Application of the eXtended Finite Element Method (XFEM) in industrial damage tolerant approaches for aerospace structures

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Abstract

In this contribution, the authors present several examples of computation of stress intensity factors (SIFs) and simulation of the propagation of cracks in aeronautical structures. The featured applications involve the use of the eXtended Finite Element Method (XFEM). The XFEM is particularly suited for the solution of complex fracture mechanics problems as it allows for introducing an a priori knowledge of the solution in the FE approximation space, e.g. the displacement discontinuity (crack opening) and the functions of the development expansion of the crack tip displacement field in a linear elastic solid, which avoids the need for a conforming mesh.

1. Introduction

The eXtended Finite Element Method (XFEM) is a numerical method which handles geometries containing singularities without the need of building a conforming mesh[1,2]. It is based on the Partition of Unity Method[3]. In addition, the Level Set Method[4] is used to represent the discontinuity into the geometry[5]. This mesh-independence property is particularly desirable in fracture mechanics which deals with solid mechanics problems with one or several cracks inside the solid. Indeed, there is no need to rebuild a mesh which conforms to the crack surface geometry as the cracks propagate (if the mesh is sufficiently fine to capture the stress variation near the front). The discontinuous displacement field is modelled with additional degrees of freedom at nodes which have their support cut by the crack.

The XFEM has been the subject of intense research efforts for an entire decade now. It has recently reached a maturity level that allows its transfer into industrial damage tolerant approaches, supplementing standard procedures based on the use of simplified solutions gathered in software such as Esacrack, Nasgro, Afgrow. It also permits to generalize the use of complete three-dimensional finite element simulations of cracked structures thanks to a significantly reduced effort devoted to meshing and CAD/CAE operations.

For instance, the following problems have been addressed and/or are still being under investigation, at least for what concerns damage tolerant approaches with quasi-static loading conditions, under the hypothesis of small perturbations and linear elastic materials (the following list is not exhaustive, it only summarizes the perception of the authors):

- **Mesh refinement**. In 3D, the XFEM must be coupled with selective mesh refinement in the vicinity of the crack in order to capture the sharp variation of the fields[6]. It must be noted that, even though the mesh does not need to be conformal, it has to be fine enough in the sense of global and local convergence of the solution. The use of the level sets helps defining size maps that are particularly suited for fracture mechanics problems.
- **Enrichment strategies**. The quality of the solution depends on the size of the zone of which the approximation space is enhanced with the a priori knowledge of the solution[7,8].
- **Error indicators and estimators**. Specific error indicators are necessary in order to characterize the quality of the solution. In fracture mechanics problems, they can either take the form of indicators based on dedicated recovery procedures[9-11] or estimators of bounds on the stress intensity factors[12]. Error maps can be used for the definition of element size maps or for adaptivity of shape function degree.
- **Reliable stress intensity factors**. With any FE method based on unstructured meshes, numerical oscillations of the stress intensity factors can be observed[13]. At least smoothing operations could be necessary. Other methods suggest for solving the stress intensity factors as a specific finite element field with proper interpolation[14].

