OPTIMISATION OF REUSABLE LAUNCHERS TRAJECTORY BY COORDINATION OF OPTIMAL CONTROL PROBLEMS

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Abstract:

Due to competitiveness and the induced launch cost reduction, the future launchers should have to be partly reusable. Also, to obtain the best solutions, the trajectories optimisation process requires optimising simultaneously the ascent trajectory (to deliver payload into orbit) and the return trajectory (to recover a part of the launcher). Due to the influence and the sensitivity of the different trajectory branches, the performances and the trajectories optimisation of the future launchers ask for increasingly sophisticated technical methods.

In this paper we present an optimisation methodology based on coordination between optimal control and optimisation problems. To improve the behaviour of the global optimisation process, the examination of the reusable launcher trajectories suggests a natural approach by breaking down the whole trajectory in different parts (ascent, re-entry, RTLS, orbital) and to optimise separately each segment with appropriated methods. Then we consider the coordination between two successive optimal control problems (Ascent phase and Return to Launch Site) with intermediary constraints (RTLS constraints and ascent trajectory constraints).

We test this approach on a TSTO vehicle taking-off at Kourou with a first stage recovery in vicinity of the launch pad (Return To Launch Site strategy (RTLS)). The optimisation process is discussed and we show that this approach is robust and simplifies the computational effort required to solve the whole trajectory and the general launcher design optimisation problem. Finally results concerning the TSTO trajectory optimisation and Launcher design are analysed.

Résumé:

En raison de la compétitivité et de la réduction induite des coûts de lancement, les futurs lanceurs devront être partiellement réutilisables. Aussi, afin d'obtenir les meilleures solutions, les procédures d'optimisation des trajectoires exigent d'optimiser la trajectoire ascensionnelle (pour mettre en orbite la charge utile) et la trajectoire de retour (pour récupérer une partie du lanceur). En raison de l'influence et de la sensibilité des différentes branches de la trajectoire, l'optimisation des trajectoires et des performances des futurs lanceurs requiert des méthodes techniques de plus en plus sophistiquées.

Dans cet article, nous présentons une méthodologie d'optimisation basée sur la coordination entre des problèmes de contrôle optimal et d'optimisation. Pour améliorer le comportement global de la procédure d'optimisation, l'examen des trajectoires des lanceurs réutilisables suggère une approche naturelle en décomposant la trajectoire complète en différentes parties(la partie ascensionnelle, la phase de rentrée, le retour sur site (RTLS)) et en optimisant séparément chacune de ces parties par des méthodes appropriées. Nous considérons ensuite la coordination de deux problèmes successifs de contrôle optimal (la phase ascensionnelle et le retour sur site) avec contraintes intermédiaires (contraintes liées à la trajectoire de retour et la trajectoire ascensionnelle).

Nous testons cette approche sur un véhicule de type TSTO décollant de Kourou avec récupération du premier étage à proximité de Kourou (stratégie de retour sur site (RTLS)). Le processus d'optimisation est discuté et nous montrons que cette approche est robuste et simplifie l'effort de calcul demandé pour résoudre le problème de l'optimisation complète de la trajectoire et la conception du lanceur. Enfin les résultats d'optimisation de la trajectoire TSTO et de conception du Lanceur sont analysés.

1 – INTRODUCTION

The trajectories optimisation and the design of the reusable launchers is generally a heavy task for traditional approach due to the interdependence and complexity of the different parts of the trajectories. The classical approach is to formulate the launcher trajectory optimisation problem as the evaluation of a single function providing the criterion and the constraints values according to all the parameters as input; then a non linear parametric optimiser intends to find the solution. For such an approach, experience shows poor convergence properties due to trajectory instabilities and ill conditioning. To improve the optimisation behaviour, the examination of the reusable launcher trajectories suggests a natural approach by decomposing the whole trajectory in different parts (ascent, re-entry, RTLS, orbital parts) and to optimise separately these last ones with appropriated methods. To take into account the interdependence of the trajectory branches a coordination method must manage the coupling parameters at system level. Several authors have followed this direction: Hargraves and Paris have proposed a multiple vehicle trajectory optimisation method based on collocation, phases segmentation and multi-shooting[6], Beltracchi proposes a decomposition procedure and a based derivatives parameters coordination to solve the all-up trajectory optimisation problem concerning a Booster and an upper stage delivering a maximal payload in a fixed orbit[7], Rahn and Schöttle have used a decomposition and coordination method applied to air breathing trajectory optimisation with non linear programming methods and parallel processing[8]. In this study we propose an approach based on the coordination of two successive optimal control problems to optimise the main design and trajectories of a fully reusable TSTO launcher.

2 – Nomenclature

α	Angle of attack
C_F	Fuel rate to thrust ratio
CU	Payload
CU_{net}	Payload without apogee motor
F	Thrust
γ	Flight path angle
γ_b	Pitch angle
GLOW	Gross Lift Off Weight
Γ_t	Transverse load factor
$H^{(pb)}$	Hamiltonian of problem pb
Hapo	Apogee Altitude
Hperi	Perigee Altitude
i	Orbit inclination
ISP	Specific impulse
ISS	International Space Station
$\lambda^{(pb)}$	Costate vector of problem pb
M_0	Initial mass at lift off
$M_{turbo\text{-}jet}$	Turbojet mass
$M\_{unitary}$	Turbojet engine unitary mass
$M_{fuel\text{-}rtls}$	Kerosene consumption during the cruise Orbiter fly-back
N_{motor}	Number of turbo-jet engines
μ	Bank angle
qi	Flow rate of stage i
$Pdyn_{sep}$	Dynamic pressure at staging
$Pdyn_{max}$	Maximum dynamic pressure
Ψ	Velocity azimuth angle (Heading angle)
ψ_0	Flight initial azimuth
RTLS	Return To Launch Site
TSTO	Two Stage To Orbit
T	Total Turbojet thrust
$T\_{unitary}$	Turbojet thrust by motor
T1	Booster burn-out time
T2	Orbiter burn-out time
V	Relative Velocity
V_{md}	Velocity at minimal drag
Xki	Structure index of stage i
W	Weight

3 – Vehicle and mission description

3.1 Mission

The Launcher is a TSTO vehicle taking-off from Kourou launch pad. The launcher recovery baseline is a RTLS strategy with a turbo-jets powered fly back for the first stage, a direct re-entry from orbit for the second stage. The reference mission will be to insert a 20 tons-payload on low orbit (LEO) to ensure the international station supply. Therefore the final aimed orbit will be the ISS orbit: (380 km x 380 km, i = 51.6 deg).

