Coarse Alignment Methods Comparison

J. F. Lima and W. C. Leite Filho
Instituto de Aeronáutica e Espaço
Praça Marechal Eduardo Gomes, 50 - Vila das Acácias
CEP 12228-904 São José dos Campos - SP - Brasil
jany@iae.cta.br, *waldemar@iae.cta.br*

Abstract

In a satellite launch vehicle, the alignment process is defined as the determination of the angular relationship C_p^n between the platform p, and the navigation n frames[1]. In self-alignment, it uses only the specific force and angular rate measurements given by the accelerometers and rate gyros that belong to the Inertial Navigation System (INS), which is to be aligned. Besides that, the information of local gravity, Earth's angular rate, latitude and flight azimuth must be also supplied to the system. Self-alignment is divided in coarse and fine alignment. The coarse alignment calculates the transformation matrix C_p^n from the platform to the navigation reference frame from the information described above, and its result will be used as the initial condition for the fine alignment. Fine alignment utilizes optimal estimation techniques, mainly the Kalman filtering, where drift, bias and attitude error are calculated with the goal of refining the C_p^n matrix previously calculated. The resulting matrix will then be used as the initial condition for the navigation, thus having its error propagated with time. Hence, it is fundamental that the self-alignment process be accurate, in order to guarantee the accomplishment of the mission. A new method is proposed in this paper and their errors are compared with methods proposed by Britting[1] and Jiang[2].

1. Nomenclature

g local gravity magnitude

Ω Earth's angular rate magnitude

L geographic latitude

A flight azimuth

n (subscript) navigation coordinate frame

p (subscript) platform coordinate frame

g_n $[-g\ \ 0\ \ 0]^T$ local gravity vector expressed in the navigation coordinate frame

ω_n $[\Omega\sin(L)\ \ -\Omega\cos(L)\sin(A)\ \ \Omega\cos(L)\cos(A)]'$ Earth's angular rate vector expressed in the navigation coordinate frame

f_p $[-f_x\ \ -f_y\ \ -f_z]^T$ specific force vector exerted on instrument set expressed in the platform coordinate frame

ω_p $[\omega_x\ \ \omega_y\ \ \omega_z]^T$ angular rate vector exerted on instrument set expressed in the platform coordinate frame

g_c $[gc_x\ gc_y\ \ gc_z]^T$ specific force vector after angle correction expressed in the platform coordinate frame

ω_c $[\omega c_x\ \omega c_y\ \omega c_z]^T$ angular rate vector after angle correction expressed in the platform coordinate frame

f_t true specific force measurements, that is, those obtained by the instruments on the assembly position

ω_t true angular rate measurements, that is, those obtained by the instruments on the assembly position

θ angle between g_n and ω_n

$\Delta\theta$	error angle
x	cross product
$\bullet$	dot product
$\lVert\cdot\rVert$	length of the vector
[navA]	$[g_n \;\; \omega_n \;\; (g_n \mathbf{x} \omega_n)]$
[navB]	$[g_n \;\; (g_n \mathbf{x} \omega_n) \;\; (g_n \mathbf{x} \omega_n) \mathbf{x} g_n]$
[platA]	$[f_p \;\; \omega_p \;\; (f_p \mathbf{x} \omega_p)]$
[platB]	$[f_p \;\; (f_p \mathbf{x} \omega_p) \;\; (f_p \mathbf{x} \omega_p) \mathbf{x} f_p]$
$\left[\tilde{C}_p^n\right]$	computational transformation matrix from platform coordinate frame to navigation coordinate frame
$[\cdot]_O$	optimal orthogonal approximation to $[\cdot]$

