

# Optimization Based Clearance of Flight Control Laws

## Using Differential Evolution

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### Abstract

At present, the clearance of modern flight control laws to take into account the many uncertainties that actually occur during flight has become a great challenge to engineers and researchers, and traditional grid-based methods have gradually been incompetent for this task. In this paper, we aim to explore solving this very difficult problem by using optimization methods. The aircraft model and flight control laws used in the present study is a twin-engine fighter with a RIDE (Robust Inverse Dynamics Estimation) controllers and an AOA limiter. Our approach is applied to the problem of evaluating a nonlinear AOA Limit Exceedence clearance criterion. By transforming the criterion into a cost function, the original clearance problem is simplified into that of searching the worst combination of values in the uncertain parameter space. This optimization problem is solved with a differential evolution algorithm. Compared with traditional grid-based ones, the optimization-based methods have two obvious advantages. One is the points between grids can be cleared; the other is the difficult from 'disaster of dimension' problems can be reduced. Furthermore, the methods mentioned above can be used for the clearance of all the present criterions.

### 1. Introduction

Flight control system (FCS) enables the pilot to control the aircraft and provides safe and economic operation, and aircraft manufactures have reached a high level of expertise and experience in flight control law design. For example, modern high performance aircrafts are often designed to be naturally unstable due to performance reasons, and can only be flown by means of a flight control systems that provides artificial stability. In addition to this, other advanced active flight control techniques are also being widely applied to the modern flight control systems. As the safety and capability of the aircraft is greatly dependent on the controller design, it must be proven to the clearance authorities that the flight control laws have been designed such that the aircraft is safe to fly throughout the whole envelope, under all parameter variability and failure conditions.

This task is a very tremendous one, especially for fighter aircrafts, because of the many different store configurations, the large number of parameter variations and uncertainties, and the large flight envelope for which a clearance has to be provided. In the current industrial clearance, all these above factors are considered as parameterized uncertainties at first, and then a grid-based method is used to find the worst-case result for each criterion. This leads to a very lengthy and expensive process. Moreover, with the increase of the uncertainties due to the enlarged envelope and the complication of the systems, the huge cost of traditional grid-based approach brings a great challenge to flight control engineers and researchers.

In Europe, industrial organizations and research institutes had realized this point, and in 1999, the Flight Mechanics Action Group 11, FM (AG11), was established to address this challenge, which focused on the applications of several advanced techniques during the clearance process of flight control laws<sup>1</sup> It was the first time to carry out a systematic research in this field and the researchers had obtained many useful results.

In this paper, an optimization based clearance method using differential evolution is proposed, which is used to clear a RIDE (Robust Inverse Dynamics Estimation) flight control law with an AOA (angle of attack) limitator, and has the capability to improve both the reliability and efficiency of current process.

We begin in Section 2 with a description of the principle for optimization based flight control law clearance. Section 3 introduces the differential evolution algorithms. Following this, the DE based clearance experiments and results are respectively presented in section 4 and section 5. And conclusions are drawn in the last section of this paper.

## 2. Optimization based clearance of flight control laws<sup>1</sup>

A clearance criterion can be described as  $c(p, FC)$ , depending on the combination of  $q$ -dimensional uncertain parameters vector  $p$  and flight condition vector  $FC$ . Based on the limiting acceptable value of  $c(p, FC)$ , namely  $c_0$ , the clearance problem for a given criterion  $c(p, FC)$  can be comprehended as a process of compare and verification. We can define a distance function as

$$d(p, FC) = c(p, FC) - c_0 \quad (1)$$

If for a fixed  $FC$ ,  $d(p, FC)$  is positive for all uncertain parameter values, then the clearance requirement is fulfilled in  $FC$  and the point  $FC$  is cleared.