At the Orbiter burn-out, the payload is released on a transfer orbit, a propulsive module circularizes this last one at apogee to reach the final circular orbit. The Orbiter is injected on a viable orbit (typically: (380 km x 90 km, i = 51.6 deg)) where it can stay several days before de-orbiting for the re-entry phase.

3.2 Vehicle

The TSTO Launcher is composed of two winged stages (the first stage as a Booster, the second stage as an Orbiter) in a mated configuration which operates in parallel during launch and initial ascent. Until the two- stage separation event, the second stage (Orbiter) uses the propellant stored in the Booster tanks thanks to a cross-feed fuel system. The use of such a system increases the thrust level at takeoff and during the initial ascent phase, thus the propulsive requirements are reduced for the Booster rocket allowing a lower constructive index. Previous studies on two-stages vehicle concepts have shown that cross feeding reduces until 23% the dry mass and 26% the gross weight[9-10].

3.2.1 Propulsive characteristics

In order to minimize the propellants storage volume while conserving a good propulsive index, a LOX/kerosene solution was adopted for the two stages. The rockets engines used are:

- R-180 engines for the Booster[1]

The RD-180 is developed by Energomash (Russia); this rocket engine is a high performance two thrust chamber. Its characteristics are[1]:
Thrust in vacuum: 4.152 MN,
ISP in vacuum: 338s, ISP at sea level: 311s,
Mass: 5.3 t
The indicated engine mass includes pneumatic and hydraulic systems, electrical panels, thrust frame. The RD-180 has a continuous throttling capacity (50 – 100%). It is flight qualified (Atlas 3) and can be reusable.

- NK 43 engines for the Orbiter[1]

Built by the considered Kuznetsov Russian Company, the Nk-43 engine shows excellent performances:
Thrust in vacuum: 1.79 MN,
ISP in vacuum: 346 s, ISP at sea level: 247,
Nozzle Area Ratio: 80,
Mass: 1.47 t
Throttle continuous Range: 55 - 104 %. An optimization for low altitudes would be recommended for TSTO.

- Turbo-jet for Booster powered fly-back

An engine derived from the Pratt Whitney F100 was selected. If a ceramic technology is retained to protect the hot parts (burner, turbine, nozzle), the mass engine is drastically reduced. Using the turbojet design tool 'Engine Sim' developed by the NASA Glenn Research Center[3], the next characteristics were obtained:
For a cruise altitude of 2000 meters and 0.3 Mach number the engine characteristics are as follows:

-Thrust (h=2000 m, Mach=0.3) = 74.04 KN (net thrust),

-Fuel consumption = 6397 Kg/h

-F/W= 11.27 N/kg (Thrust to Weight ratio)

-TSFC=86.4 mg/N.h (Fuel rate to Thrust ratio)

-Mass air Intake=89.74 kg/s,

-Engine weight: 671. kg,

-C_F=2.4 10^{-5} kg/s/N (Fuel rate to thrust ratio)

This engine will be used as a base for the turbo-jet propulsion Launcher design

- Propulsive Orbital module

A solid rocket apogee motor has been selected to circularize the final orbit
ISP = 300 s,
constructive index = 18% (classical values)

3.2.2 Mass characteristics

The rules applied to estimate the mass characteristics are based on classical relationships between the structural index and the propellant mass for a given stage [2].

	Booster	Orbiter	Orbital Module
Propulsive Constructive Index*	**9%**	**10.5%**	**18%**
Aerodynamic surfaces (Mass) (t)	**4.**	**6.2**	
TPS (Mass) (t)	**4.1**	**6.2**	
Turbo jet Mass (4 x 0.7) (t)	**2.8**		
Landing system	**1.5**	**1.5**	
Rocket engine Mass (t)	**16.2 (3 RD 180)**	**3. (2 NK43)**	
Range of Lox/kerosene	**700 – 900 t**	**200 – 350 t**	**0.5 – 0.6 t**
Range of Kerosene for RTLS	**10 – 35 t**		
Range of Tanks + structure + rocket engines	**63 – 81 t**	**21 – 36.7 t**	**0.09 – 0.11 t**
Total Mass Range At Lift-off	**785 - 1030**	**235 – 400**	**0.59 – 0.70**

*** A low Constructive index is obtained thanks to the cross-feed fuel system**

The Gross Lift Off Weight (GLOW) is located between 1040 t and 1450 t (with the 20-tons payload)

3.2.3 Aerodynamic characteristics

The aerodynamic data comes from previous internal studies on similar configurations. The aerodynamics of the vehicle is tabulated as a table of lift and drag coefficients versus angles of attack and Mach numbers. Three aerodynamic tables have been used for the mated configuration, the Booster alone, the Orbiter alone.

3.3 Flight sequence

The trajectory is naturally broken in three phases: the ascent phase until the Booster-Orbiter separation, the Orbiter phase until payload insertion, the RTLS phase for the Booster. Additionally sub phases may be added: a cruise phase for the Booster, a circularization phase for the payload delivering into the aimed final orbit. The Orbiter reentry phase will be not treated in this study because this phase is well uncoupled from the other phases.

