Formulation comparison in Multi Disciplinary Optimization

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Abstract

This article presents several MDO formulations which are used in complex system design. Their features, advantages and drawbacks are exposed. Then, in order to compare their performances, they are theoretically experimented on a test-case concerning the conceptual design of aSuperSonic Business Jet (SSBJ).

keywords: Multi Disciplinary Optimization, feasibility studies, algorithms.

Introduction

Research of the best performance, the best quality at the lowest cost, is a key point in the field of aerospace. It is compulsory to master design and sizing process, since any error leads to huge consequences to an operational or economical point of view.

A global perception of multi-disciplinary optimization is given in [1] and [2]. A natural example is aircraft design. A lot of disciplines are involved, such as structural mechanics, aerodynamics or propulsion. There is also aeroacoustics with the aim of reducing the noise.

Aircraft design is a complex process due to the wide number of disciplines involved. Several parameters can be managed by only one discipline or can be shared. There are also interactions among disciplines : outputs can be injected as inputs in other disciplines. Moreover, disciplinary objectives are often conflicting.

In optimal design research, engineer skills and know-how is no longer sufficient. One has to develop design and optimization techniques which will enable to integrate disciplinary methods. This will allow the research of a global optimum or a Pareto front. Starting from these results, experts will be able to discuss in order to find a compromise.

ONERA, the french aerospace lab, has started a project in order to carry out studies on Multi-Disciplinary Optimization (MDO). Several methods are investigated, such as model reduction, aerodynamics and structures optimization, acoustics and MDO formulations. MDO formulations comprise the different ways to cope with a multidisciplinary optimization problem.

Technical background is described in section 1. Then 4 MDO formulations, which are called MDF, IDF, IDF sequential and CO, are presented in section 2. Finally, we test them with a Super Sonic Business Jet test-case in section 3.

1 Technical background

In this part, the tools that are used to set up MDO formulations are described. Concepts such as disciplines, different kinds of design variables, interactions between disciplines, and optimization, are defined and explained.

1.1 Disciplines

The process of complex system design can be divided into several sub-processes, presently disciplines, in order to make it easier to solve. For example, in the case of aircraft design, disciplines are currently structural mechanics, aerodynamics and propulsion. Each discipline taken separately is well understood by specialists and produces a computer program which describes aircraft behavior from a disciplinary point of view. It takes into account several variables which are separated into three main classes : private, shared, and coupling variables.

. i stands for discipline i.

. X_i : discipline i private variables. It acts only on i. Let X be $\bigcup_i X_i$.

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- Z : shared variables. They are common for at least two disciplines.
- $Y_{ji}^{\wedge}$: predicted coupling variables, input for i and originated from j.

  Let $Y_i^{\wedge}$ be $\bigcup\limits_j Y_{ji}^{\wedge}$: all the coupling variables that act as input for i.

  Actually, $\bigcup\limits_j Y_{ij}^{\wedge}$ contains all the coupling variables originated from i that act as input for other disciplines.

  All these variables can be gathered in $Y^{\wedge} = \bigcup\limits_i Y_i^{\wedge}$.

Considering these variables, discipline i computes outputs Y_i.

$$[Y_i, Y_i^*] = Y_i(X_i, Z, Y_i^{\wedge})$$

- $Y_i()$ refers to discipline i computer program.
- Y_i output vector.
- Y_{ij}^* is calculated by i and destined for j.

  $Y_i^* = \bigcup\limits_j Y_{ij}^*$ are the computed coupling variables originated from i and destined for other disciplines.

  In effect, $\bigcup\limits_j Y_{ji}^*$ stands for the calculated coupling variables computed by other disciplines and destined for i.

Figure 1 illustrates how a discipline works.

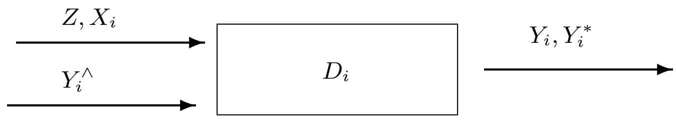

Figure 1: discipline i

In the next part, the differences between :

- $Y_i^{\wedge}$ and $\bigcup\limits_j Y_{ji}^*$
- Y_i^* and $\bigcup\limits_j Y_{ij}^{\wedge}$

are exposed.

