

A Quick-Look Analysis Tool for the Impulse Performance of Spacecraft Propulsion Systems

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Abstract

The latest up-date of a software program is presented, which is a quick-look help-tool for the performance analysis of spacecraft jet propulsion systems. The software is based on the 'system-specific impulse', I_{ssp} , which allows a more accurate determination of the propulsive performance of spacecraft propulsion systems than the commonly used 'specific impulse', I_{sp} .^[1] The program is intended to be a high level, quick-look tool for a preliminary selection of propulsion systems for spacecraft missions of given delta-v requirements. In addition, for tutorial purposes, the program is of exceptional help to provide a basic understanding of the impulse performance capability of today's common propulsion systems.

1. Introduction

In general, when selecting a spacecraft propulsion system for given mission impulse demand, most important is its achievement of the impulse requirement with highest possible spacecraft payload mass. At first glance this will be achieved by selecting a propulsion system with high thrust engine performance 'specific impulse', I_{sp} . With regard to the 'Rocket Equation', this will result in a mission final high spacecraft mass, which means high payload mass, because of lower propellant mass consumption during the spacecraft mission. However, propellant storage systems, and especially for electric propulsion, electric power supply and power processing systems may form a major 'dead' dry mass of the overall propulsion system mass. Therefore, the choice and sizing of propulsion systems is not always clear on the basis of I_{sp} alone.

Because of the complex interaction between I_{sp} , mission impulse demand and propulsion system mass, a more comprehensive propulsion related figure of merit is required than the I_{sp} alone. A most useful reference number comprising the entire propulsion system performance is the 'system-specific impulse', I_{ssp} , which defines the total impulse, I_{tot} , delivered by the system, divided by the total propulsion system mass, m_{PS} , including propellant:

$$I_{ssp} = \frac{I_{tot}}{m_{PS}} \left[\frac{\text{Ns}}{\text{kg}} \right]. \quad (1)$$

The I_{ssp} allows a more accurate determination of the propulsive performance of spacecraft propulsion systems than the commonly used I_{sp} , which only takes into account the propellant and the thrust engine characteristics, but not associated system equipment like propellant tank, valves, piping, etc.

The software program presented in this paper is based on the related subject of propulsion systems performance analysis, which was first described in [1]. The underlying analysis tool is outlined in the Appendix 1 to this paper while it is detailed in the tutorials of the Issp-program.

The Issp-program can be downloaded from the Swedish Space Corporation (SSC) website <http://www.ssc.se/> ('ISSP - QUICK-LOOK PROGRAM': look for "SYSTEMS & SERVICES", "Satellite Systems", "SATELLITE SUBSYSTEMS" and "Propulsion Systems").

2. Notes on the Software Program

The software program, developed at the Swedish Space Corporation, SSC, was presented earlier in its preliminary configuration as a 'demo-version'.^[2] However, moreover based on comments and recommendations of users, the program has been further developed with major improvements to its present version 2.1 as summarised below.

Examples of system performance analysis results, together with values of I_{ssp} of actual built propulsion systems, are presented in Table 1 to confirm the validity of the quick-look program.

An example of system performance analysis is presented to illustrate the application of the quick-look program.

2.1 Program Features and Operations

The purpose of the program is to be a support tool for the performance evaluation of spacecraft propulsion systems. The program contains a software part for system performance evaluations and a tutorial part to support the understanding of the underlying theoretical basis of the program.

The theory, which controls the propulsion system analysis, is summarised as usable equations without derivation in Appendixes 1 and 2 to this paper.

A most important consideration for the selection of a suitable propulsion system for given mission impulse requirements will be the trade-off between its velocity change capability and propulsion system mass.

Therefore, a preliminary selection of propulsion systems is performed with the help of the overall propulsion system mass fraction:

$$\frac{m_{PS}}{m_{SC}} = \frac{v_e}{I_{ssp}} \left[1 - \exp\left(-\frac{\Delta v}{v_e}\right) \right] \quad (2)$$

Here:

m_{PS} = mass of propulsion system (including propellant) [kg]

m_{SC} = total mass of spacecraft at start time [kg]

I_{ssp} = system specific impulse [Ns/kg]

v_e = propellant exhaust velocity [m/s]

Δv = mission velocity change requirement [m/s]

Equation (2) has been derived from the 'Rocket Equation' in combination with the definition of the I_{ssp} ; see Appendix 1.

The program plots the system mass fraction, m_{PS}/m_{SC} , as a function of velocity change Δv , with exhaust velocity v_e and system specific impulse I_{ssp} as parameters. This allows the selection of propulsion systems where e.g. the system mass shall not exceed a certain percentage of the overall spacecraft mass for given mission Δv requirements.