<u>First Ascent phase</u>

After a vertical lift-off from Kourou the launcher performs a manoeuvre consisting of a pitch manoeuvre followed by a null flight path angle phase (gravity turn) in order to minimize the structural loads. In this phase the pitch angle (γ_b) and the flight azimuth (ψ_0), determining the pitch plan orientation, are optimized. The Booster burn-out time (T1) will be also a parameter.

A constraint on the maximum dynamic pressure is imposed: $Pdyn_{max} < 50kPa$

At Booster burn-out the Booster and Orbiter are separated. After separation the Orbiter executes a pull-up manoeuvre to move away from the Booster.

The dynamic pressure at separation is constrained: $Pdyn_{sep} < 10$ kPa

Number of Constraint: 2 (inequalities) Number of parameters: 3 (γ_b, ψ_0, T1)

The first ascent phase ends at the "staging point" (separation point between the Booster and the Orbiter).

<u>RTLS phase</u>

This phase can be cut out into two sub-phases: the reentry phase and the cruise phase. After separation (at staging point) the Orbiter executes a fly back manoeuvre by curving its trajectory thanks to lift orientation and angle of attack control. During this phase the transverse force acting on the vehicle is constrained: Transverse Load factor max < 4 g. Two other path constraints will be also considered: heat flux < 130 kW/m^2, dynamic pressure < 70 kPa

At the end of the purely aerodynamic phase (tf_{aero}), final conditions for starting the powered cruise flight are required: H (t_{faero}) = 2000 m, V (tf_{aero}) = Velocity at minimal drag (V_{md}), γ (tf_{aero}) = 0.

Next the turbo-jets are turned on, and the Orbiter cruises at constant Lift coefficient and velocity until Kourou launch pad (2 equality constraints on final latitude and longitude). Cruising at minimal drag has been chosen to minimize the number of turbo-jets. In fact the velocity Orbiter has to be greater or equal than V_{md} to ensure flight stability. This strategy minimizes the turbojet thrust to compensate the drag force.

The RTLS phase will be optimized thanks to a numerical optimal control method (steepest ascent method). The optimal control was preferred to parametric optimization because the re-entry trajectory of such a vehicle is very sensitive to the controls and any satisfactory simple parametric representation of the control is well known actually.

Orbiter phase

This phase starts at staging point after the Booster separation. During this phase, the Orbiter thrust orientation is optimized by linear piece wise controls (8 parameters). The Orbiter burn-out time (T2) will be also a parameter. To ensure Orbiter controllability and structural loads, two constraints are required:

Angle of attack constraint (until h = 70 km)
$\alpha < 10$ deg
Transverse load factor :
$\Gamma_t < 1$ g

At the end of the Orbiter flight the next constraints are imposed:

Altitude of the apogee: Hapo = 380 km
Altitude of the perigee: Hperi > 90 km (to ensure a viable orbit)
Orbit inclination: i = 51.6 deg
Number of Constraint: 5
(2 equalities and 3 inequalities)
Number of parameters: 9

Circularization phase

After the Orbiter and the supply ISS vehicle separation, this last one coasts until the apogee where the orbit is circularized thanks to a propulsive module. Then the net payload is deduced.
Constraint:
net payload CU_{net} = 20 tons
Number of Constraint: 1

4- Optimisation Methodology

4.1 Optimisation formulation

4.1.1 Criterion selection

A simultaneous methodology to optimize the whole trajectory and the Launcher design is proposed. The optimization problem of the RLV and the trajectory optimisation may be posed in several ways, therefore the global optimization may be considered as a multi-objective problem, various criteria may be enumerated:

1) To obtain a realizable trajectory

This is a necessary condition. The result of the trajectory optimisation is to comply with the mission requirements and to verify the imposed constraints all along the trajectory.

2) To design the best vehicle both for the Booster and the Orbiter

The lowest fuel consumption during RTLS may be wished. To minimize the number of turbo engines required for the RTLS cruise phase, the minimisation of the total Booster weight at burnout may be selected. The mass of Orbiter propellants required to insert the payload on the transfer orbit should have also to be minimized.

3) To design the best Launcher

The criterion for the Launcher can be the minimization of the gross weight at lift-off; the criteria enumerated above contribute to attain this objective. To be compliant with the above criteria several parameters are added:

- The Gross Lift Off Weight (GLOW),
- The kerosene consumption during the cruise Orbiter fly-back
- The Booster propellant mass for the ascent phase
- The Orbiter propellant mass for the orbital phase

4.1.2 Selected methods

The optimization problem is formulated as a non linear parametric optimization problem for the ascent and Orbiter phases. The Booster RTLS phase will be optimized thanks to an optimal control method.

4.1.3 Parameters and constraints

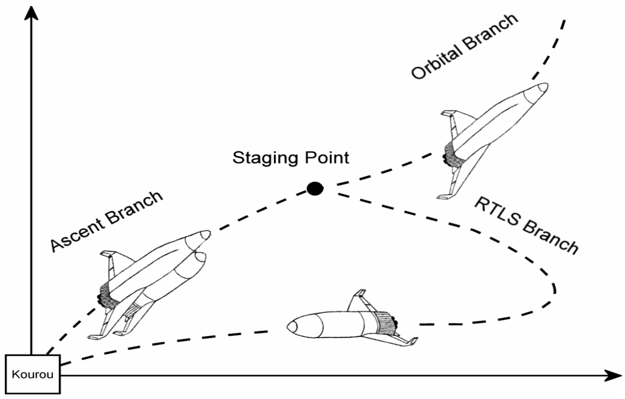
For the parametric part of the problem we have to manage 14 parameters in order to verify 3 equalities and 5 inequalities constraint (§3.2 above)
For the optimal control part (RTLS) we have to verify three path constraints (transverse load factor, maximal dynamic pressure and heat flux), 3 intermediary constraints (at the start of the powered cruise), and two final constraints (final position).

