Modeling and Parameter Estimation of a Quadrotor MiniUAV

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Abstract
In this work, a miniature-sized, radio controlled quadrotor is modeled and identified using real-time flight data. The quadrotor that is used is this work, is equipped with a telemetry circuit to collect real-time data. Due to the lack of motor speed sensing circuitry, a nonlinear motor-propeller model is obtained and identified. This motor model is used to obtain the motor speeds of the recorded flight data. Euler angles versus motor speeds data is used to identify the nonlinear rotational subsystem of the quadrotor via nonlinear grey-box approach. The identified model performances are satisfactory.

1. Introduction

Quadrotors are very popular because of their ability to hover and vertical takeoff and landing (VTOL). As they are commonly remote-controlled vehicles, a modeling and control problem arises. It is possible to say that the first mini quadrotor concept works started to appear in early 2000s and then it turned into a development race. In 2000, Kroo et al. [1] from Stanford University has started Mesicopter Project and built a centimeter scale quadrotor which is controlled by a passive control mechanism. Altuğ et al. [2] controlled a quadrotor helicopter using a stationary camera to give visual position and orientation feedback; applied backstepping and feedback linearization techniques to a cross-type quadrotor model. Researchers from Stanford University developed STARMAC [3] based on Draganflyer III quadrotors, replaced the onboard electronics of the Draganflyer with a dual PIC microcontroller based controller board, which has also inertial measurement unit (IMU), GPS and ultrasonic range sensor, then applied sliding-mode feedback controller. Bouabdallah et al. [4] [5] [6] evaluated a Newton-Euler based quadrotor model as two subsystems and tried Lyapunov based control law as well as integral backstepping and feedback linearization based control approaches, then equipped their quadrotor with a controller board equipped with camera and sonar distance sensors for autonomous flight. Some researchers studied identification and parameter estimation of quadrotors. Stanculeanu et al. [7] collected real-time attitude angles versus command input data from a closed-loop quadrotor, and then identified the quadrotor dynamics as a linear state space model using prediction error method (PEM). Li et al. [8] worked on an AR. Drone 2.0 quadrotor in a lab equipped with camera based motion capture system and used attitude angles versus throttle input data to identify a linear model using PEM.

2. Dynamical model of quadrotor

A quadrotor model can be evaluated in two parts: the motor dynamics and the body dynamics. The body dynamics can be divided into another two: rotational and the translational dynamics. This approach is given in the figure 1.

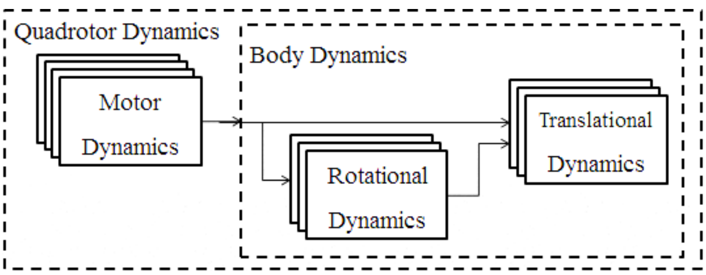

Figure 1: Quadrotor dynamic model overview

2.1 Reference coordinate frames

Basically there are two different approaches for quadrotor modeling: Euler-Lagrange and Newton-Euler. To express these models, coordinate frames and Euler angles have to be described.

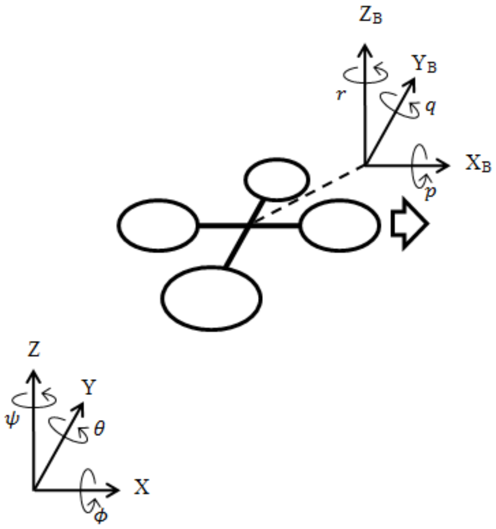

Figure 2: Reference frames, rotation angles and rotation rates

The position and orientation of the quadrotor will be given relative to a fixed coordinate frame, which is called the inertial frame. The X and Y axes of the frame are placed parallel and coincident to the ground while Z axis is pointing upwards in a right handed configuration. This configuration is also described by East, North, Up (ENU) coordinates.

$$\zeta \triangleq (x, y, z) \in \mathbb{R}^3 \tag{1}$$

The mobile frame placed to the center of gravity of the quadrotor is called the body frame. The X_B axis points the forward direction of the quadrotor, the Y_B axis points to the left and the Z_B axis points up in a right-handed configuration.