1.2 Interdisciplinary interactions

Interdisciplinary interactions can be illustrated with an aero-structural example. By taking into account the wing shape, aerodynamics algorithms can compute the pressure field. This pressure deforms the wing. New wing shape produces a new pressure field, and so on. In effect, any disciplinary changes will affect other disciplines. The physical problem is solved when the wing shape matches the pressure field.

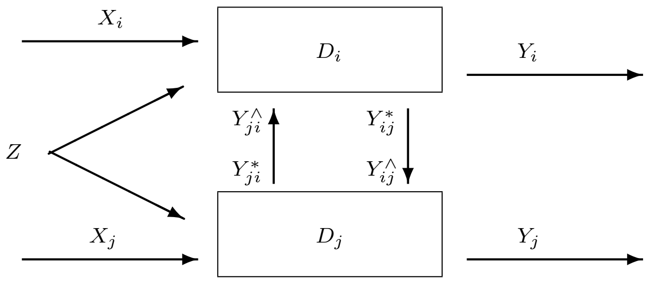

Figure 2: interactions between two disciplines

Interactions are shown on figure 2.

One starts from configuration $[X, Z, Y_{ji}^{\wedge}]$.
A new $Y_{ij}^* = Y_{ij}^*(X_i, Z, Y_{ji}^{\wedge}])$ is calculated.
Then, $Y_{ij}^{\wedge} \leftarrow Y_{ij}^*$: this variable is injected in j, which computes $Y_{ji}^* = Y_{ji}^*(X_j, Z, Y_{ij}\wedge)$.
In this two disciplinary case, one has $Y_i^{\wedge} = Y_{ji}^{\wedge}$ and $\bigcup\limits_k Y_{ki}^* = Y_{ji}^*$. Thus, $\bigcup\limits_k Y_{ki}^*$ becomes different from initial configuration $Y_i^{\wedge}$.
In reality, $Y^{\wedge}$ values are searched for minimizing the quantity of

$$|Y^{\wedge} - Y^*(X, Z, Y^{\wedge})|$$

That is to find $Y^{\wedge}$ which matches Y^*.

- In some cases, where the system has good properties, it can be solved with iterative methods, like Fixed Point Algorithm or Gauss-Seidel Method.
- In other cases, Gauss-Newton Method can be used.
- This design step is called Multi Disciplinary Analysis (MDA).
- Another way to deal with interdisciplinary coherence, within an optimization process, is to add equality constraints such as

$$Y^{\wedge} - Y^*(X, Z, Y^{\wedge}) = 0$$

With all these variables, we have a large configuration set to explore in order to find the best aircraft, according to the optimization objective and the fidelity level of the disciplinary models. We need optimization tools which are able to carry out this task, in an automatic and efficient way.

1.3 Optimization

Optimization problem can written as follows:

$$\begin{cases} \min\limits_x F(x) \\ G(x) \leq 0 \\ H(x) = 0 \\ x_l \leq x \leq x_u \end{cases}$$

Vector x includes all design variables.

. F is the objective function. It could be the aircraft total weight or range.

. G is the inequality constraints vector. Discipline i has its own requirements which are expressed by G_i. Let $G = \bigcup_i G_i$

. H is the equality constraints vector and represents interdisciplinary coherence constraints.

. Each variable has a lower bound and an upper bound, in order to perform optimization within a reasonable configuration set.

From a practical point of view, we use SQP algorithm, since it matches non-linear optimization problems well. Multiple local optima problems can be addressed by an exploration stage, and several optimizations which start with the best configurations found previously.

MDO formulations can be now set up with all these tools.

2 MDO formulations

In this section, different MDO methods are described. Their characteristics, advantages and drawbacks are pointed out, as well as their similarities and differences. These methods are explained in more details in [3] and [4].

2.1 Multi Disciplinary Feasible

This approach is also called All In One or Fully Integrated Optimization. It is the reference for MDO formulations. Diagram 3 represents this formulation.

Starting from configuration $[X, Z]$, the system carries out a coupled analysis and determines outputs. Then, Y is given to the optimizer who chooses a new point to test.

In effect, variables X, Y and inequality constraints G are considered at the same level. Coupling variables $Y^\wedge$ and Y^* are only considered during coupled analysis.

++ Each optimization point fulfills interdisciplinary coherence, that will lead to reliable results. Moreover, system analysis can be seen, from an optimizer point of view, as a black box.

- - There is a lack of modularity. System analysis is made of one compact block and it is hard to adapt to any change.

MDA can be highly time-consuming. It is carried out for each tested point, even when gradient is calculated with finite differences.