4.1.4 Problems coupling

The parametric part and control optimal part are coupled via the conditions at the Booster-Orbiter separation and the Kerosene mass to be reserved for the powered fly back and the dry Booster mass (depending on the Booster propellants mass allocation for the ascent phase).
The Orbiter part and the ascent phase are coupled also via the conditions at the Booster-Orbiter separation.

4.1.5 Decomposition approach

A natural approach is to decompose the global problem in two or three sub problems:



4.2 Solving methodology

4.2.1 Problem Decomposition

We consider the decomposition into two problems:

1) An upstream problem : the Ascent and the Orbiter optimisation

For this problem, the criteria will be to minimize the Launcher mass at lift-off (M_0). Because this problem is initially uncoupled with the RTLS problem, if hypothesis are taken about the required kerosene mass for the RTLS trajectory ($M_{fuel\text{-}rtls}$) and Turbojet mass ($M_{turbo\text{-}jet}$), the whole ascent trajectory can be optimised until payload delivery. For instance we can firstly take fixed guessed values or use a more sophisticated model to take into account the RTLS phase.
A particular output data resulting from the solution of the upstream problem are the staging point conditions (position, velocity, mass)

2) A downstream problem: the Booster RTLS optimisation

For this problem, the criteria will be to minimize the kerosene mass required to return at Launch pad, and also to determine the necessary turbo-jet power.

Re-entry phase
The re-entry phase starts at the staging point. This part of the trajectory is a no propelled gliding phase controlled by the optimisation of the angle of attack (α) and the heading angle (μ). The re-entry phase is subject to three path constraints (load factor, heat flux, dynamic pressure).

Turbo_jet cruise strategy
During cruise, the lift force (L) is supposed to compensate for the weight (mg), so:

$$\mathrm{L} = \mathrm{mg} = \frac{1}{2}\rho V^2 C_L$$

The turbo-jet thrust (T) must compensate the drag (D), so: $\mathrm{T} = \mathrm{D} = \frac{1}{2}\rho V^2 C_D$

Then the relation $T = mg\left(\frac{C_L}{C_D}\right)^{-1}$ is deduced.

In order to minimize the thrust, we have to choose a maximal lift-to-drag ratio. The lift and drag coefficients depend on the angle of attack and the mach number. The turbo-jet thrust depends on the cruise altitude and velocity.The problem of the thrust minimisation can be formulated as follow:

$$\max_{\alpha,h,V}\left(\frac{C_L}{C_D}\right) \quad \text{with the constraints :}$$
$$\frac{1}{2}\rho(h)V^2 C_L(V/a,\alpha) = mg$$
$$T(V,h) = \frac{1}{2}\rho(h)V^2 C_D(V/a,\alpha)$$
$$(a = a(h) : \text{sound velocity})$$

The solution depends on the booster weight (mg). For mg = 100 tons, we found h ≈ 2000 m, V/a = 0.3, L/D = 3.08. For these conditions, the unitary turbo-jet fuel rate to thrust ratio is C_F=2.4 10^{-5} kg/s/N and the thrust T= 74.04 kN.

Cruise climb strategy:
During cruise, the weight (mg) falls. To respect the equation (L = mg) , we choose to fix C_L and V, so we have to let ρ decreasing (or h increasing) : this is the climb cruise strategy. With this strategy, we can expressed the range (R) by:

$$R = \left(\frac{V}{g\,C_F}\frac{L}{D}\right)\log\left(\frac{M_0}{M_f}\right) \quad \Longrightarrow \quad M_{fuel\_RTLS} = M_0\left(1 - e^{-\left(\frac{V}{gC_F}\frac{L}{D}\right)}\right)$$

The turbo-jet thrust level $\left(T = mg\left(\frac{C_L}{C_D}\right)^{-1}\right)$ gives the turbojet mass: $M_{turbo\text{-}jet} = N_{motor} \,.\, M_{motor\_unitary}$

Where $N\_{motor} = \dfrac{T}{T_{unitary}}$ is the required number of turbo-jet engines (N_{motor} can be considered as a motorization level and can be fractional)

4.2.2 Coordination of the two problems

The downstream problem (RTLS phase) can be formulated as a control optimal problem:

$$\min_{u,t_f}\left[\phi(x(t_f)) + \int_{t_c}^{t_f} L_2(x,u,t)dt\right]$$

$$= \min J_2(u,tf)$$

$$= \min (M_{fuel\_RTLS} + M_{turbo\_jet})$$

$$x(t_c) = \chi(t_c) \qquad \dot{x} = f(x,u,t)$$

and $D_i(x,u,t) \le 0$ for $t_c \le t \le t_f$,

$$\psi_i(x(tf),u(tf),tf) = 0$$

The classical co-state equations:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \quad with\ H = \lambda^T f + J_2$$

where t_c is the date of booster separation and $\chi(t_c)$ is the state vector at booster separation.
The optimal control solution provides the initial co-state vector: $\vec{\lambda}(0)$
It is well-known that:

$$\vec{\lambda}(0) = \frac{\partial(M_{fuel\_RTLS} + M_{turbo\_jet})}{\partial(\vec{\chi}(tc))},$$

So the initial co-state vector of the downstream problem provides the sensitivity of the kerosene mass and turbojet mass with respect to the staging point .
Therefore, we propose to coordinate the two problems by taking the following model for the upstream problem:

$$\boxed{\begin{aligned}&M_{fuel\_RTLS} + M_{turbo\_jet} = \\ &(M_{fuel\_RTLS} + M_{turbo\_jet})_0 + \vec{\lambda}(0)(\vec{\chi}(tc) - \vec{\chi}_0)\end{aligned}} \quad (*)$$

where $(M_{fuel\_RTLS} + M_{turbo\_jet})_0, \vec{\lambda}(0), \vec{\chi}_0$ correspond to the previous solution of the downstream problem.