2.2 Euler angles and rotations

The most important problem in quadrotor control is its orientation in space. The well-known Euler angles representation is suitable for this purpose. The Euler angles ϕ (roll), ψ (pitch) and ψ (yaw) are the rotation angles about the axes X, Y and Z respectively (shown in figure 2).

$$\eta \triangleq (\psi, \theta, \phi) \in \mathbb{R}^3 \tag{2}$$

The yaw-pitch-roll (YPR or ZYX) composite rotation matrix [9] that transforms an orientation from the body frame to the inertial frame is given below.

$$R = R_z R_y R_x = \begin{bmatrix} Cos\theta Cos\psi & Cos\psi Sin\theta Sin\phi - Cos\phi Sin\psi & Cos\phi Cos\psi Sin\theta + Sin\phi Sin\psi \\ Cos\theta Sin\psi & Cos\phi Cos\psi + Sin\theta Sin\phi Sin\psi & -Cos\psi Sin\phi + Cos\phi Sin\theta Sin\psi \\ -Sin\theta & Cos\theta Sin\phi & Cos\theta Cos\phi \end{bmatrix} \tag{3}$$

The angular velocities (attitude rates) of the quadrotor: p, q and r are shown in figure 2.

$$\Omega \triangleq (p, q, r) \in \mathbb{R}^3 \tag{4}$$

2.3 Thrusts and torques

All motor on the quadrotor contribute to the main thrust (lift) relative to their angular velocities. It is possible to say that the motor M_i produces the force f_{Mi}, which is proportional to the square of the angular speed and the total thrust is the sum of the individual thrusts. This is given below where k is the lift constant and ω is the speed of motor Mi.

$$f_{Mi} \triangleq k\omega_{Mi}^2 \tag{5}$$

$$F_B = \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k\sum\omega_{Mi}^2 \end{bmatrix} \tag{6}$$

The torques that act about the roll and the pitch axes are given below.

$$\tau_\phi = f_{M4} - f_{M2} \tag{7}$$

$$\tau_\theta = f_{M3} - f_{M1} \tag{8}$$

The torque about the yaw axis is different from the ones above, and expressed as follows where b is the drag constant.

$$\tau_\psi = b(-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) \tag{9}$$

$$\tau_B = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} k(\omega_{M4}^2 - \omega_{M2}^2) \\ k(\omega_{M3}^2 - \omega_{M1}^2) \\ b(-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) \end{bmatrix} \tag{10}$$

The torques and thrusts are shown in figure 3.

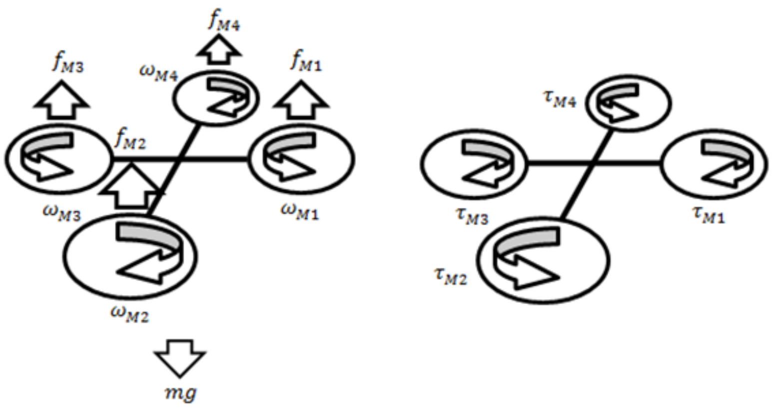

Figure 3: Torques and thrusts

2.4 Equations of quadrotor body

This work only covers the rotational dynamics of quadrotor due to the lack of precise motion capture system.
It can be said that a traditional quadrotor body is symmetrical for XB, YB and ZB axes. Thus, its moment of inertia matrix will be symmetrical.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{11}$$

The Newton-Euler based model approach is given below, where I_r represents the total moment of inertia of a rotor.