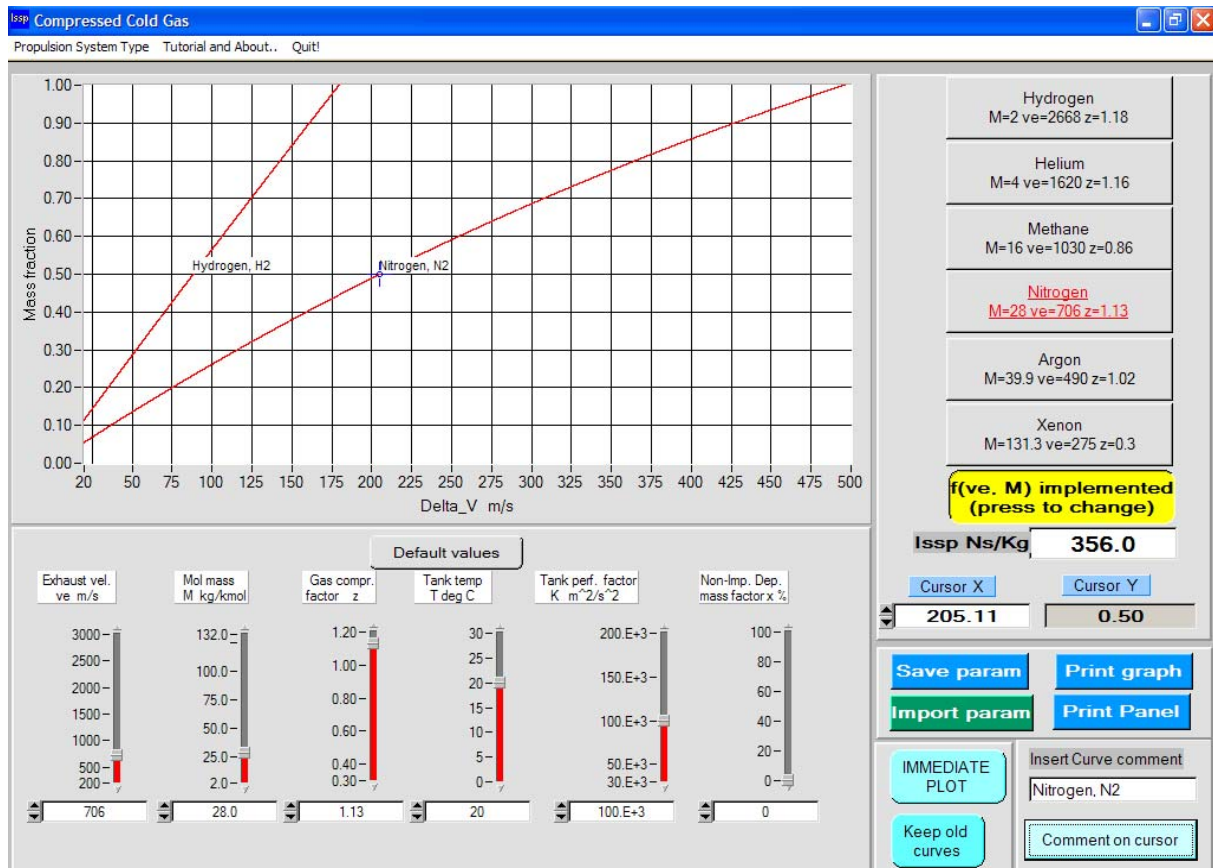


Figure 1: Compressed cold gas propulsion system panel

The calculation is being done by changing parameters in the mathematical formulas noted in Appendix 2, which evaluate the I_{ssp} of the different propulsion system designs. The parameters can be altered in realistic pre-set limits, depending on the kind of propulsion system. The I_{ssp} is different for different propulsion systems and depends on a number of parameters. Therefore each specific system is given its own input panel where all parameters can be set with numerical sliders. The panels are selected from the menu 'Propulsion System Type', comprising compressed cold gas, vaporising liquids, solid propellants, monopropellant hydrazine, bipropellants and electric propulsion (gas and liquid propellants).

When a specific propulsion system is selected for performance evaluation, the corresponding panel is shown, in this case "Compressed Cold Gas", see Figure 1. The lower part of the panel shows a number of numeric inputs (slides) that can be moved within realistic limits. When moved, the graph responds immediately.

On the right side of the panel, a line of buttons allows to set (when clicked upon) typical values for the indicated propellant.*

Saving and importing parameters can be done by clicking the indicated buttons that give the standard windows "save as..." and "open..." dialogs. The parameter log files, which are saved this way, are ordinary text-files that can be used by any type of text editor.

The panels of the various kinds of propulsion systems look very similar; the program is self explanatory and strictly modular and can easily be expanded to include any number of propulsion systems.

To support the program operations, a tutorial text can be called up. This text explains the propulsion system analysis content of the software program, it notes modular and expandable list of data for system specific impulses for various propulsion systems and it presents a very short description of the supplementary interactive C-program that does all calculations.