The current industrial clearance approach depends on an exhaustive search on a grid for both flight conditions and variable parameters. For instance, we have  $n$  flight conditions to be cleared, and choose  $m$  values for each uncertain parameter. Then we have to check the value of  $d(p, FC)$  for  $n \times m$  times to complete the clearance. This approach has two main difficulties<sup>1</sup>. First, there are tremendous costs involved when simultaneously checking the robustness for many uncertain parameters. Because the computation cost increases exponentially with the number of parameters  $q$ , which is known as ‘disaster of dimension’, this approach is not suitable for large scale and complex problems. Second, the values of parameters and flight conditions between grid points actually are not tested and cleared. The grid-based approach has no guarantee of this case.

From the optimization point of view, both of these aspects can be easily improved by considering the clearance problem as a constrained minimization in the space of uncertain parameters and flight conditions. For a specific criterion, the optimization object is to find the worst-case and the corresponding combination of uncertainties. The general steps of optimization based clearance of flight control laws can be concluded as follows:

- a) Choose flight condition to be cleared, and be sure of the perturbation ranges of uncertain parameters.
- b) Change the criterion into a cost function for optimization.
- c) Check the nominal condition, if the result is satisfied with the need of criterion, go on to the next step, or else turn step f).
- d) In the search space of uncertain parameters, minimize the cost function obtained in step b).
- e) Check the worst-case, if the result is satisfied with need of criterion, this flight condition point is cleared, or else is not cleared.
- f) Turn next flight condition.

Obviously, the key to realize above steps lies in how to change the criterion into a suitable form for function optimization, and how to select or design efficiency optimization algorithms. We focus on the second point.

There are many optimization algorithms, and evolutionary algorithms have received many attentions due to their potential for solving a wide range of difficult optimization problems. One of the most promising novel evolutionary algorithms is the Differential evolution (DE) introduced by Storn and Price a few years ago<sup>2-4</sup>, which is a very simple, efficient, and robust evolutionary inspired search technique for global optimization. The following research of this paper is based on a differential evolution algorithm. Compared with other global optimization algorithms, such as genetic algorithm (GA), particle swarm optimization (PSO), and so on, differential evolution has some advantages in the speed and quality of convergence.

### 3. Differential evolution algorithms

DE is very simple to understand and to implement, and it has only a few control variables, which remain fixed throughout the entire optimization procedure.

Considering a  $q$ -dimensional optimization problem and taking the DE/rand/1 scheme for example, the realization of DE algorithm can be described as following basic steps.

a) Randomly initialize the population in the search space with the size of  $N$ , and each individual  $X_i$ ,  $i=1,2,\dots,N$  in the population is a feasible solution of the problem, can be represented as  $X_i=(x_{i1},x_{i2},\dots,x_{iq})$ .

b) Compute the cost function derived from the optimization problem to evaluate the fitness for those members in the population.

c) For each individual, perform a mutation operation to get offspring generation:

$$v_i = X_{r1} + F(X_{r2} - X_{r3}) \quad (2)$$

where  $r_1, r_2, r_3$  are randomly chosen mutually different integers in the bound of  $[1, N]$ , and  $F$  is a positive real and constant factor between  $[0, 2]$ , which controls the amplification of the differential variation ( $X_{r2} - X_{r3}$ ). Note that the selection of  $F$  is of importance to the performance of the algorithm: if  $F$  is too large, it is slow to get rid of the difference between the individuals to converge; if  $F$  is too small, the algorithm converges fast and be easy to get premature.

In addition, a greedier search strategy is adopted to accelerate the convergence sometimes, which uses the best individual  $X_{best}$  in the father generation to replace  $X_{r1}$  in the equation (2).

d) With the probability  $CR$ , Crossover the corresponding individuals of both father and offspring generations to get a new offspring generation.

e) Use the cost function to evaluate each individual in the new offspring generation and compare the result with the one in the father generation. If the offspring is better, replace the corresponding one of father generation with the new one, or else keep the father generation one reserved.

f) Repeat those above steps until the satisfied solution is obtained, or until the termination condition is fulfilled.