$$\omega \triangleq -\omega_{M1} + \omega_{M2} - \omega_{M3} + \omega_{M4} \tag{12}$$

$$I\dot{\Omega} = \tau_B - \Omega \times I\Omega - I_r\Omega \times \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \triangleq \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \tag{14}$$

$$\dot{p} = \frac{U_1}{I_{xx}} + \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{I_r}{I_{xx}} q\omega \tag{15}$$

$$\dot{q} = \frac{U_2}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{I_r}{I_{yy}} p\omega \tag{16}$$

$$\dot{r} = \frac{U_3}{I_{zz}} + \frac{I_{xx} - I_{yy}}{I_{zz}} pq \tag{17}$$

If the Euler angles ϕ, θ, ψ assumed to be small, these body frame model can be generalized to the inertial frame. The gyroscopic forces are also considered small and they can be neglected. Finally, the following nonlinear equations of the rotational dynamics are obtained.

$$\ddot{\phi} = \frac{U_1}{I_{xx}} \tag{18}$$

$$\ddot{\theta} = \frac{U_2}{I_{yy}} \tag{19}$$

$$\ddot{\psi} = \frac{U_3}{I_{zz}} \tag{20}$$

2.5 Equations of the motor model

The motor model is also important for a quadrotor. A brushed DC motor model will be sufficient for most of the quadrotors. The model is nonlinear because of the nonlinear aerodynamic load of the propeller ($k_L = k + b$).

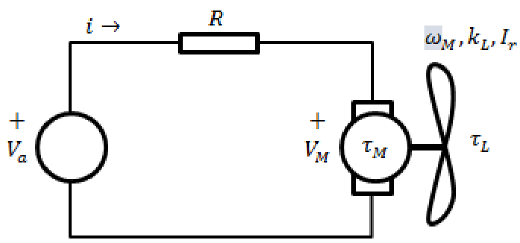

Figure 4: Model of the motor

The parameterized model is given below, where u is the armature voltage and the state variable x is the motor speed.

$$\dot{x} = a_0 u - a_1 x - a_2 x^2 \tag{21}$$

Finally, the pulse width modulation (PWM) duty can be converted to the armature voltage as given below.

$$0 \leq PWM_{Duty} \leq 1, \qquad V_a = V_{Supply} * PWM_{Duty} \tag{22}$$

3. Quadrotor platform

The goal of this work is collecting real-time flight data and identifying the model parameters. Crazyflie quadrotor is used for this purpose. Crazyflie is a miniature quadrotor that enables the implementation of low-level software to run on its onboard microcontroller. It has a radio communication link to a computer. It can be controlled using a generic USB joystick.

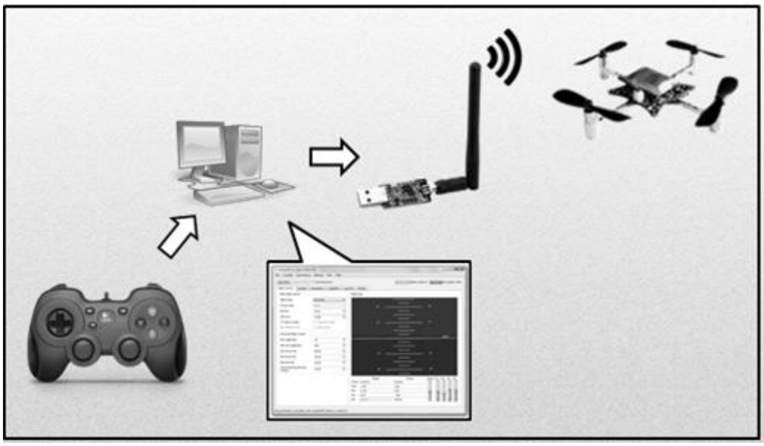

Figure 5: The operation scheme of the Crazyflie quadrotor

Closed-loop flight data is collected from the quadrotor, because open-loop flight is not possible. The collected data consists of the Euler angles versus the PWM signals that applied to the open-loop motors. The battery voltage is also collected during flight, and used to translate the PWMs into the armature voltages. The motors of the Crazyflie are driven open-loop and there is no motor speed feedback. The sampling time is 2ms.

4. Estimation of the model parameters

In this chapter, the motor and the quadrotor parameters are found consequently.

4.1 Estimation of the motor parameters

To identify the motors, a special speed sensing circuit is implemented to one of the motors. The motor speed, the motor PWM and the battery voltage are recorded on the ground, exciting the motor giving throttle (in open-loop). Then the PWM is converted to the armature voltage using the battery (supply) voltage. The graphs are given below.