A solution to this last drawback is called Global Sensitivity Equation. This method [5] enables less computer

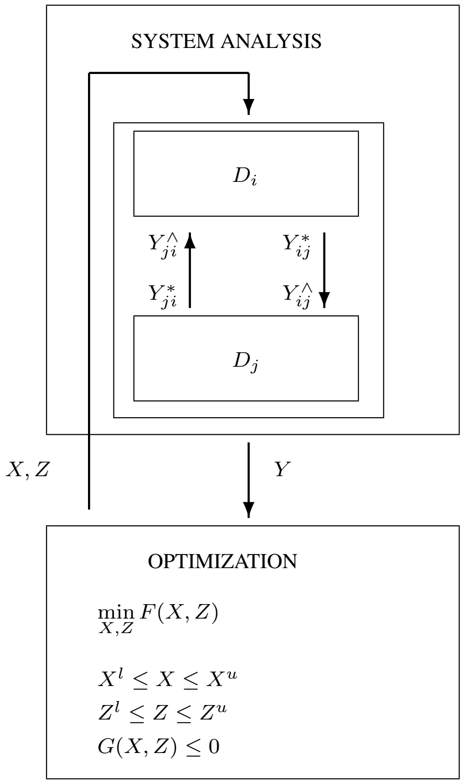

Figure 3: MDF formulation

time consumption. In fact, it operates on gradient calculation with finite differences. First, interdisciplinary coherence is established at one point. Then, sensitivities are computed for each discipline taken separately. Finally, the gradient at this point is given by solving a linear system.

Another way to cope with this problem is to treat interdisciplinary coherence differently, as we will see in next sections with IDF and CO.

2.2 Individual Disciplinary Feasible

This formulation described in figure 5 is also called All At One or Simultaneous Analysis and Design. In this method, disciplinary exchange is not direct, but exists through the optimizer. Interdisciplinary coherence is ensured with equality constraints, such as:

$$Y^\wedge - Y^*(X, Z, Y^\wedge) = 0$$

In fact, variables X, Z, and $Y^\wedge$ and inequality constraints G are considered at the same level. Starting from configuration $[X, Z, Y^\wedge]$, disciplinary outputs $[Y, Y^*]$ are generated and then given to the optimizer.

++ MDA is never carried out.

- - Outputs are not reliable since interdisciplinary coherence is not always effective. Therefore it can lead the optimization process to nowhere. It is also difficult for the optimizer to deal with a large number of nonlinear equality constraints.

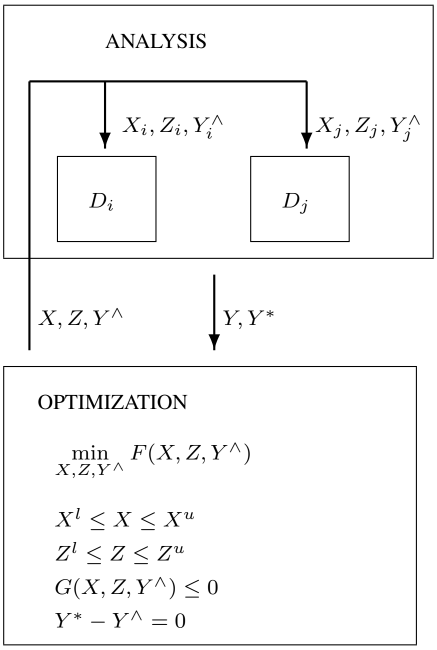

Figure 4: IDF formulation

In the next part, a formulation known as IDF sequential, which reduces the number of equality constraints, is presented.

2.3 Individual Disciplinary Feasible sequential

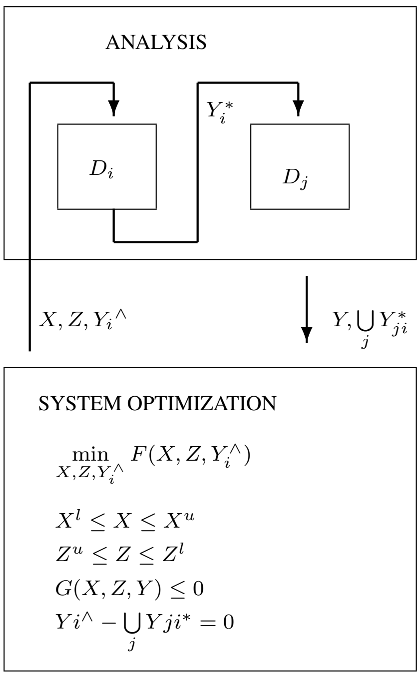

Figure 5: IDFS formulation

IDF sequential is also called IDFS. As it is illustrated in figure 5, we reduce the set of predicted coupling variables controlled by the optimizer to $Y_i^\wedge$.