*N.B. Figure 1 gives a first impression of the program analytical features. It shows that for compressed cold gas propellants, although hydrogen, H_2 , has the highest I_{sp} performance, it has a lower I_{ssp} -performance with higher system mass fraction than e.g. nitrogen, N_2 . Consequently, contrary to the prediction of the 'Rocket Equation', propulsion systems with H_2 will result in lower available spacecraft payload mass than with N_2 .

2.2 Software Program Description

The program is written in C and uses the LabWindows environment from National Instruments. The basic program structure is shown in Fig.2 below. The program runs in an infinite loop and the result is instantaneously plotted.

Future versions of the program may be extended to, and serve as worked examples for testing open interprocess protocols and database- intranet connectivity.

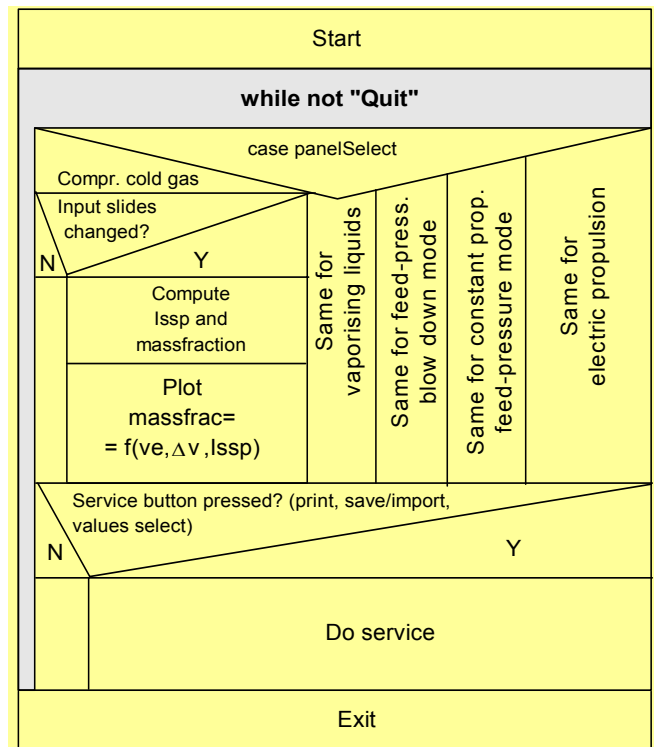


Figure 2: Program structure

2.3 Program Improvements

Based on comments and recommendations of users, the program has been further developed from its configuration as a 'demo-version'^[2] now to version 2.1 with major improvements as follows:

- Consideration of the 'non-impulse dependent' propulsion hardware mass. In general, attempts to evaluate potential increase in propulsion performance have been concentrated on those parameters, which characterise system's propulsive performance capabilities. Initially, comparison of propulsion performances had been therefore based mainly on impulse and power dependent parts of propulsion systems, like propellant and its corresponding tankage for chemical propulsion, while for electric propulsion, in addition the combined mass of the power supply and power processing systems had been considered too. Now, also the non-impulse and non-power dependant parts of propulsion systems are included, such as thrusters, valves, piping, harness, electric control boxes, etc.; for details see Appendix 1.
- The up-dated program will now adapt automatically to different screen sizes.
- For the comparison of different propulsion systems performances, a 'multigraph facility' with redesigned system input panels has been introduced. To compare different propulsion systems, the system and curve of interest can be saved in separate small text files, that at a later time can be imported to the 'Multiplot' panel, where they are plotted and all parameters are shown in the corresponding text box, see Fig. 3.