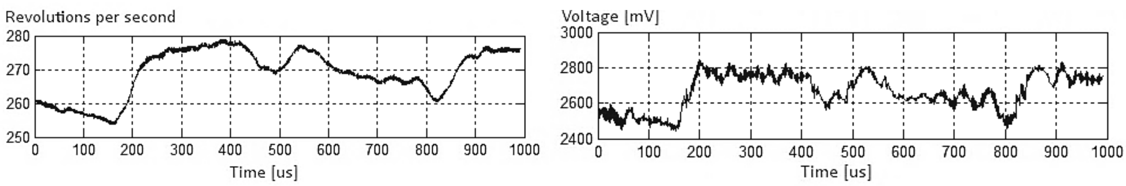

Figure 6: Motor speed and armature voltage

The nonlinear motor model is defined as a nonlinear grey-box model in Matlab and its parameters are found using PEM function in Matlab. The validation result (fit=91.73%) is given below.

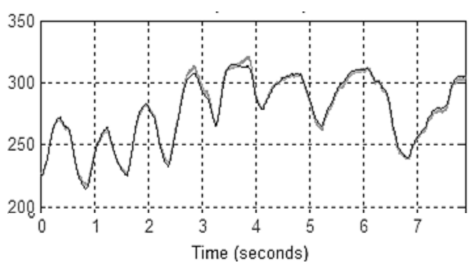

Figure 7: Validation of the identified motor model

The estimated parameters are given below.

$$\dot{\omega}_{Mi} = 0.862446V_a - 6.814507\omega_{Mi} - 0.006149\omega_{Mi}^2 \tag{23}$$

4.2 Estimation of the quadrotor model parameters

The identified motor model is used to obtain motor speeds from the PWMs of the recorded flight data. The same model is used for all of the motors, assuming that they are identical. The simulated motor speeds are given below.

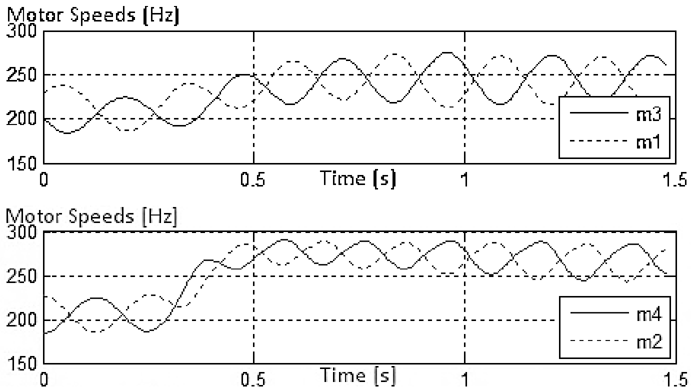

Figure 8: The simulated motor speeds

The recorded Euler angles (continuous lines) versus the pseudo-torques (dashed lines) are given below.

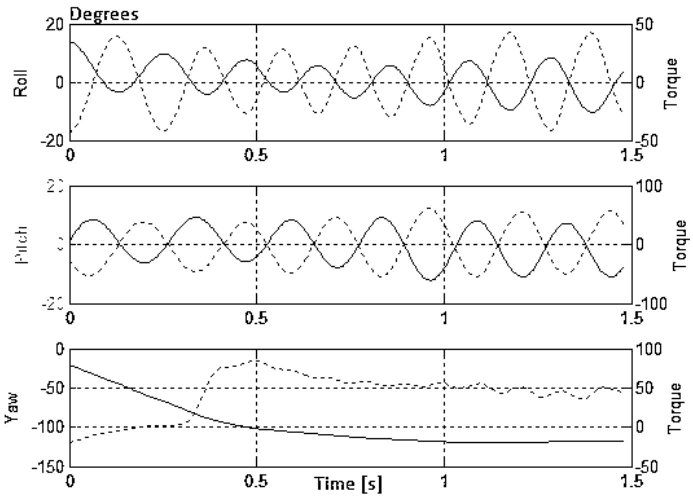

Figure 9: The recorded Euler angles (flight data)

Now, the model equations are used to construct a nonlinear grey-box model in Matlab. Estimation of the parameters is done using PEM. The results for the roll axis are given below.