Starting from configuration $[X, Z, Y_i^\wedge]$, discipline i calculates its outputs $[Y_i, Y_i^*]$ and feeds other disciplines with Y_i^*. Once disciplinary analyses are done, outputs $[Y, \bigcup_j Y_{ji}^*]$ are given to the optimizer.

++ Compared with the IDF method, the optimizer has to manage a small number of predicted coupling variables and inequality constraints.

- - It is not obvious that, $|Y_i^\wedge - \bigcup_j Y_{ji}^*|$ convergence leads to a the global $|Y^\wedge - Y^*|$ convergence.

2.4 Collaborative Optimization

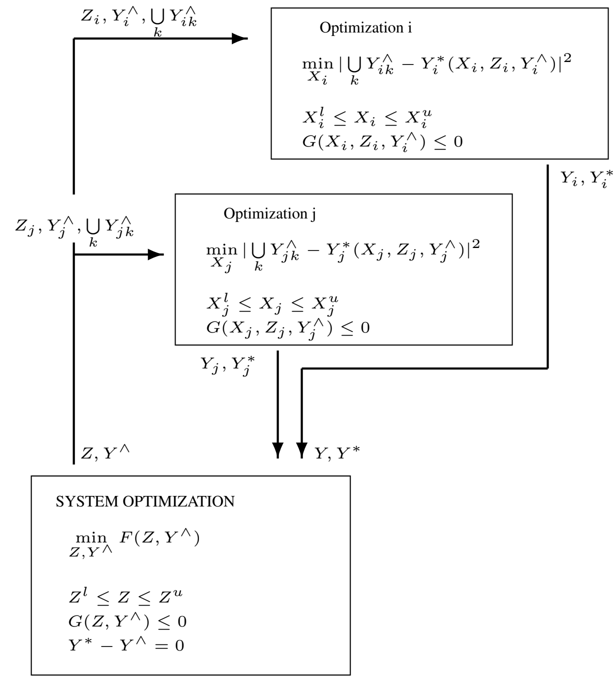

Figure 6: CO formulation

CO formulation is shown in diagram 6.

In this method, as in IDF, interdisciplinary coherence is ensured by equality constraints. Moreover, there is a bi-level optimization.

Actually, $Y_i^\wedge$ is given to discipline i as a set local objectives. Then, i has to settle its X_i in order to match these targets, while satisfying its disciplinary inequality constraints G_i. That is to solve the following minimization problem :

$$\left\{ \begin{array}{c} \mathbf{given[Z_i, Y^\wedge]} \\ \min_{X_i} |\bigcup_k Y_{ik}^\wedge - Y_i^*(X_i, Z_i, Y_i^\wedge)|^2 \\ G_i(X_i, Z_i, Y_i^\wedge) \leq 0 \\ X_i^l \leq X_i \leq X_i^u \end{array} \right.$$

To illustrate this method, structures is discipline 1 and aerodynamics is discipline 2. Y_{12} is the wing shape, and Y_{21} the pressure field. The system asks structures to compute a wing shape close to the target $Y_{12}^\wedge$, by taking into account the shared variables Z, and the targeted pressure field $Y_{21}^\wedge$.

Then discipline 1 has to find its X_1 in order to minimize

$$|Y_{12}^{\wedge} - Y_{12}^{*}(X_1, Z_1, Y_{21}^{\wedge})|^2$$

This is the same with aerodynamics, where X_2 is searched to minimize

$$|Y_{21}^{\wedge} - Y_{21}^{*}(X_2, Z_2, Y_{12}^{\wedge})|^2$$

In reality, some values of Z and $Y^{\wedge}$ do not enable the satisfaction of the disciplinary constraints G_i. In this case, the positive values of vector G_i are given to the system optimizer. Then, it can find out the right values of Z and $Y^{\wedge}$ that will allow admissible disciplinary configuration.

In this paper Z and $Y^{\wedge}$ are controlled by the system optimizer, and X is driven by disciplinary optimizers. Concerning the original version of CO that is described in [6], Z is driven by disciplinary optimizers and the system optimizer.