- **Robust level set propagation algorithms**. The level set representation of cracks requires specific update algorithms, which can be either fully numerical, fully geometrical or mixed[15]. Knowing that the level sets are used in the definition of local coordinates systems at the crack tip and in the definition of the virtual crack extension vector field in the evaluation of the *J* integral, they must be carefully evaluated, updated and orthogonalized.
- **Solvers**. XFEM leads to sparse, symmetric definite positive matrices like the FEM. If the size of the problem is sufficiently small (hundreds of thousands of degree of freedom), direct solvers are able to solve the system without difficulty. If the system is larger, one must be used an iterative solver, which requires the use of a particular preconditioning due to conditioning of the stiffness matrix which is badly affected by the near-tip enrichment[8].
- **Availability for industrial crack analysis procedures**. The XFEM has been implemented in various research codes which are not - and will probably not be – used in the industrial software environment. The short term availability of the XFEM can only be achieved through the provision of plug-ins, connected to commercial FE software. For instance, implementation of user defined extended finite elements, or of substructuring approaches[6,16-18] are very efficient solutions. A mid-term prospect could be hard-coding the method in the commercial FE software, depending on the demand of the industrials in the field of crack propagation in aerostructures and aeroengine components.
- **Multi-scale approaches**. The XFEM is also used for studying very small defects in large structures. For instance, in the case of thin-walled structures modelled with shell elements, it is now possible to look at the propagation of small initial three-dimensional non-through cracks[6,19]. Other research has introduced the XFEM in scale transition approaches[20].
- **Multi-physics applications**. Most of the fracture mechanics problems involve complex boundary conditions and loads. For instance, it is not rare that mechanical loads, thermal loads, residual stresses and other body forces interact and influence the stress concentration and the stress intensity factors. The XFEM must be adapted, as well as all the post-processing capabilities (e.g. the interaction integral) [21].
- **3D validation**. While numerous 2D validation cases are available in the literature, it is only recently that 3D applications with both verification and validation have been published[22], for instance by the authors[6, 17-19,23]. It is of utmost importance to propose exhaustive validation of the method in order to promote its use in the industrial software environment and crack analysis procedure.

Other major research topics include the introduction of contact and friction on the crack faces, extension to multi-materials, extension to non-linear materials, application to other types of loading, damage to fracture transition, among numerous works.

The present contribution summarizes the approaches followed by the Group of Multiscale Modeling of Materials and Structures of Cenaero for fracture mechanics problems, using the XFEM. It also shows interesting applications of 3D crack propagation.

The two following approaches are used:

- The first one mixes the classical FEM and the XFEM through a substructuring approach (the S-FE/XFE method)[6,16-18]. In this case, the structure is decomposed into cracked and uncracked domains which are treated by an in-house XFE-code called Morfeo (developed by Cenaero) and the commercial FE software Samcef™, respectively. The interface problem between the two domains is solved using the Finite Element Tearing and Interconnecting method[24]. Among many advantages, the method allows for handling mixed-dimensional problems.
- The second relies essentially on the in-house XFE-code Morfeo used in standalone, i.e. the entire fracture mechanics problem is solved by a unique XFE-code.

2. Applications

2.1 Crack in a Compressor drum of an airplane engine subjected to centrifugal forces

In the first application, the S-FE/XFE method is applied to a static crack analysis in a section of a compressor drum of a turbofan engine. The results obtained with the S-FE/XFE method are compared with those obtained with a standard FE computation. The standard FE problem is also solved with the FETI method using the same decomposition as the S-FE/XFE problem in order to assess the influence of the XFEM on the solver behaviour. The influence of mesh refinement at the crack front is studied. The initial mesh is chosen so as to ensure convergence of the strain energy for the non-cracked standard FE problem.

The computation sequences for the standard FE and the S-FE/XFE crack problems are explained hereafter.

- Crack definition. A through-the-thickness crack is inserted into the model at the centre hole of the structure. The direction of the crack is radial. There are two different methods for introducing the crack depending on the method used:
  - standard FEM: the crack is introduced into the CAD by inserting a closed surface leading to the renumbering of all the geometrical entities of the CAD (depending of the software used for generating the CAD and the mesh). The related data (materials, boundary conditions,...) must be updated due to the modification of the CAD;
  - S-FE/XFE method: the crack is introduced by means of its “level sets". The CAD is not modified and the previous dataset can be used without any changes.
- Mesh generation:
  - In the standard FEM case, the mesh generation is decomposed in two steps: first, the mesh is generated on the whole domain and the crack surface is also meshed; second, the nodes on the crack surface are duplicated and the mesh is split.
  - For the S-FE/XFE method, the mesh is arbitrarily generated on the whole domain.