The upstream problem is formulated as follow:

Min **M0**
Under
Trajectory constraints & Orbital constraints

$$\left|\vec{\chi}(tc) - \vec{\chi}_0\right| \le \Delta\vec{\chi}_{max}$$

with:
(the variation on staging point is limited to avoid no-linearities)

$$M_0 = \sum M_{ergols} + \sum M_{inerts} + \sum M_{compl} + M_{stage3}$$
$$+ Payload + (M_{fuel\_RTLS} + M_{turbo\_jet})$$

where $(M_{fuel\_RTLS} + M_{turbo\_jet})$ is computed thanks to the relation (*)

<u>Design parameters</u>: Propellant masses repartition of the Booster & Orbiter

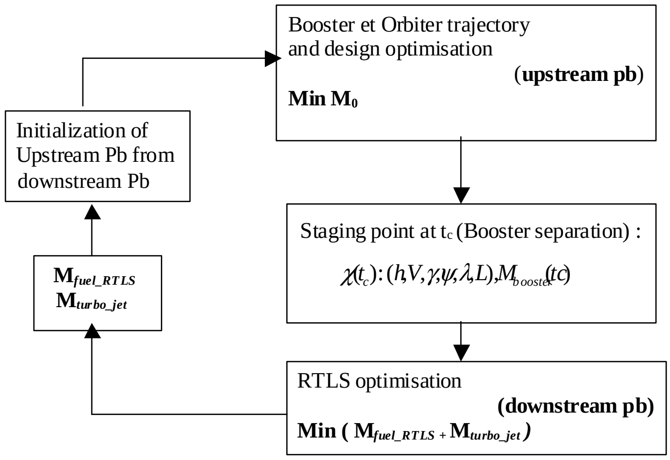


Figure 2: diagram of the coupling

<u>Algorithm</u>
Therefore we propose the algorithm where the vector $\vec{\lambda}(0)$ is taken as coordination parameters:
At step k , we have $(\vec{\lambda}(0))^{k-1}$ from the previous solution of the downstream problem

1) The upstream problem is solved with the formulation (*) to compute ($M_{fuel\_RTLS} + M_{turbo\_jet}$)
   $\chi^k(t_c)$ and t_c^k are obtained

2) The downstream problem is solved with
   $\chi(t_c) = \chi^k(t_c)$ and $t_c = t_c^k$
   $\vec{\lambda}(0)^k$ is obtained

3) The algorithm stops when:

$$\left|\chi^k(t_c) - \chi^{k-1}(t_c)\right| < \varepsilon_\chi \quad , \quad \left|t_c^k - t_c^{k-1}\right| < \varepsilon_{tc}$$

$$\left|M_{fuel\_RTLS} + M_{turbo\_jet} - (M_{fuel\_RTLS} + M_{turbo\_jet})_{k-1}\right| \le \varepsilon_M$$

And when the following optimality conditions hold (Kuhn and Tucker conditions):

If μ_i are the Lagrange parameters associated with the upstream problem constraints (Ci) , then:

$$\frac{\partial M_0}{\partial p} + \mu_0 \vec{\lambda}^k(0) \frac{\partial \vec{\chi}^k}{\partial p} + \sum_{i>0} \mu_i \frac{\partial C_i}{\partial p} = 0 \qquad (**)$$

for each parameter p of the upstream problem

Else, return to 1) with a new $\vec{\lambda}(0)$

Initialisation of the upstream from downstream problem solution

From the optimality conditions above (**), it follows:

$$\delta M_0 + \mu_0 \vec{\lambda}^k(0).\delta\vec{\chi} + \sum_i \mu_i \left(\sum_j \frac{\partial C_i}{\partial p_j} \delta p_j\right) = 0$$

If only equality or saturated inequality constraints are considered, then:

$$\boxed{\delta M_0 = -\mu_0\, \delta(M_{fuel\_RTLS} + M_{turbo\_jet})} \qquad (***)$$

This exchange relation between M_0 and ($M_{fuel\_RTLS} + M_{turbo\_jet}$) allows to re-initialise the upstream problem.

First M_0 and $M_{fuel\_RTLS} + M_{turbo\_jet}$ are updated:

$M_0^{k+1} = M_0^{k+1} + \delta M_0$ *and* $M_{fuel\_RTLS} + M_{turbo\_jet}$ provided by the downstream solution

Then the next equations are solved (at least mean square sense):

$$\sum_j \frac{\partial C_i}{\partial p_j} \delta p_j = -\frac{\partial C_i}{\partial M_0} \delta M_0 - \frac{\partial C_i}{\partial M_{fuel\_RTLS}} \delta M_{fuel\_RTLS}$$

for all equality or saturated inequality constraints C_i and parameter p_j of the upstream problem.

Therefore the solution (δp_j) is used to update the parameters p_j of the upstream problem.

