the surface of a hemisphere under test conditions

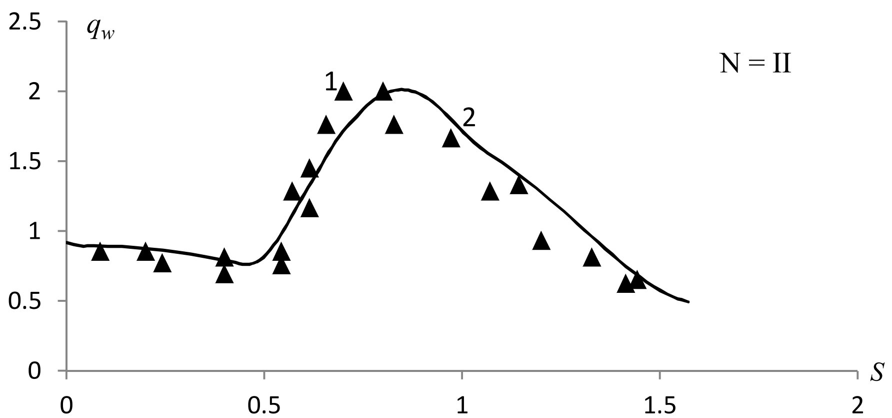
Here, 1- experimental data; 2 - standard Cebeci--Smith model; 3 - the effective length method; N – test mode number from the Table 1.

From the analysis of these data, it follows that both calculation approaches quantitatively correctly describes the experimental data, but they tend to overestimate the predicted thermal load with increasing of Reynolds number. The reason of this problem for standard Cebeci--Smith turbulence model could be the fact that it was developed for plate surfaces.
Thus, for application of this model in blunt bodies, especially for conditions with high Reynolds numbers, standard Cebeci--Smith turbulence model should be modified in the way that could ensure the minimum of the root-mean-square discrepancy between the predicted and the experimental data obtained in [3].
The modification was made by variation of empirical constants of the model: von Karman constant κ contained in the turbulence intensity in the internal part of the boundary layer and calculated by Eq. (2); constants α and β contained in Klebanov's function γ used in the external part of the boundary layer and calculated by Eq. (3). The optimal solution was found with using of extremum searching algorithm for multidimensional function [11].
In the beginning it were changed all three constants, but the analysis of the calculation had shown that for good results it was enough to vary only von Karman constant [12].
For application of this modification in cases when spherical blunting is connected with palate surface the value of von Karman constant should fluently change from the optimal magnitude (on the critical point of the blunting) to the normal value 0.41 on the plate surface.
This could be achieved with using of a such dependence:

$$\kappa(s) = \kappa_0 + (0{,}41 - \kappa_0) \cdot \sin^{2n}(s) \cdot [3 - 2\sin^n(s)]$$
$$\kappa_0 = 0{,}22 \qquad n = 3{,}099$$

The obtained optimization results are shown in the figure 2.





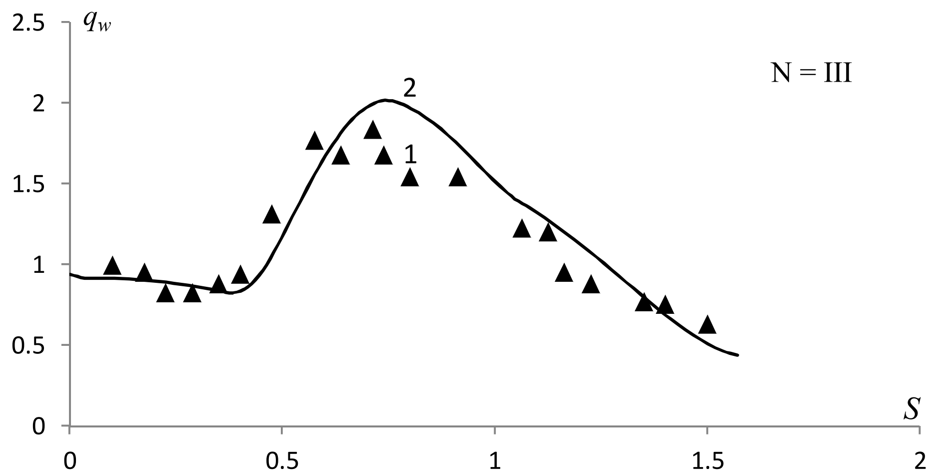


Figure 2. 1 – experimental data, 2 – modified turbulence model of Cebeci-Smith; N – test mode number from the Table 1.

From the figure it follows that application of the above modified turbulence model permits a considerably more accurate description of experimental data on the distribution of the specific heat flux over the surface of a hemisphere in the entire analyzed variation range of Reynolds numbers.

4. Conclusion

1. It is shown that both the standard Cebeci-Smith turbulent viscosity model and the effective length method correctly describe the analyzed amount of the experimental data, but, they tend to overestimated the specific heat flux by about one and a half times at high Reynolds number values.
2. It is proposed a modification of Cebeci-Smith turbulence model. The modification allows for turbulence in boundary layers on the blunt surfaces and enables a satisfactory description of the entire scope of the experimental data.

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