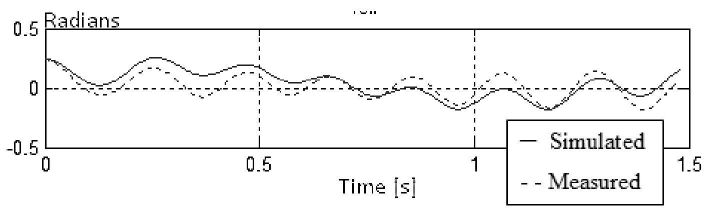

Figure 10: Roll axis identification data fit, RMS error = 0.0847

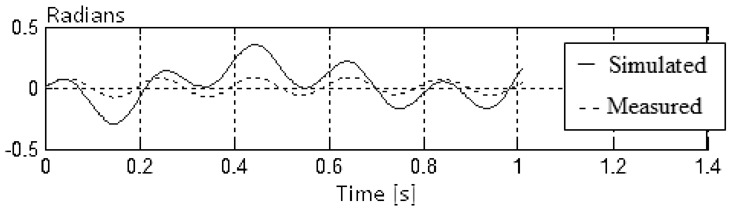

Figure 11: Roll axis validation data fit, RMS error = 0.1209

It is reasonable to use the roll axis model for also the pitch axis thanks to the structural symmetry. Performance of the roll axis model with the pitch axis data is given below.

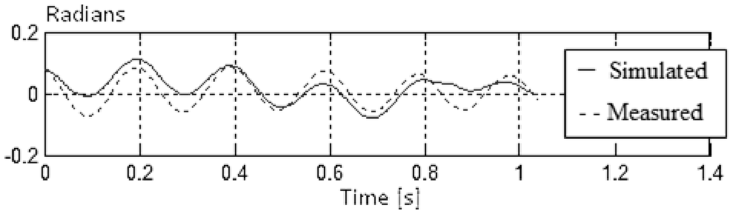

Figure 12: Roll axis model applied to the pitch axis validation, RMS error = 0.0345

The estimation results for the yaw axis are given below.

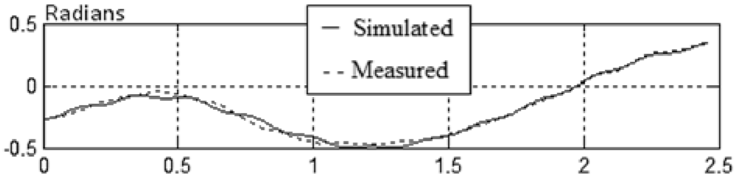

Figure 13: Roll axis identification data fit, RMS error = 0.0230

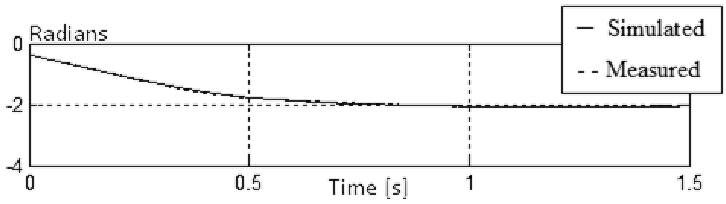

Figure 14: Roll axis validation data fit, RMS error = 0.0181

The identified model equations are explicitly given below.

$$\ddot{\phi} = 0.0136(\omega_{M4}^2 - \omega_{M2}^2) \quad \textbf{(24)}$$
$$\ddot{\theta} = 0.0136(\omega_{M3}^2 - \omega_{M1}^2) \quad \textbf{(25)}$$
$$\ddot{\psi} = 0.0005(-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) \quad \textbf{(26)}$$

4.3 Black-box approach

As a black-box approach, adaptive neuro-fuzzy inference system (ANFIS) is used but the results are not satisfactory. This is due to the improper training data, because the closed-loop system cannot be excited enough.

5. Conclusions

This work shows that a quadrotor model can be identified using flight data with satisfactory results. The online simulation of the motor dynamics via an identified motor model also works well. The identification of a quadrotor model without motor speed sensors is possible. From this point, it can be said that the cascade modeling approach that is shown in figure 1 is applicable. The prediction error method (PEM) performs well to estimate the unknown parameters of a closed-loop system

On the other hand, noticeable identification and validation errors occurred. The assumption of small Euler angles, the neglected gyroscopic forces and the errors that come from the identified motor model can be counted as the reasons. One of the reasons of the errors is the non-identical motor and propeller dynamics. There are also some aerodynamic effects that cannot be modeled.

In the future works, the position measurement will be done using motion capture cameras, thus the translational subsystem will be identified and autonomous flight will be studied.

References

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