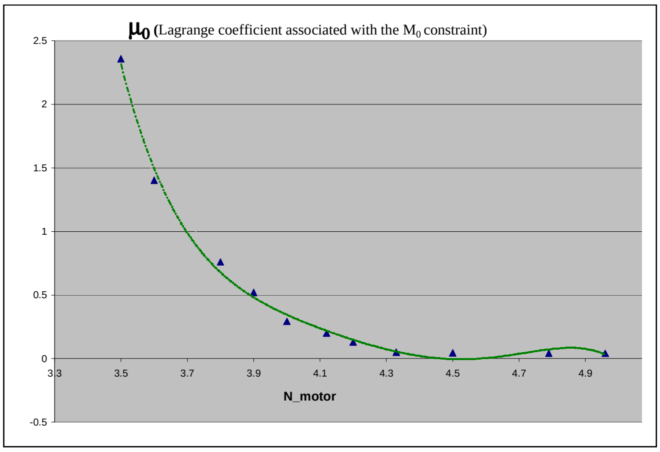


Figure 3 : Lagrange coefficient μ_0 wrt $N_{\_motor}$

In the figure 3, μ_0 has been computed (it is an output of the upstream problem solution) for different values of N_motor. For N_motor greater than 4, μ_0 is slightly positive: a change of M_{fuel_RTLS} does not modifies greatly M_0. On the other hand, for lowest values of N_motor, a change of M_{fuel_RTLS} leads to a significant change on M_0, in this case the re-initialisation of the upstream problem is very efficient to accelerate the convergence. The curve highlights also the no-linearity of the problem.

5 – Results

Two numerical optimisation methods have been used to solve the global TSTO problem.

The first method is based on the use of a parametric optimiser (non linear reduced gradient method) and a semi-analytical approach for the trajectory integration, this method allows a fast and accurate integration and optimisation of launchers trajectory[5].

The second method is a control optimal method based on a backward sweep algorithm and steepest ascent[4]. Although the control optimal method is more time consuming than the former method, the control optimal method is recommended when the control is not easily to model (by instance for aerodynamic trajectory shaping), it allows a good convergence towards the optimal solution.

We have tested the coordination process described in the section 4.2.2 with the objectives of minimizing the Gross lift-off Weight (GLOW) for the upstream problem and to minimize the required fuel for RTLS (M_{fuel_RTLS}) for the downstream problem.

Coordination method convergence

The tests results show good convergence properties of the coordination process. Leaving initially free the number of turbo-jet engines and starting from a distant solution (initialized by hand), the method converges in 10 iterations towards an optimal solution (minimal GLOW) with an accuracy less than 10 kg for M_{fuel_RTLS}.

The optimal turbo-jet number ($N_{\_motor}$), for minimal M_0, is 4.33 , M_{fuel_RTLS}= 22.4 t and M_0 = 1081.3 t (GLOW).

Now, starting from this optimal solution and forcing $N_{\_motor}$ = 4, the coordination process converges easily (3 iterations) towards the optimal solution ($N_{\_motor}$ = 4, M_0 = 1082 t , M_{fuel_RTLS}= 16.7 t) despite large changes at staging point (Δh = -2.2 km , ΔV = -165 m/s, Δ M_{fuel_RTLS} = -5.7 t).

The figures below illustrates the convergence of the method.

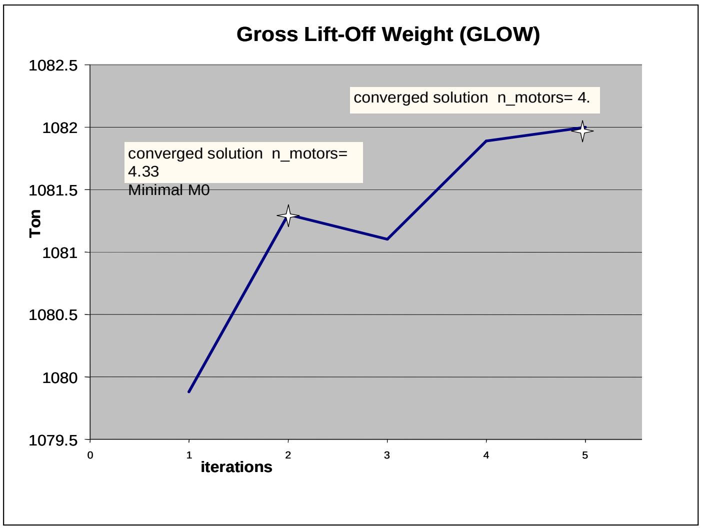


Figure 4: Convergence of the coordination process

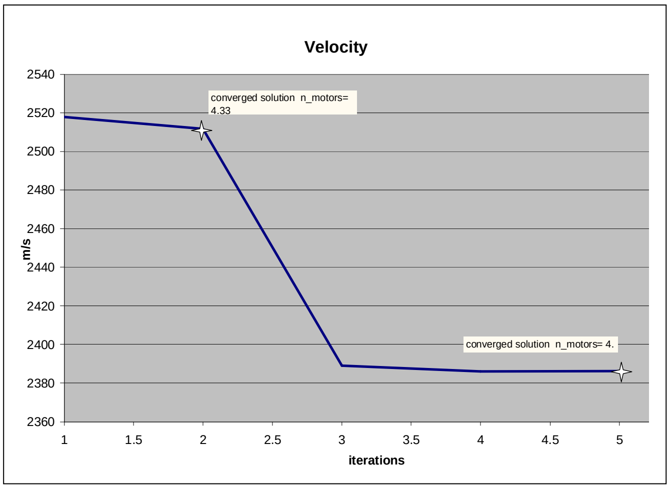


Figure 7: Staging point convergence (Velocity)