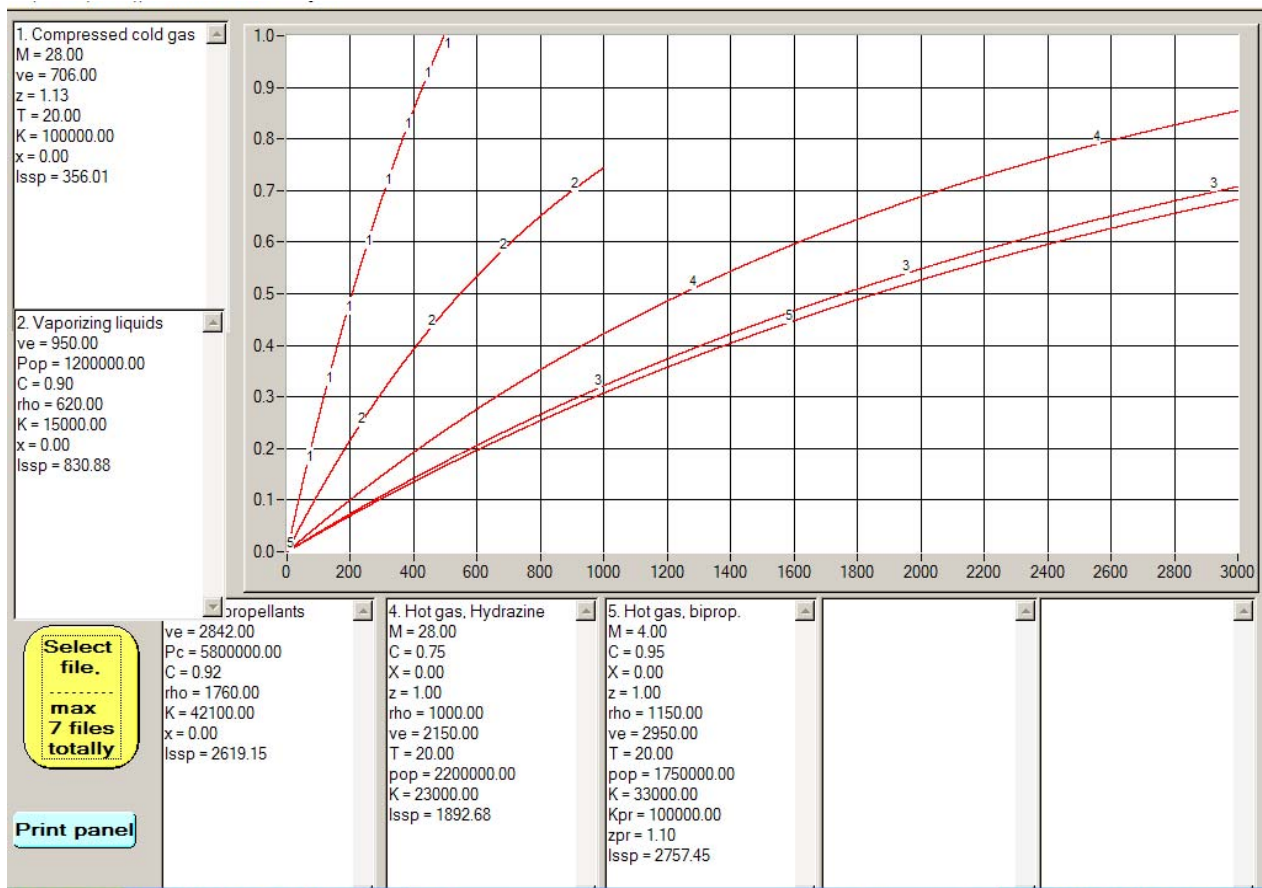


Figure 3: Example of 'Multiplot' for comparison of different propulsion system performances

2.4 Comparison of Analysis with Results for Actual Systems

Values of I_{ssp} calculated for systems operating with cold gas, solid propellant, monopropellant hydrazine and with bipropellants are noted together with assumed typical mission average values of thruster I_{sp} in Table 1. For comparison, values of I_{ssp} of examples of actual spacecraft propulsion systems are also presented in Table 1, showing especially for higher total impulse missions an overall good agreement with calculated values of I_{ssp} .

Table 1: Comparison of propulsion system performances: calculated and actual built systems
(Note: Listed data are examples and therefore only indicative)