### 4. DE based flight control law clearance

#### 4.1 Aircraft model and flight control laws

The aircraft model used in this paper is an augmented aerodynamic model of a supersonic twin-engine heavy fighter aircraft, which add a large number of considerations of a variety of parameter uncertainties, and users can set values of those parameters within specified ranges.

The aircraft dynamics can be described as a set of twelve first-order coupled nonlinear differential equations, which are given in the general form as below:

$$\dot{x}(t) = f(x(t), u(t), \Delta) \quad (3)$$

where  $x(t)$  is the state vector with twelve components, including three coordinates of position, three attitude angles, velocity, angle of attack (AOA), sideslip angle, and three angular rates.  $\Delta$  represents the uncertain parameters.  $U(t)$  is the control input vector composed of symmetric horizontal tail deflection angle, aileron deflection angle, and rudder deflection angle. The output response of the aircraft is given by

$$y(t) = h(x(t), u(t)) \quad (4)$$

where  $y(t)$  is the output vector. And the control input is determined by

$$u(t) = g(x(t), y_{ref}(t)) \quad (5)$$

The above equation (5) represents the flight control law, and  $y_{ref}(t)$  is the expected reference demand. Equations (3), (4) and (5) together constitute the close loop system with the flight control law in the loop.

For the convenience of the study, only the pitch channel of the aircraft is considered. A flight control law to be cleared is designed to track the pitch rate demand by using a RIDE (robust inverse dynamics estimation) method, and at the same time, an AOA limiter is adopted to avoid entering the dangerous situations with very high AOA. Both of these techniques can be consulted from reference 5.

#### 4.2 DE based clearance

The clearance criterion considered here is the AOA Limit Exceedence Criterion<sup>1</sup>, which is used to verify whether the AOA limiter works effectively under all parameter variability and failure conditions. For this criterion, it uses a pitch stick pull command to identify whether the maximum AOA response exceeds the limits.

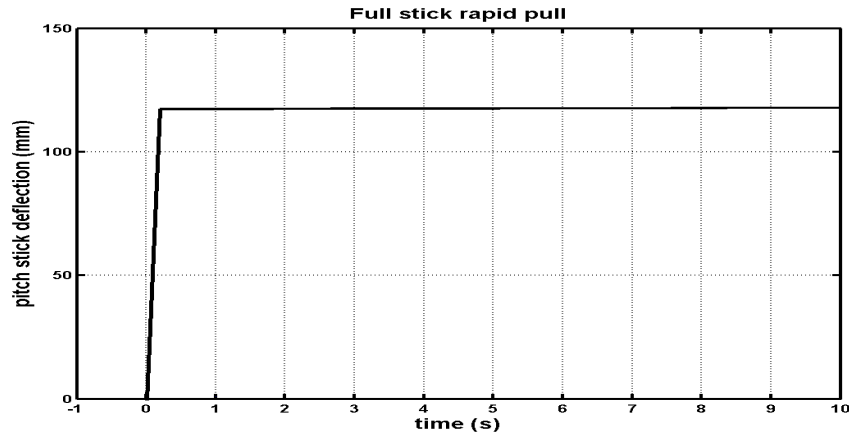


Fig.1 Full Pitch Stick Rapid Pull Command

Figure 1 above shows the specified pitch stick command, which is a step full pull signal passes a human-sense filter to approximate pilot's real rapid full pull. The command is with a pitch stick deflection to the upper limit 120mm and held for 10 seconds.

On the view of optimization, it aims to find the maximum AOA response of the above command under all combinations of the uncertain parameters, and then check whether the worst-case meets the need of AOA limits. So we can easily get the cost function for optimization as below:

$$\alpha_{\max} = \max(\alpha(t)) \quad t \leq 10s \quad (6)$$

where  $\alpha$  represents angle of attack..

The flight condition with altitude 5000 meters and Mach number 0.6 is taken to be an example. At this point, the limit of AOA is designed as 24 degrees. Considering the pitch channel uncertain parameters listed below in Table 1, DE is used to find the worst combination in the parameter space.