++ Disciplinary objectives are dynamically allocated by the system optimizer. This approach matches well the company's organization, in which an entity oversees the design and the other disciplines trying to reach targets with their own means.

- - As for IDF, it is hard to deal with a lot of equality constraints. In addition, disciplinary optimizations can be time-consuming, since whenever discipline i is called, X_i is optimized.

In the next section, these formulations are illustrated an compared with the mean of an aircraft conceptual design test-case.

3 Application

3.1 The SSBJ test-case

This test-case is provided by Sobiesznanski-Sobiesky and widely used in literature [7]. It was created especially for testing MDO formulations and enables the conceptual design of an SSBJ.

Incidentally, models are easy to dispose in order to organize the different formulations and they are not time consuming.

Figure 7 represents SSBJ dependency diagram.

There are four disciplines. Three of them represent design, such as structural mechanics, aerodynamics and propulsion. The fourth is dedicated to aircraft performance, taking into account the outputs of the other disciplines, will determine SSBJ range. There are interactions between them and they calculate inequality disciplinary constraints.

There are twelve disciplinary constraints.

Shared variables are :

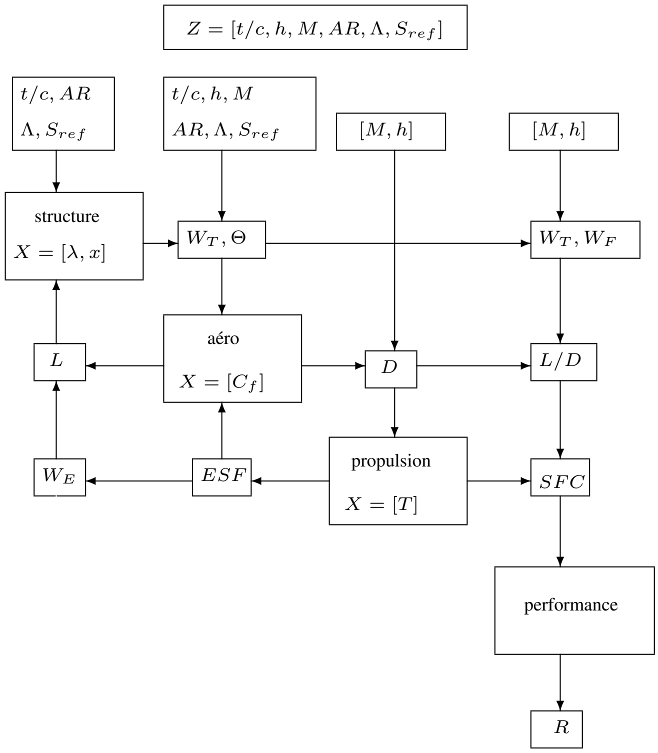

Figure 7: SSBJ test case

- Z

t/c	thickness/chord
h	cruise flight altitude
M	Mach number
S_{REF}	wing reference area
Λ	wing sweep
AR	aspect ratio

Now, private variables X_i, shared variable subset Z_i, and coupling variables in input $Y_i^{\wedge}$ and output Y_i^{*}, are exposed for each disciplines.

- Discipline 1 : Structure

X_1	λ	taper ratio
	x	box section
Z_1	AR, Λ	
	$t/c, S_{REF}$	
$Y_{21}^{\wedge}$	L	lift
$Y_{31}^{\wedge}$	W_E	engine weight
Y_{12}^{*}	W_T	total weight
	Θ	wing twist
Y_{13}^{*}	W_F	fuel weight
Y_{14}^{*}	W_T, W_F	

- Discipline 2 : Aerodynamics

X_2	C_f	skin friction coefficient
Z_2	AR, Λ, M	
	$t/c, S_{REF}, h$	
$Y_{12}^{\wedge}$	W_T	total weight
	Θ	twist
$Y_{32}^{\wedge}$	ESF	engine scale factor
Y_{21}^{*}	L	lift
Y_{23}^{*}	D	drag
Y_{24}^{*}	L/D	lift to drag ratio

- Discipline 3 : Propulsion

X_3	T	thrust
Z_3	h, M	
$Y_{23}^{\wedge}$	D	drag
Y_{31}^{*}	W_E	engine weight
Y_{32}^{*}	ESF	engine scale factor
Y_{34}^{*}	SFC	specific fuel consumption

- Discipline 4 : Range

Z_4	h, M	
Y_{14}	W_T	total weight
	W_F	fuel weight
Y_{24}	L/D	lift to drag ratio
Y_{34}	SFC	specific fuel consumption
Y_4	R	range

The aim of optimization is to maximize the range (Y_4), and to respect all disciplinary constraints.