PROPELLANT	THRUSTER SPEC.-IMPULSE I_{sp} (mission average) (Ns/kg)	TOTAL IMPULSE I_{tot} (Ns)	PROPULS. SYSTEM MASS m_{PS} (kg)	SYSTEM SPEC.- IMPULSE I_{ssp} (Ns/kg)	REMARKS/REFERENCES <u>Calculated</u> : I_{ssp} values are calculated with help of the 'Issp software program' and typical propulsion system performance data. <u>Actual propulsion systems</u> : I_{ssp} values are derived from values of I_{tot} and m_{PS} [3] as listed below if not otherwise noted
Nitrogen (N ₂)	706	-	-	291	<u>Calculated</u> : tank material: Ti 6Al 4V, $x=0\%$ Vela III [4] COS-B [5]
	706	845	4.4	193	
	706	6780	24	281	
Ammonia (NH ₃)	950	-	-	700	<u>Calculated</u> : tank with vaporiser NLR Explorer 30 (1965)
	800	4450	6.8	654	
Monopropellant Hydrazine; N ₂ H ₄	2150	-	-	1860	<u>Calculated</u> for tank with diaphragm, $C=0.75$, $K=2 \cdot 10^4 \text{ m}^2/\text{s}^2$, $x=0\%$ ECS ERS-1 EURECA GEOS GIOTTO HIPPARCOS MARECS TELECOM-1
	2170	$2.61 \cdot 10^5$	142	1838	
	2134	$6.4 \cdot 10^5$	375	1707	
	2237	$1.34 \cdot 10^6$	740	1811	
	2110	$6.41 \cdot 10^4$	38	1687	
	2163	$1.49 \cdot 10^5$	80	1862	
	2178	$6.97 \cdot 10^4$	40	1743	
	2168	$2.36 \cdot 10^5$	130	1815	
	2168	$3.05 \cdot 10^6$	167	1820	
	2168	$3.05 \cdot 10^6$	167	1820	
Bi-Propellant; MMH/MON	2950	-	-	2730	<u>Calculated</u> for surface tension tank, $C=0.95$, $K=2.6 \cdot 10^4 \text{ m}^2/\text{s}^2$, $x=0\%$ DFS EUROSTAR EUTELSAT-2 OLYMPUS TV-SAT/TDF1/TELE-X TELECOM-2
	2963	$2.22 \cdot 10^6$	849	2615	
	2962	$2.89 \cdot 10^6$	1101	2625	
	2900	$3.10 \cdot 10^6$	1170	2650	
	2930	$5.05 \cdot 10^6$	1839	2746	
	2960	$3.05 \cdot 10^6$	1147	2659	
	2962	$3.34 \cdot 10^6$	1253	2666	
Solid Propellant Composite	2842	-	-	2620	<u>Calculated</u> : $P_c = 5.8 \cdot 10^6 \text{ N/m}^2$, $C=0.924$ $K=4.21 \cdot 10^4 \text{ m}^2/\text{s}^2$, $\rho=1.76 \cdot 10^3 \text{ kg/m}^3$, $x=0\%$ Orbus-6 Inert.Upper Stage Motor [6] MAGE 1S Apogee Kick Motor [7] MAGE 2 Apogee Kick Motor [7]
	2842	$7.73 \cdot 10^6$	2960	2613	
	2852	$1.17 \cdot 10^6$	447	2617	
	2880	$1.41 \cdot 10^6$	528	2670	
Electric Propulsion Xenon	15100	-	-	11343	<u>Calculated</u> : Xe-Propellant, $K=1 \cdot 10^5 \text{ m}^2/\text{s}^2$ $\gamma=82 \text{ W/kg}$, $\eta=50\%$, $\tau=5000\text{h}$, $x=10\%$; SPT SMART-1 [8], [9]; PPS 1350 (SPT) GALS [10]; Station. Plasma Thruster SPT-100
	15107	$1.2 \cdot 10^6$	111	10811	
	15700	$8.16 \cdot 10^5$	128	6375	

This confirms that the analytical tool, which forms the basis for the software program, describes very well those design parameters, which characterise the system's propulsive performance capabilities.

With regard to electric propulsion systems, when compared to chemical propulsion, still only limited data of built systems are available. Further evaluation of the I_{ssp} needs to be performed as soon as more spacecraft with electric propulsion have been built.

3. Example of System Performance Analysis

To illustrate system performance analysis, an example for a propulsion parametric investigation is presented from [11]. This example has been chosen to demonstrate the investigation of potential increases of electric propulsion performances. For electric propulsion systems, their performance is primarily dictated by optimum values of thruster exhaust velocity, v_{e-opt} , see I_{ssp} formulas (6.1) and (6.2) in Appendix 2. With the assumption that the system is operated with xenon-gas as propellant, v_{e-opt} becomes:

$$v_{e-opt} = \sqrt{2\eta\gamma\tau \left(1 + \frac{zRT}{KM}\right)} \quad (3)$$

Here, high values of I_{ssp} , that is high values of v_{e-opt} , will be achieved mainly for high values of overall specific power γ , overall power conversion efficiency η and thrust operation time τ . Parameters of the xenon gas storage system, like gas compressibility factor z , tank performance factor K , gas storage temperature T and gas molecular mass M will have only a secondary impact on values of I_{ssp} . The overall specific power γ contains mainly those of the power supply system, power processing system and thrusters. The impact of these propulsion system parameters on system performance is outlined in examples of parametric investigations below. The parametric investigations have been

performed by altering overall system specific power γ and thrust time τ , considering an overall system power efficiency of $\eta = 0.5$. In order to demonstrate the impact of these parameters on I_{ssp} with resulting values of ‘propulsion system mass fraction’, parameters have been combined for extreme cases of γ and τ as follows.^[11] Combined overall system values of γ containing electric power generators, power processing systems and thruster have been assumed in the range of $\gamma = 7.2$ to 83 W/kg, with a main impact by the electric power generators, which can vary e.g. from $\gamma = 7.65$ W/kg for Radioisotope Thermoelectric Generators (RTG) based on Sterling technology^[12] to $\gamma = 175$ W/kg for highly efficient solar panels with solar concentrators.^[13] Thruster operation time have been assumed for max. life of thrusters, ranging from 7000 h to 15000 h.^[14] Details of the parametric investigations are depicted in Fig. 4 for optimum values of thruster exhaust velocity v_{e-opt} .