Table 1 Aircraft Uncertain Parameters

| Parameter   | Bound               | Description                                                             |
|-------------|---------------------|-------------------------------------------------------------------------|
| $dmass$     | $[-2000, 9000][Kg]$ | Variation in aircraft mass                                              |
| $dx_{cg}$   | $[-0.06, 0.06][m]$  | Variation in position of center of mass                                 |
| $dMz_{dtz}$ | $[-5, 5][\%]$       | Uncertainty in pitch moment due to symmetric horizontal tail deflection |
| $dCy_{dtz}$ | $[-5, 5][\%]$       | Uncertainty in lift due to symmetric horizontal tail deflection         |

|           |                |                                                                |
|-----------|----------------|----------------------------------------------------------------|
| $dI_{zz}$ | $[-20,20][\%]$ | Uncertainty in aircraft inertia around z-axis from nominal one |
|-----------|----------------|----------------------------------------------------------------|

The DE algorithm used in this study is the DE/best/2 scheme, which is defined that the population size is 40,  $F$  is 0.4, and  $CR$  is 0.7.

## 5. Simulation results

The optimization process is simulated in MATLAB 6.5 environment: aircraft model and flight control laws are built as SIMULINK models, and the DE algorithm is programmed in a .m document.

In the nominal condition of level flight, each uncertainty in the Table 1 is set to zero. The AOA response of the command is shown in figure 2, and the maximum value of AOA is 22.4 degrees. Using the DE defined above, after 100 generations of iteration, we can easily get the worst-case  $[dmass, dxcg, dMz\_dtz, dCy\_dtz, dI_{zz}] = [9000, 0.06, -5, 5, -20]$  with the maximum value of AOA 23.25 degrees, and the AOA response of the command is shown in figure 3.

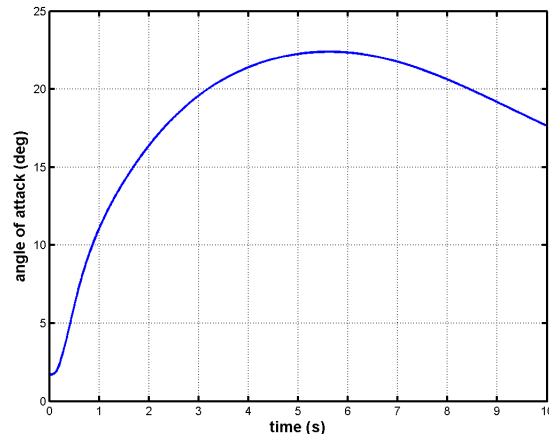


Fig.2 AOA response in the nominal condition

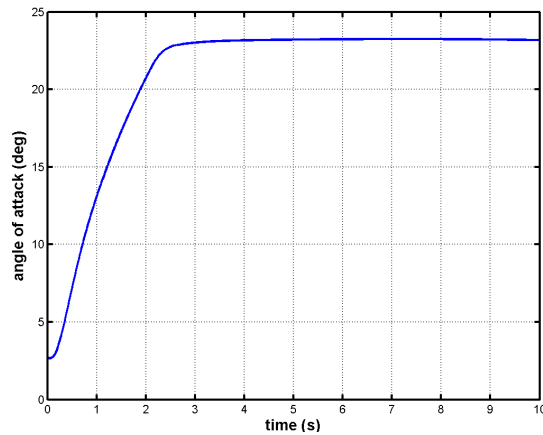


Fig.3 AOA response in the worst-case condition

From the above test, we can find that the AOA limiter can work well in the worst-case condition, and the value of AOA does not exceed the 24 degrees, which means this flight condition is cleared to the criterion.

To study the influences of the uncertain parameters on the final result, a sensitivity of each parameter is analyzed in the worst-case condition, and the results are shown as figure 4. Note that all parameters have been normalized in the range  $[-1, 1]$ .