Even if this SSBJ example expresses disciplinary organization well, it does not represent the complexity encountered in industry. Moreover these semi-empiric models are not always physically reliable.

In the next section, MDO formulations concerning this test-case are implemented, with the mean of Model Center Software developed by Phoenix Integration (www.phoenix-int.com).

3.2 Results

Model Center enables the testing of MDO formulations on complex system cases, by wrapping easily disciplinary modules and linking them with other disciplines or optimizers, and building the corresponding workflow.

Tables 1 and 2 present variables and results obtained.

var	**MDF**	**CO**	**IDF**	**IDFS**
shared				
t/c	0.05989	0.05505	0.05997	0.05995
h	**60000**	**60000**	**60000**	**60000**
M	**1.4**	**1.4**	**1.4**	**1.4**
AR	**2.5**	**2.5**	**2.50001**	**2.5**
Λ	**70**	**70**	69.4145	**70**
S_{REF}	**1500**	1483.05	1477.88	**1500**
structure				
λ	0.25626	0.24901	0.27917	**0.1**
x	0.76418	1.0389	0.88459	**0.75**
aero				
C_f	**0.75**	**0.75**	**0.75**	**0.75**
propulsion				
T	0.15624	0.13599	0.15622	0.15624

Table 1: Optimal variables as a function of the MDO formulation used

Shared variables are roughly similar. The largest differences are for variable S_{REF}. From a physical point of view, wing surface should be as large as possible for maximizing the fuel capacity and reducing the induced drag. This enables to maximize the range.

Concerning structural variables, except for IDF sequential, λ are quite similar. x is more dispersive. Variables $[\lambda, x]$, which are obtained with IDF sequential, reach their lower bounds. C_f is always minimal. T is quite similar, excepted for CO where it has a low value.

outputs	**MDF**	**CO**	**IDF**	**IDFS**
range				
R	3494.52	3248.76	3451.65	3505.46
L/D	13.3528	14.0934	12.7454	13.1514
W_T	74253.3	74971	70778.1	73161.7
W_F	19317.9	17651.2	18959.9	19334.6
Sfc	0.92393	0.93454	0.92395	0.92393
call number				
struct	239	639	104	460
aero	239	717	120	367
prop	239	440	64	51
range	45	77	136	184
total	762	1873	424	1062

Table 2: Results as a function of the MDO formulation used

Range is calculated with the Breguet-Leduc formula:

$$R = \frac{M(L/D)661\sqrt{\theta}}{SFC} \ln\left(\frac{W_T}{W_T - W_F}\right)$$

Actually, θ is an internal variable of the discipline 4. It has no link with the Θ variable which is used in the structures discipline.

IDF sequential has a better range than MDF, due to its lower total weight W_T , larger fuel weight W_F, and better L/D ratio. This can be explained with variables $[\lambda, x]$ which are smaller in IDF sequential.

The best number of calls is reached by IDF. Assuming that each module call costs alike, IDF seems to be more time efficient than others.

3.2.1 MDF

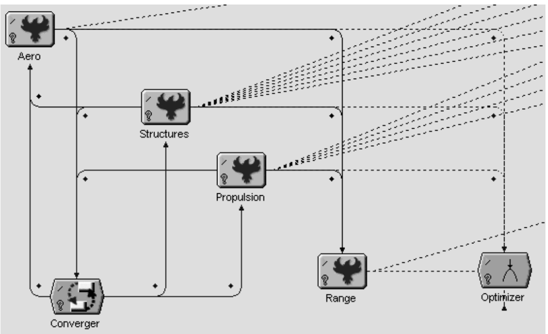

Figure 8: MDF

Figure 8 shows MDF implementation in Model Center.

MDA calls each discipline 4 to 8 times and the coupling variables converge with a 10^{-5} accuracy.

Results obtained with this formulation are considered as a reference for comparison.