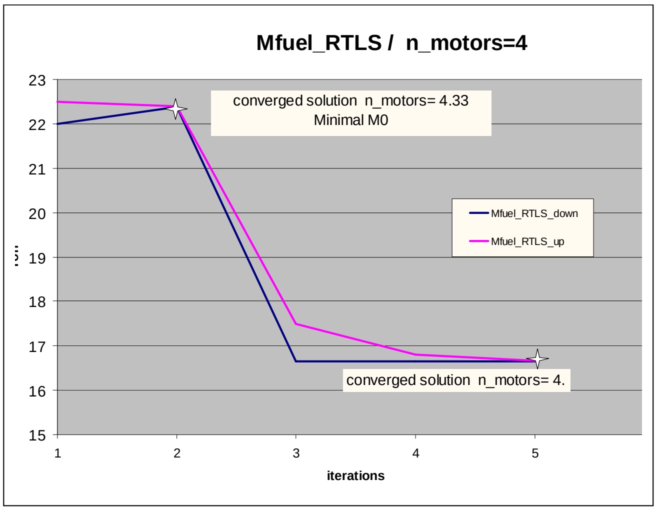


Figure 5 : Convergence from N_motor = 4.33 to N_motor = 4

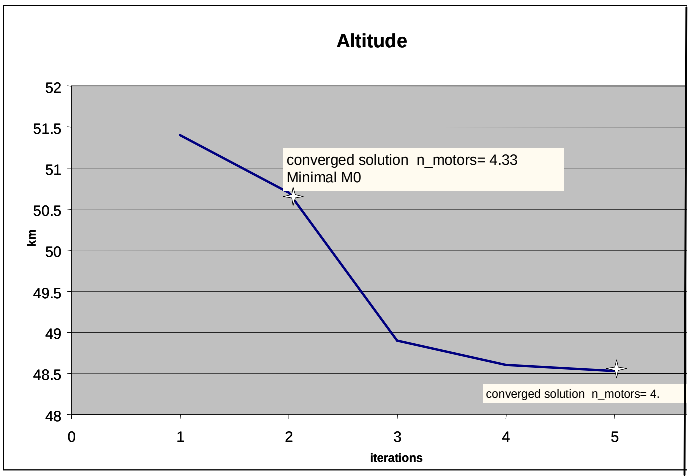


Figure 6: Staging point convergence (Altitude)

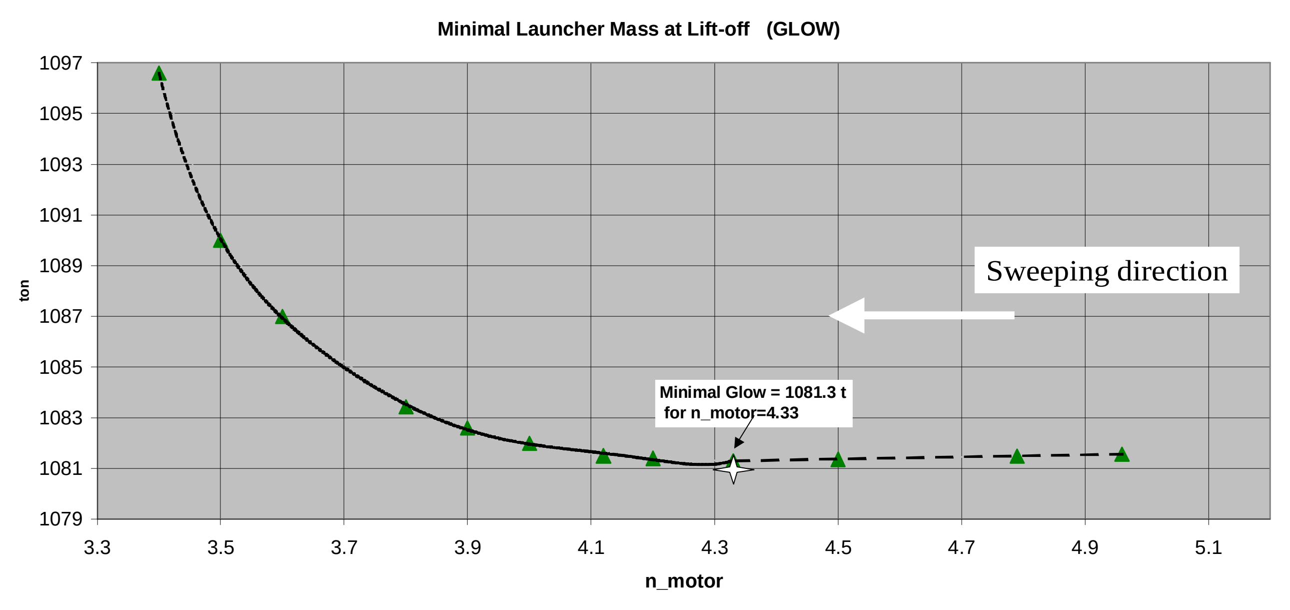


Figure 8: sweeping process wrt N_motor

The sweeping process with respect to the turbojet motorization level (N_motor) shows a flat optimal solutions area for N_motor > 4.3. On the other hand for N_motor < 4, the Lift-off mass (M_0) increases strongly and M_{fuel_RTLS} decreases as N_motor decreases until N_motor = 3.4. This last solution corresponds to the minimal number of engines to ensure the return on site (no solution has been found for N_motor < 3.4). A good compromise should be to choose a turbo-jet motorization level between 3.9 and 4.9.

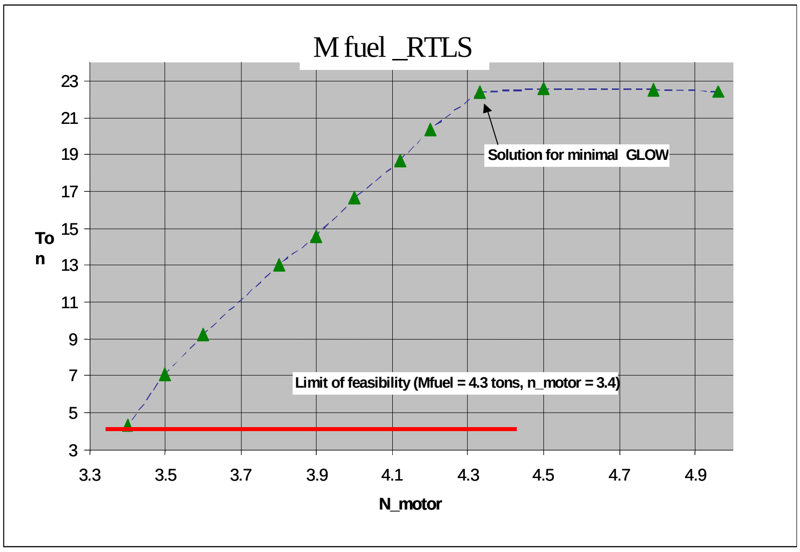


Figure 9: sweeping process wrt N_motor (M_{fuel_RTLS})