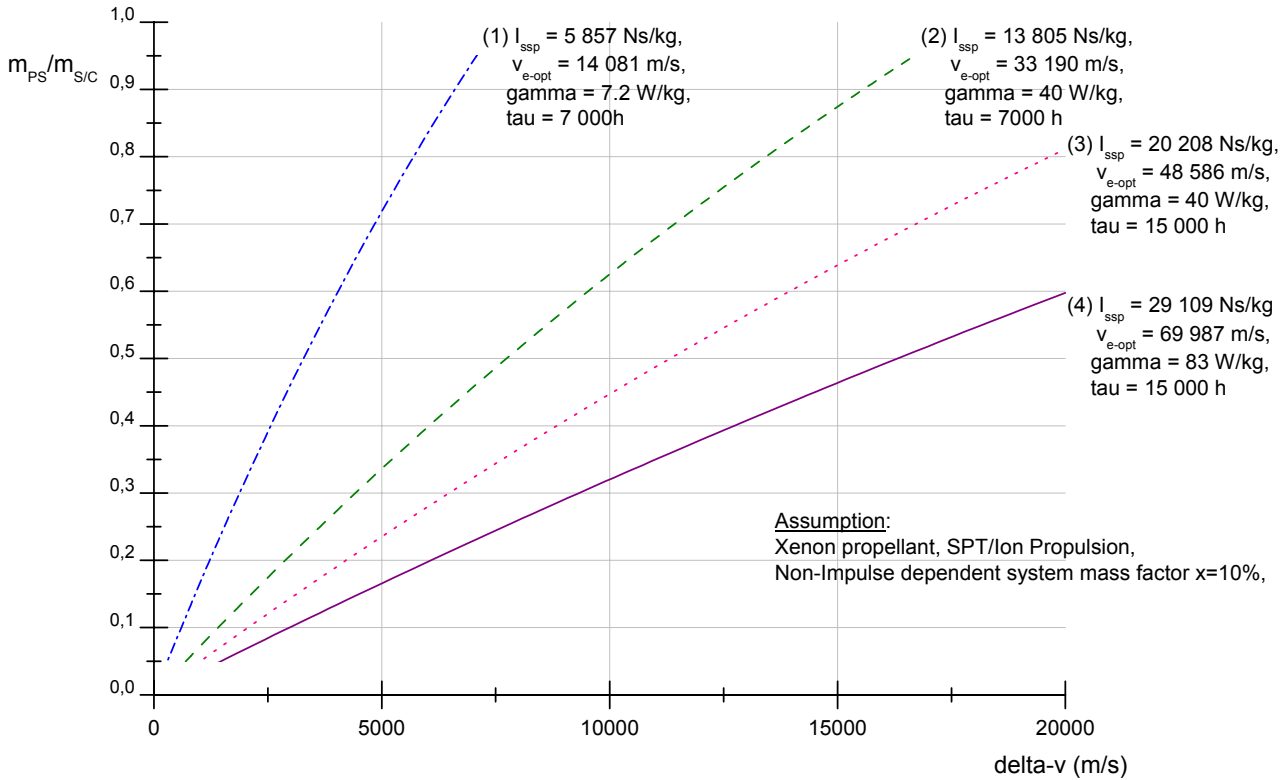


Figure 4: Parametric investigations in Δv -performance of electric propulsion

The parametric investigation performed by altering γ and τ shows the importance of the large range of specific power γ mainly caused by the electric power generators which have a major impact on the overall value of γ . Systems powered for deep space operations by RTG's will have lowest value of γ . The thrust operation time τ will be mainly dictated by mission manoeuvre operating times and/or max. life of thrusters. Hence, thrust operation time τ should be always a maximum within the permissible frame of mission manoeuvre time.

4. Conclusions

Based on the system reference number I_{ssp} a quick look tool in form of a software program has been developed, which gives a first and important indication for the selection of propulsion systems. It will be of particular benefit for the feasibility study phase of a spacecraft program and for propulsion study purposes in general. It is nearly self-explanatory, which results in a very user-friendly software program. Hence, it is also particularly suited for tutorial purposes. The program is of exceptional help to provide a basic understanding of the impulse performance capability of today's common propulsion systems. Therefore, the program allows the handling of those parameters, which are most important for the determination of the basic system impulse performances.

Values of I_{ssp} , which have been mainly evaluated for chemical propulsion systems, are in overall good agreement with those of actual systems. With regard to electric propulsion systems, limited data of built systems are available. Further evaluation of the I_{ssp} needs to be performed as soon as more spacecraft with electric propulsion have been realised. However, for both electric as for chemical propulsion systems, the I_{ssp} will be indispensable for an objective comparison of system performances and system performance analysis.