2. Introduction

The INS self-alignment of a launcher is intended to establish the coordinate transformation matrix from the platform to the navigation reference frame C_p^n from measurements from the accelerometer and rate gyros. The information of local gravity, Earth's angular rate, latitude and flight azimuth are supplied to the system. Britting[1] uses this information to form the [navA] and [platA] matrices and in this way calculate the $\left[\tilde{C}_p^n\right]$ matrix. Due to the uncertainties on the instruments measurements, the calculated matrix is non-orthogonal. So, its orthogonalisation becomes necessary, since C_p^n is orthogonal by definition. Jiang[2] follows the procedure described above, using [navB] and [platB] instead of [navA] and [platA]. This change, of simple procedure, results in a significant reduction of the coarse alignment errors[2]. However, Jiang notes that the choice of different basis to set up the reference navigation frame will result in distinct coarse alignment errors. Britting as Jiang choose the geographic coordinates system (North, East and Down axes) for both the navigation and the platform reference frames. In this way, the ideal coordinate transformation matrix, if there were no assembly errors and measurement uncertainties, is the identity matrix. When the geographic coordinate system is chosen for navigation and platform coordinate frames, the method described by Jiang is better because its east level error is not corrupted by gyro uncertanty[2]. The method described in this paper is based on the fact that the vectors lenghts as the angle between them remain constant independent of the coordinate system they are described. In the alignment process the vectors in question are the local gravity (g_n), Earth's angular rate (ω_n), and the angle θ between them for a specific latitude. However, due to measurements uncertainties these vectors are modified on the three directions (x, y and z), thus causing an error in length and angle. The measured angle formed by the specific force (f_p) and angular rate (ω_p) are $\theta + \Delta\theta$. The proposal is to correct such measurements based on the statistical weighting of the error on the angle $\Delta\theta$. The specific force and angular rate vectors are normalized, and the correction for the angle is rated between both in order to obtain the best result, making that the angle between g_c (specific force vector after correction) and ω_c (angular rate vector after correction) is equal to θ. Section 1 presents the nomenclature that is used throughout this work. On section 3, the methods used by Britting and Jiang are exposed. Then, the new method is presented as its implementation form. On section 4, it is explained concisely how the simulation was performed, and the comparison parameter is established. On Section 5, the results are presented and on Section 6, the conclusion.

3. Methods Discussion

3.1 Coarse alignment by Britting[1]

It is known that the measurements resolved on the platform reference frame are expressed in the navigation reference frame through the relation:

$$[navA] = \left[\tilde{C}_p^n\right] * [platA] \quad \textbf{(1)}$$

As the coordinate transformation matrix is orthogonal by definition, the equality $\left[\tilde{C}_p^n\right]^T = \left[\tilde{C}_p^n\right]^{-1} = \left[\tilde{C}_n^p\right]$ is used and the matrix is calculated:

$$\left[\tilde{C}_p^n\right] = \left[[navA]^{-1}\right]^T * [platA]^T \quad \textbf{(2)}$$

However, the calculated matrix is non-orthogonal due to the sensors errors and measurements uncertainties. Thus, an orthogonalisation procedure becomes necessary. It is obtained by forming the matrix product:

$$\left[\tilde{C}_p^n\right]_O = \left[\tilde{C}_p^n\right] * \left\{\left[\tilde{C}_p^n\right]^T * \left[\tilde{C}_p^n\right]\right\}^{-1/2} \quad \textbf{(3)}$$

3.2 Coarse alignment by Jiang[2]

Jiang proposes the same transformation, yet performed with the [navB] and [platB] matrices. This matrix is formed by three orthogonal vectors among themselves, which point to the Down (g_n), East (g_n x ω_n) and North (g_n x ω_n) x g_n directions.

$$\left[\tilde{C}_p^n\right] = \left[[navB]^{-1}\right]^T * [platB]^T \quad \textbf{(4)}$$

As his method the measurements on the geographic coordinates system and arranges the vectors on these same directions (North, East and Down), each error is maintained on its own axis, not being reflected to the others. The matrix resulting from this procedure is also non-orthogonal, thus it needs to go through (3) before this result can be used on the fine alignment.

3.3 New approach to coarse alignment by using error angle for correction

3.3.1 Vectors f_p and ω_p normalization

It is known that due to the measurements uncertainties the vectors f_p e ω_p are modified on the three directions (x, y and z), causing an error in length and angle. The first step is to normalize the vectors allowing the correction only in angle.

$$g_m = \frac{f_p}{\left\| f_p \right\|} \tag{5}$$

$$\omega_m = \frac{\omega_p}{\left\| \omega_p \right\|} \tag{6}$$

3.3.2 Calculation of θ and Δθ

The second step is to calculate the θ values (angle that the vectors must keep) and the Δθ (error in angle due to the erroneous sensors measurements). The angle θ can be directly calculated in function of the latitude or then through the scalar product, as per (7). The error in angle Δθ is then calculated from the difference between the angle formed by the erroneous sensors measurements, normalized, and the angle θ.