3.2.2 IDF

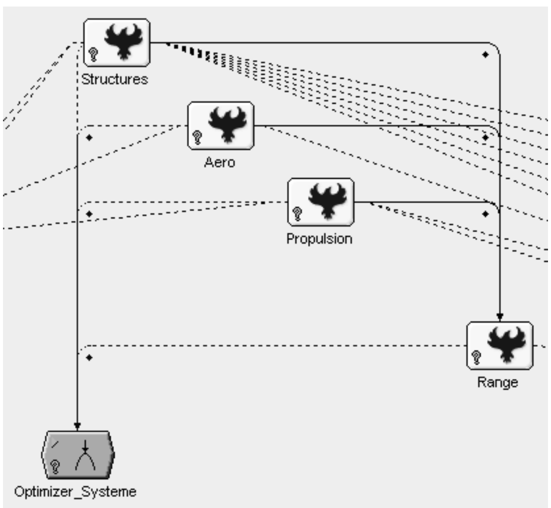

Figure 9: IDF

Figure 9 describes IDF in Model Center.

Optimization is highly sensitive to constraint sensibility parameters. The number of discipline calls for IDF is lower than that of MDF. MDA, carried out with optimal values $[X, Z]_{IDF}$, produces outputs that are slightly different from Y_{IDF}. This formulation works faster but is less robust than MDF.

Figures 10 and 11 present convergence for relative equality constraints

$$|\frac{Y^\wedge - Y^*(X, Z, Y^\wedge)}{Y^\wedge}|^2 = 0$$

In figure 10, the initial point satisfies equality constraints, and in figure 11, the initial point is a random point.

In the first case, maximum error is in the order of 0.1, in the second case, it is less than 0.01.

Convergence is better in the second case. In fact, the further the point is from a coupled configuration, the stronger the attraction is to this configuration. This is due to the quadratic nature of the constraint.

Interdisciplinary coherence is in general reached at the end of optimization.

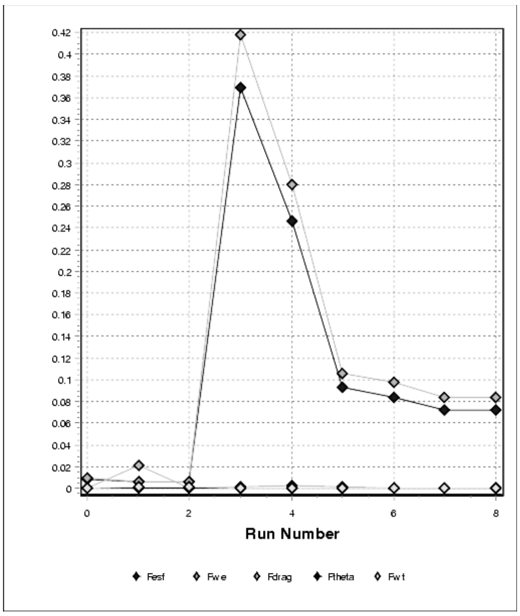

Figure 10: Interdisciplinary coherence constraints evolution starting from a reasonable point

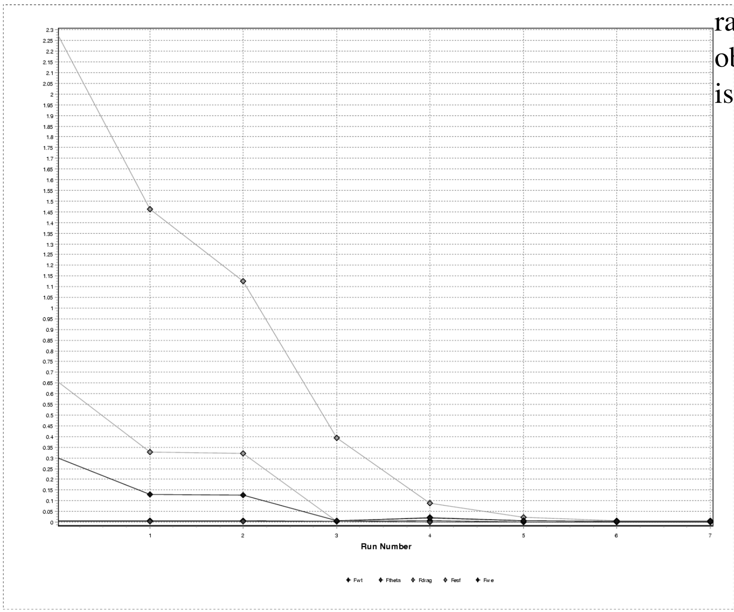

Figure 11: Interdisciplinary coherence constraints evolution starting from a random point