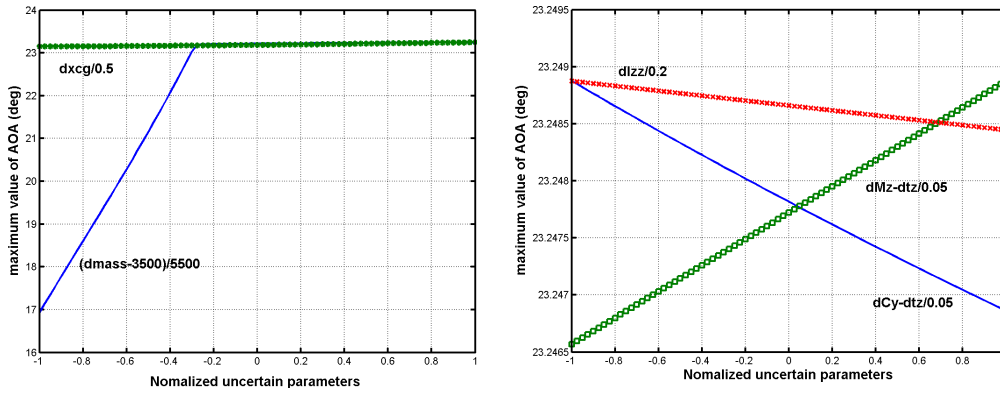


Fig.4 Sensitivity plots in the worst-case condition

Obviously, when uncertain parameters change one by one, variations in aircraft mass and position of center of mass have relatively bigger influences on the AOA response than other parameters.

For a further test, a more serious case with larger uncertainties is considered, which extends the uncertain bound of  $dx_{cg}$  from  $[-0.06, 0.06]$  to  $[-0.12, 0.12]$ , and bounds of other parameters keep unchanged. The following figure 5 gives the worst-case AOA response in this condition, and the worst-case is  $[dmass, dx_{cg}, dMz_{dtz}, dCy_{dtz}, dlzz]=[8984.24, 0.12, -5, 5, 20]$ .

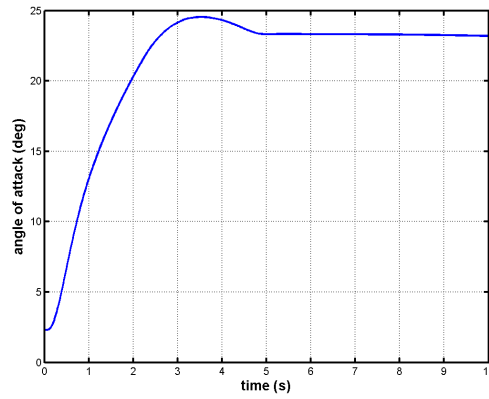


Fig.5 AOA response in the worst-case condition with extended uncertain bound

From figure 5, it can be seen clearly that the AOA response has exceeded the limit with the maximum value 24.54 degrees. If this occurs in the real verification process, the flight control law should be further optimized and redesigned to meet the need of criterion. Note that four of the uncertain parameters in this case are on the upper or lower bound and  $dmass$  is inside its bound.

Although the simulation results show that the DE based approach appears to work well, it needs to be pointed out a drawback that the optimization methods will only find a maximum (or minimum) value of the cost function. If the control law is not cleared for a given flight condition, the analysis will not reveal all combinations of uncertainties that violates the criterion, which may have useful information for redesigning the flight control law. This is a fairly difficult problem, and is left for future further investigation.

## 6. Conclusions

This paper has introduced an advanced technique based on optimization strategy for the clearance of flight control laws, which is performed by using a differential evolution algorithm. In contrast to traditional grid-based approaches, the proposed approach reduces the two main difficulties mentioned in Section 2. First, it can clear the points between grids. Second, it can avoid the ‘disaster of dimension’ problems in some sense. In addition, this approach can

efficiently cope with the clearance for all other defined criterions, including linear or nonlinear, and frequency-domain based or time-domain based ones. Generally speaking, although the optimization algorithm can't absolutely guarantee that it would not miss the worse case in any condition, this approach is a rather promising solution for the engineering application to improve the efficiency of clearance process.

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