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Appendix 1: Analysis Tool

An important measure of propulsion system performance is its velocity change capability, Δv , in relation to propulsion system mass. Therefore, a most important consideration for the selection of a suitable propulsion system for given mission impulse requirements, will be the trade-off between its Δv -capability and propulsion system mass. Hence, a preliminary selection of propulsion systems is performed with the help of the overall 'propulsion system mass fraction' = m_{PS}/m_{SC} . The dependence of the propulsion system mass fraction on mission velocity change, Δv , has been derived from the 'Rocket Equation' in combination with the definition of the 'system-specific impulse', I_{ssp} , as follows:

The first equation (A1) below is obtained from the rocket equation. The second equation (A2) is just the definition of I_{ssp} , and the final expression (A3) follows from the first two.

$$I_{tot} = v_e m_p = v_e m_{SC} \left[1 - \exp\left(-\frac{\Delta v}{v_e}\right) \right] \quad (A1)$$

$$I_{ssp} = \frac{I_{tot}}{m_{PS}} \Rightarrow I_{tot} = I_{ssp} \cdot m_{PS} \quad (A2)$$

$$\frac{m_{PS}}{m_{SC}} = \frac{v_e}{I_{ssp}} \left[1 - \exp\left(-\frac{\Delta v}{v_e}\right) \right] \quad (A3)$$

Here:

I_{tot} = total impulse delivered by the propulsion system

I_{ssp} = system-specific impulse [Ns/kg]

m_p = mass of mass of contained propellant in the system at start of mission [kg]

m_{PS} = mass of propulsion system (including propellant) [kg]

m_{SC} = total mass of spacecraft (vehicle) [kg]

v_e = propellant exhaust velocity [m/s] (mission average values) \Rightarrow numerical equal to I_{sp} [Ns/kg]

Δv = mission velocity change requirement [m/s]

The program plots the system mass fraction as a function of Δv , with exhaust velocity v_e and system specific impulse I_{ssp} as parameters.

Indeed, I_{ssp} is a very useful tool, but its practical application requires a very clear definition of what is included in "total mass of propulsion system", m_{PS} . The I_{ssp} can be directly derived from actual spacecraft propulsion systems by determining the total impulse delivered by the mass of contained propellant, divided by the mass of the propulsion system. On the other side, the I_{ssp} can be derived analytically, in order to facilitate the preliminary selection of propulsion systems (chemical, electrical) for spacecraft missions of given impulse and velocity change requirements. Therefore the I_{ssp} has to be further defined for the two different main kind of systems applied commonly for spacecraft propulsion according to the source of energy with resulting system configuration shown in Figures 1A and 2A below.

- A) Propulsion Systems with self-contained energy in propellants, comprising cold gas and hot gas systems (chemical propulsion):

$$I_{ssp} = \frac{I_{tot}}{m_{H/W} + m_{PSS}} \quad (A4)$$

with m_{PSS} , the mass of propellant and corresponding tankage (propellant storage system), which is proportional to propulsion impulse, and $m_{H/W}$, the propulsion hardware mass, such as thrusters, valves, piping, etc., which is independent of propulsion impulse.

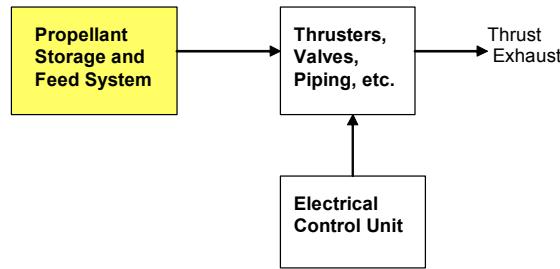


Figure 1A: Schematic of Chemical Propulsion Systems
(Coloured box: impulse dependant parts of propulsion system: m_{PSS})

- B) Propulsion Systems with externally supplied energy to propellant, comprising e.g. electric propulsion, where the electric power supply, power processing system, and electric thruster assembly, m_{EI} , has to be added to the mass of the propulsion system:

$$I_{ssp} = \frac{I_{tot}}{m_{H/W} + m_{PSS} + m_{EI}} \quad (A5)$$

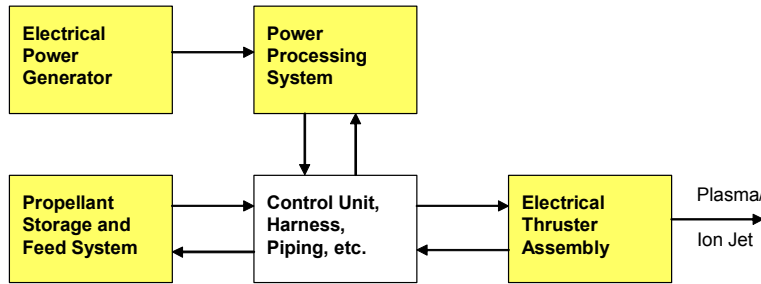


Figure 2A: Schematic of Electrical Propulsion Systems
(Coloured boxes: impulse/power dependant parts of propulsion system: $m_{PSS} + m_{EI}$)