3.2.3 IDF sequential

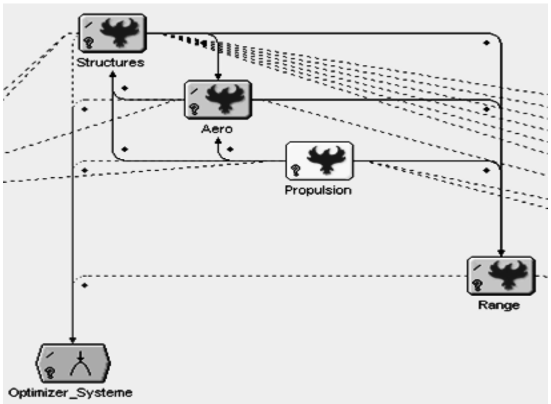

Figure 12: IDF sequential

IDF sequential is shown in figure 12. The best range of 3505 Nm is reached. MDA, carried out with optimum values $[X, Z]_{IDFS}$, gives a range of 3500 Nm. It is more robust than IDF, but it requires twice as many calls. Model Center computes outputs only if it was not carried out before. In the one hand, with IDF sequential, each change in propulsion will invalidate aerodynamics and structures outputs. In the other hand, with IDF, propulsion changes will only affect this discipline.

3.2.4 CO

The worst range of 3249 Nm is reached with this method. This formulation is also very sensitive to optimization parameters. The large number of disciplinary calls is a consequence of the local optimizations. MDA, carried out with optimum values $[X, Z]_{CO}$, gives a range of 3240 Nm If we compare CO outputs with those obtained with MDA, we observe some differences. CO is not robust and is time consuming.

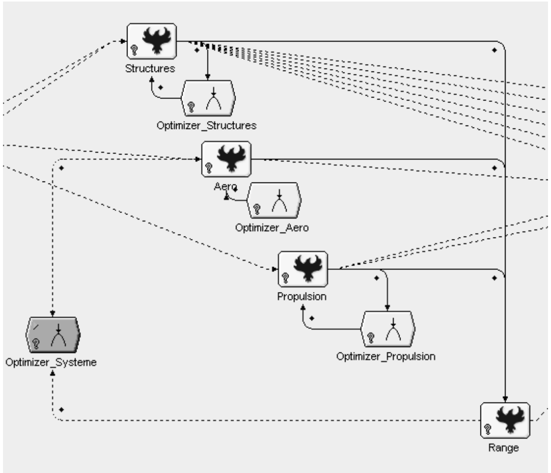

Figure 13: CO

To sum up with MDO formulations:

. MDF can be considered as the most reliable.

. IDF is the fastest but lacks of robustness.

. IDF sequential has the best range with a good robustness, but requires a large number of calls.

. CO does not work well with this test-case.

Conclusion

The SSBJ test case allows us to test and compare MDO formulations according to their reliability and efficiency. We have demonstrated techniques which are used to implement them.

Even if this test expresses well the organization of aircraft design process, it does not represent its complexity. MDO formulations should be implemented with a more complex example.

One will have to use reduction model techniques in order to integrate accurate computer programs in the global design process.

Individual discipline can also compute its own reduced model, where outputs are optimized according to the private variables X. These concepts can lead bi-level methods such as CO or DIVE [8] to be more efficient.

References

[1] M. Masmoudi, *Conception collaborative*, Rapport Technique MIP UPS Toulouse 3 (2004).

[2] R. Sobieszczanski-Sobieski Jarolaw ans Haftka, *Multidisciplinary Aerospace Design Optimization : Survey of Recent Developments*, AIAA (1996).

[3] J.-C. Blondeau C Schotté, *Méthodologies MDO*, Rapport Technique ONERA (2005).

[4] N. Brown, *Evaluation of MDO Techniques Applied to a Reusable Launch Vehicle*, Rapport Technique Giorgia Institute of Technology (2004).

[5] P. Hajela, C. Bloebaum et J. Sobieszczanski-Sobieski, *Application of Global Sensitivity Equations in Multidisciplinary Aircraft Synthesis*, Journal of Aircratf, (12) (1990), pp. 1002–1010.

[6] A. N. M. et L. R. Michael, *Analytical and Computational Aspects of Collaborative Optimization*, Rapport Technique NASA Langley Technical Report Server (2000).

[7] Sobieszczanski-Sobieski, *Bi-Level Integrated System Synthesis (BLISS)*, NASA/TM (1998).

[8] Y. Parte et M. Masmoudi, *Disciplinary Interaction Variable Elimination (DIVE) Approach for MDO*, Dans *ECCOMAS CFD 2006* (2006).

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