$$\theta = \cos^{-1}\left[\frac{g_n}{\left\| g_n \right\|} \bullet \frac{\omega_n}{\left\| \omega_n \right\|} \right] \tag{7}$$

$$\Delta\theta = \cos^{-1}\left(g_m \bullet \omega_m \right) - \theta \tag{8}$$

3.3.3 Corrected ω_c vector from the ω_m vector.

The first correction in angle will be performed on the vector ω_m, creating from this the corrected vector ω_c. This will be corrected in accordance with three conditions. The first (8) is that ω_c must keep its length equal to 1, so the correction will not be inserting error length to the new vector. The second (9) is that the angle between ω_c e ω_m will have to be (1-N)*Δθ. The weighing factor (1-N) indicates that for N=0, all correction will be made on the angular rate vector. This can be easily observed on the Figure 1. For N=0, the angle between ω_c and ω_m is Δθ, and the angle between ω_c and g_m is θ. Therefore, the correction is complete because we have module equal to 1 and a vector between angles equal to θ. For N=1, we would have coincident ω_c and ω_m, hence the correction will have to be made entirely on the vector g_m. We insert the rated correction (1-N) because each INS has a set of sensors with different characteristics, and for each characteristic a weighing N can obtain a better result. The correction above can be made on the wrong direction if the third condition (10), where the angle between ω_c and g_m should be θ + N*Δθ, is not accomplished. This condition imposes that the vector has to be moved in the correct direction, keeping the same plane determined by g_m and ω_m, leaving the N*Δθ correction for the g_c vector (corrected g_m vector).

$$\left\| \omega_c \right\| = 1 \tag{8}$$

$$\left(\omega_c \bullet \omega_m \right) = \cos((1-N) * \Delta\theta) \tag{9}$$

$$\left(\omega_c \bullet g_m \right) = \cos(\theta + N * \Delta\theta) \tag{10}$$

Figure 1 shows the procedure described above.

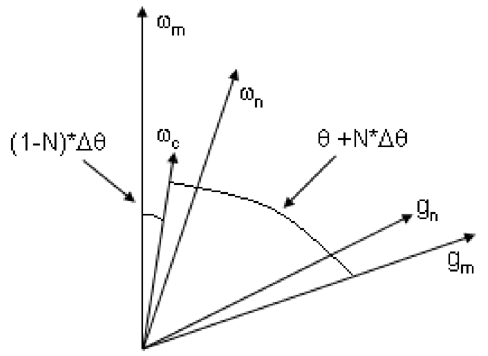

Figure 1 – Vector ω_c

3.3.4 Corrected g_c vector from the g_m vector.

The correction will be based on the weighing N*Δθ, as described on item 3.3.3. To such correction, it must be imposed the three conditions described on (11), (12) and (13). It can be noted that equation (13) makes the new vectors g_c and ω_c to have the angle θ between them.

$$\|g_c\| = 1 \qquad \textbf{(11)}$$

$$\left(g_c \bullet g_m\right) = \cos(N * \Delta\theta) \qquad \textbf{(12)}$$

$$\left(\omega_c \bullet g_c\right) = \cos(\theta) \qquad \textbf{(13)}$$

Figure 2 shows the procedure described above.

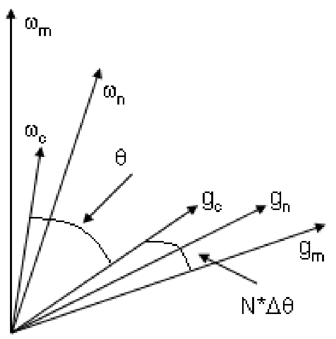

Figure 2 – Vector g_c

3.3.5 Coordinate transformation matrix $\left[\tilde{C}_p^n\right]$.

From the corrected measurements, the matrix can be found in accordance to what was explained on equations (2) and (4) or as described below in 14:

$$\left[\tilde{C}_p^n\right] = [nav] * [plat]^{-1} \quad \textbf{(14)}$$

We can normalize g_n and ω_n vectors before forming [plat] or to use corrected vectors g_c and ω_c pre-multiplied by g and Ω, respectively, for [nav]. As the vectors were corrected in lenght and angle, the resulting matrix is orthogonal, hence no need to go through the procedure exposed in (3).