Equations (A4) and (A5) for the I_{ssp} of the various propulsion systems have in common the same numerator, representing the total impulse, I_{tot} , delivered by the propellant, m_p , contained in the propellant tank, which is:

$$I_{tot} = m_p v_e \quad (A6)$$

while the denominator in (A4) and (A5) varies with the kind and design of propulsion systems. Detailed derivations of usable equations are presented in the tutorials of the software program. With regard to the hardware mass $m_{H/W}$, in general it is assumed, that for high impulse mission requirements, $m_{H/W}$ is small compared to $m_{PSS} + m_{EI}$ and is disregarded or, if properly known, included as a fraction 'x' of $m_{PSS} + m_{EI}$. E.g. $m_{PS} = m_{PSS} + m_{EI} + m_{HW} = (m_{PSS} + m_{EI}) \cdot (1+x)$, with 'x' = 'Non-impulse/non-power dependent system mass factor' (e.g. 0% - 100%) of m_{PSS} .

Appendix 2: Propulsion System-Specific Impulse Equations

$$I_{ssp} = \frac{I_{tot}}{m_p + m_T + m_{HW}} = \frac{v_e}{\left[1 + \frac{zRT}{KM}\right](1+x)} \quad (1) \text{ COMPRESSED GAS}$$

$$I_{ssp} = \frac{I_{tot}}{m_p + m_T + m_{HW}} = \frac{v_e}{\left[1 + \frac{P_{op}}{C\rho K}\right](1+x)} \quad (2) \text{ VAPORISING LIQUID}$$

$$I_{ssp} = \frac{I_{tot}}{m_p + m_{case} + m_{HW}} = \frac{v_e}{\left[1 + \frac{P_c}{C\rho K}\right](1+x)} \quad (3) \text{ SOLID PROPELLANT}$$

$$I_{ssp} = \frac{I_{tot}}{m_T + m_p + m_{pr} + m_{HW}} = \frac{v_e}{\left[1 + \frac{P_{op}}{C\rho K} \left(1 + \frac{K(1-C)M}{zRT}\right)\right](1+x)} \quad (4) \text{ MONOPROP. HYDRAZINE (blow-down mode)}$$

$$I_{ssp} = \frac{I_{tot}}{m_T + m_p + m_{pr} + m_{Tpr} + m_{HW}} = \frac{v_e}{\left[1 + \frac{P_{op}}{C\rho K_p} \left(1 + \frac{K_p(1-C)M}{z_p RT}\right) + \frac{1.1 P_{op} M}{\rho z_p RT} \left(1 + \frac{z_{pr} RT}{K_{pr} M}\right)\right](1+x)} \quad (5) \text{ BI-PROPELLANT (pressure constant mode)}$$

$$I_{ssp} = \frac{I_{tot}}{m_p + m_T + m_{El} + m_{HW}} = \frac{v_e}{\left[1 + \frac{zRT}{KM} + \frac{v_e^2}{2\eta\gamma\tau}\right](1+x)} \quad (6) \text{ ELECTRIC PROPULSION}$$

Propellant: gas

with: (6.1) Optimal exhaust velocity

$$v_{e_{opt}} = \sqrt{2\eta\gamma\tau \left(1 + \frac{zRT}{KM}\right)}$$

$$I_{ssp} = \frac{I_{tot}}{m_p + m_T + m_{El} + m_{HW}} = \frac{v_e}{\left[1 + \frac{P_{op}}{C\rho K} + \frac{v_e^2}{2\eta\gamma\tau}\right](1+x)} \quad \text{Propellant: liquid}$$

with: (6.2) Optimal exhaust velocity

$$v_{e_{opt}} = \sqrt{2\eta\gamma\tau \left(1 + \frac{P_{op}}{C\rho K}\right)}$$

Nomenclature

C	tank filling ratio (V_p/V_T)
I	impulse [Ns]
K	tank performance factor ($P_{op} \cdot V_T / m_T$) [m^2/s^2]
m	mass [kg]
M	molecular mass [$kg/kmol$]
P	pressure [N/m^2]
R	gas constant 8.314 [$kJ/^\circ K/kmol$]
T	temperature [$^\circ K$]
v	velocity [m/s]
x	non-impulse dependent system mass factor [%]
z	gas compressibility factor
γ	specific power [W/kg]
η	overall energy conversion efficiency (N_e/N)
ρ	specific mass of propellant [kg/m^3]
τ	thrust time [s]

Subscripts

c	motor chamber
$case$	motor case
e	exhaust (effective)
El	electric (system)
HW	hardware
op	operating
opt	optimal
P	propellant
pr	pressurant (gas)
sp	specific
ssp	system-specific
T	tank
tot	total



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