During the optimisation process, the Launcher design parameters are also optimised (the propellant masses of the booster and the orbiter are optimised). The figure below shows the Launcher design for different values of N_motor. One can note that the choice of $N_{\_motor}$ influences especially the orbiter mass (ΔM= 50 t (relative mass variation ≈20%) between N_motor = 3.4 and N_motor = 4.33)

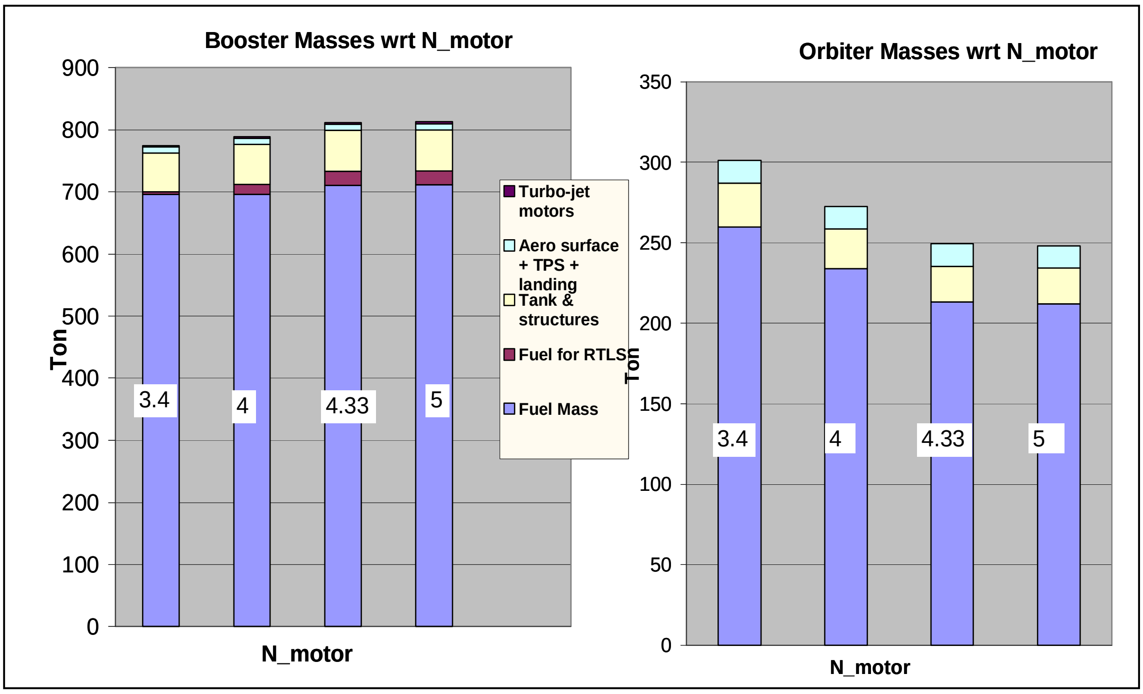


Figure 10 : Launcher design wrt N_motor

	Motorization level N_motors (turbo_jets)	3.4	4	4.33	5
Booster	Propellant mass (total)	700	712.47	733.05	734
	Fuel for RTLS	4.26	16.65	22.38	22.33
	Tanks and structure mass	63	64.12	65.97	66.1
	Aero surface + TPS + landing system	9.6	9.6	9.6	9.6
	Turbo jet mass	2.24	2.64	2.86	3.3
	Dry mass	74.9	76.36	78.4	79
	Booster mass	774.9	788.83	811.48	812.9
Orbiter	Propellant mass	259.8	234.01	212.87	211.85
	Tanks and structure mass	27.28	24.57	22.35	22.24
	Aero surface + TPS + landing system	13.9	13.9	13.9	13.9
	Dry mass	41.18	38.47	36.25	36.14
	Orbiter mass	300.95	272.48	249.13	248
Apogee motor	Propellant + tank + rocket engine	0.7	0.7	0.7	0.7
Payload		20.	20	20.	20
Launcher mass at lift off		1096.6 tons	1082. tons	1081.3 tons	1081.6

Figure 11 : details of the Launcher design wrt N_motor

4 – Conclusion

In the present research an optimisation method based on the coordination of a control optimal problem and a parametric optimisation problem is proposed. As subject for this study, a fully reusable TSTO launcher with a powered turbo jet RTLS strategy for the first stage is selected. A natural decomposition of the trajectory in three branches is proposed (Ascent phase, RTLS, Orbiter branch).

A non-linear parametric optimiser was used for ascent and Orbiter trajectory optimisation while a control optimal approach, more suitable for atmospheric trajectory shaping, was selected.

The optimisation problem is posed like a minimization of the gross lift-off weight (M_0), the minimisation of the kerosene mass (M_{fuel_RTLS}) and the turbo-jet engines mass for RTLS.

The coordination method between the RTLS control optimal problem and the ascent optimisation problem has been tested, its efficiency has been proven (four or five coordination iterations are sufficient to reach accurately the optimum from a design point to another design point).

Moreover the tests have shown that the global Launcher design, led simultaneously with the trajectory optimisation, is efficient. The method is low time consuming and sufficiently economic to explore the design space and to perform easily multi-objectives optimisation.

These first results are promising and a direction of research would be to extend the methods discussed here to a broader field of application as interdisciplinary Launcher optimisation.

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