4. Methods procedure comparison

4.1 Measurements obtainment

It is named ideal platform performed measurements those that we would obtain if there were not assembly errors (named here true misalignment) and measurements uncertainties. From these measurements, it was added the true misalignment error, multiplying the ideal values by a small rotation matrix. In this simulation the chosen sequence was 3-1-2, with rotation angles of 0.7°, 0.2° and –1°, respectively. To these measurements, it was added the on-off bias as well measurement noise (white noise) for the accelerometers and on-off drift plus measurement noise (white noise) for the rate gyros. However, the measurements that we look for are the true measurements f_t and ω_t, that is, those obtained by the instruments on the assembly position. For the –23.214° latitude, 3° flight azimuth and 3-1-2 sequence, we have that the true attitude angles between the platform and navigation frames are γ = -17.9334°, α = 86.6366° and β = 17.2612°.

4.2 Simulation

It was performed 100 runs, where to each simulation it was used the average of a sequence of 10000 measurements. From this data, the vectors g_c and ω_c were obtained in accordance to what is described for equations (8) to (13). Then, the matrices [platA] and [platB] were calculated and the equation (14) was used to obtain the $\left[\tilde{C}_p^n\right]$ matrix in accordance to the three proposed methods.

4.3 Results comparison

The coordinate transformation matrix from the platform to navigation reference frame, resulting from the coarse alignment, will be used as initial condition for the Kalman filtering, for the fine alignment. Hence, it is important to notice the nine elements of the matrix, and not only the 5 elements usually taken to find attitude angles. One way to test the complete matrix (9 elements) is to make the transformation of a vector described in the true platform reference frame (the one with the assembly error, which is the true measurement that the accelerometer and gyro rate must give) to the navigation reference frame. As the measurements of the local gravity and Earth's angular rate vectors are well known in the navigation reference frame, we shall establish the comparison through the norm of the difference between g_n and the true specific force vector pre-multiplied by the matrices found by the three methods under study $|g_n - C_p^n * f_t| \equiv |\Delta C * f_t|$. The same can be repeated to the difference between ω_n and the value obtained by the multiplication of the calculated matrix and the true angular rate vector $|\omega_n - C_p^n * \omega_t| \equiv |\Delta C * \omega_t|$. The results are presented on Section 5.

5. Results

The results are presented on the Table 1 on the units [mg] e [º/h]. The 3 methods were tested for different values of drift and bias. For the method proposed on this article, using equations (2) and (4) yield the same results.

Table 1: Methods comparison

Rate gyro drift Accelerometer bias		Method	$\|g_n - C_p^n * f_t\|$ [mg]	$\|\omega_n - C_p^n * \omega_t\|$ [º/h]
1 -	**0.5º/h 1mg**	**N = 0**	**0.62072**	**0.1787**
		Britting	31.5621	0.5077
		Jiang	**0.62072**	**0.1787**
2 -	**0.5º/h 500µg**	**N = 0**	**0.31048**	**0.1788**
		Britting	31.56	0.5077
		Jiang	**0.31048**	**0.1788**
3 -	**0.1º/h 1mg**	**N = 0**	**0.62072**	**0.0359**
		Britting	6.3281	0.1013
		Jiang	**0.62072**	**0.0359**
4 -	**0.1º/h 500µg**	**N = 0**	**0.31048**	**0.0357**
		Britting	6.305	0.1013
		Jiang	**0.31048**	**0.0357**
5 -	**0.05º/h 1mg**	**N = 0.1**	**0.58792**	**0.0181**
		Britting	3.2007	0.0507
		Jiang	0.62072	0.0184
6 -	**0.05º/h 500µg**	**N = 0**	**0.31048**	**0.0179**
		Britting	3.1597	0.0506
		Jiang	**0.31048**	**0.0179**
7 -	**0.01º/h 1mg**	**N = 0.2**	**0.54171**	**0.0037**
		Britting	0.83496	0.0104
		Jiang	0.62072	0.0060
8 -	**0.01º/h 500µg**	**N = 0.1**	**0.27267**	**0.0037**
		Britting	0.68441	0.0120
		Jiang	0.31048	0.0044
9 -	**0.005º/h 1mg**	**N = 0.4**	**0.54155**	**0.0023**
		Britting	0.63034	0.0054
		Jiang	0.62072	0.0052
10 -	**0.005º/h 500 µg**	**N = 0.2**	**0.2719**	**0.0022**
		Britting	0.41599	0.0053
		Jiang	0.31048	0.0033
11 -	**0.001º/h 1mg**	**N = 0.7**	**0.54655**	**0.0020**
		Britting	0.5478	0.0021
		Jiang	0.62072	0.0050

12 -	**0.001°/h 500 μg**	**N = 0.5** **Britting** **Jiang**	**0.27807** 0.28408 0.31048	**0.0018** 0.0019 0.0029

For the proposed method, there were used the [navA] and [platA] and then the [navB] and [platB] matrices for the calculation of the $\left[\tilde{C}_p^n\right]$ matrix, in accordance to equations (2) and (4). For both cases, the same result was obtained. Hence, on table 1 only one value is shown, even that two calculations were made to each situation. For all the cases presented, the method proposed by Jiang yields the same results that the method by correction in angle when N=0 is used, that is, all the correction is made on the angular rate vector. For the cases 1 to 4 (set 0.5°/h and 1mg; 0.5°/h and 500μg; 0.1°/h and 1mg; 0.1°/h and 500μg) our method yields the same results that by Jiang. Both are superior to the results obtained by Britting. However, when more accurate rate gyros are employed, the proposed method presents superior results to Britting and Jiang. These results are obtained rating the error between specific force and angular rate vectors, which did not occurred before, where all the error was corrected only on the angular rate vector. This is expected, because for better rate gyros, we also have better angular rate measurements, so it is expected that the correction to be weighed, imposing it also to the specific force vector. Therefore, for the cases 5 to 12 (set 0.05°/h and 1mg; 0.05°/h and 500μg; 0.01°/h and 1mg; 0.01°/h and 500μg; 0.005°/h and 1mg; 0.005°/h and 500μg; 0.001°/h and 1mg; 0.001°/h and 500μg) the proposed method presents as superior. We also can note that by maintaining the rate gyros but changing the accelerometers from 1 mg to 500 μg, the weighing was reduced. It shows that the method is sensitive to this change and imposes less weigh to the accelerometer correction. It yielded that, for the case 5 (set 0.05°/h and 1mg), the best value was obtained for N=0.1, and for the case 6 (set 0.05°/h and 500μg) the best result was for N=0 (hence having the same results as Jiang). For the other cases described above, we obtained better results for N = 0.2 and N = 0.1, N = 0.4 and N = 0.2, N = 0.7 and N = 0.5. From the results we can observe that is a relationship between sensors characteristics and weight N. Therefore, it is recommended that a previous simulation be made in order that the best weighing N is chosen for the accelerometers/rate gyros set.

6. Conclusion

The method described in this paper is based on the fact that the vectors lenghts as the angle between them remain constant independent of the coordinate system they are described. In the self-alignment process the measured angle formed by the specific force (f_p) and angular rate (ω_p) is $\theta + \Delta\theta$, instead of θ. The proposal is to correct such measurements based on the statistical weighting N of the error on the angle $\Delta\theta$. The specific force and angular rate vectors are normalized, and the correction for the angle is rated between both in order to obtain the best result, making that the angle between g_c (specific force vector after correction) and ω_c (angular rate vector after correction) is equal to θ. However, when more accurate rate gyros are employed, our method presents itself superior to Britting and Jiang. These results are obtained rating the error between specific force and angular rate vectors, which did not occur before, where all the error was corrected only on the angular rate vector. Hence, the proposed method allows correcting specific force and angular rate vectors by choosing of N.

References

[1] Britting, K. R. Inertial Navigation System Analysis. New York: Wiley Interscience, ch. 9, 1971.

[2] Jiang, Yeon Fuh. Error Analysis of Analytic Coarse Alignment Methods. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, n.1, January, 1998.

[3] Mortensen, R. E. Strapdwon Guidance Error Analysis. IEEE Transactions on Aerospace and Electronic

Systems, vol. AES-10, n.4, July 1974.

This page has been purposedly left blank