From the user point of view, the mesh generation works like a black-box and the operations are exactly the same in the two cases. Nevertheless, the time spent during this step is lower in the case of the S-FE/XFE method. Indeed, the crack surface does not need to be meshed and the nodes are not duplicated and split. The mesh has been refined at the crack front in both cases in order to improve the accuracy of the solution at the crack front.

- Elementary stiffness generation. The elementary stiffness generation for the S-FE/XFE method is made with both the FE-software and the XFE-code. The time spent during this step depends on the size of the XFE-domain. For the standard FE problem, the elementary stiffness matrices are created only by the FE-software.

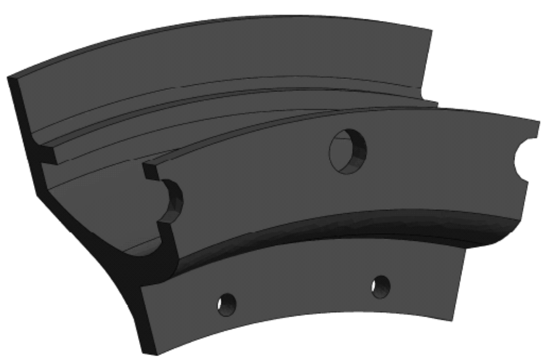

Figure 1 : Section of the drum studied

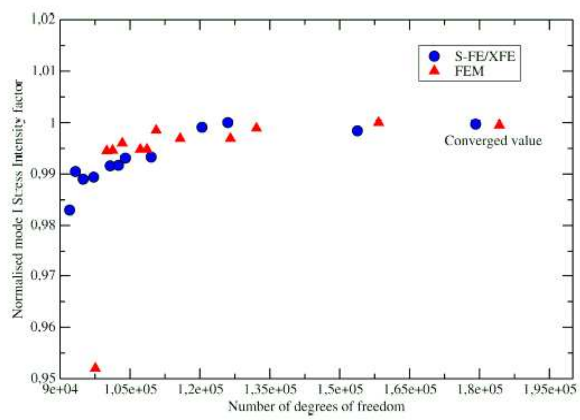

Figure 2: Comparison of Mode I SIFs computed with the S-FE/XFE method and a standard FE method

Figure 2 shows the variation of the average mode I stress intensity factors K_I as a function of the number of degrees of freedom. These values are obtained for various level of mesh discretisation along the crack front. The accuracy of the stress intensity factors is obviously improved when increasing the number of degrees of freedom. The first point corresponds to the original mesh size. With the original mesh, the error calculated from the difference to the converged solution (i.e. number of degrees of freedom > $1.8\ 10^5$) is equal to only 1.7% with the S-FE/XFE while it is equal to 4.8% with the standard FE method.

2.2 Crack propagation at the oil port of an hydraulic cylinder

The second application is part of a project financed by the European Commission (PROHIPP) that aims to improve the design and manufacturing of high-pressure fluid products, and in particular hydraulic cylinders. This kind of mechanical components is typically subject to a cyclic loading. One aim of the project is then to predict the total number of cycles of the service life, knowing the amplitude of the cyclic loading.

We consider the hydraulic cylinder illustrated in Figure 3. Its end cap is fixed and an internal pressure is applied. The maximum stress in an uncracked cylinder is reached at the inner radius of the cylinder at stress concentration points at the holes, by which the oil enters and exits the cylinder. These weakest points are indicated in Figure 4 on a zoom on one of the oil ports. These points are located in the symmetry plane of the piece. Cracks may initiate at these points and propagate inside the mirror plane towards the outer radius and thus lead to the ruin of the cylinder. This is observed experimentally. We simulate this propagation by inserting quarter-circular corner cracks centered on the weakest points of growing radius. The aim of the present work is simply to compute the SIFs along the front for the different radii. The SIFs values are not used to determine the growth rate along the front and the cracks are assumed to remain quarter-circular. The relevant dimensions of the cylinder in this study are the wall thickness t = 5mm and the inner radius ri = 35mm. The Young modulus is 210GPa and the Poisson ratio is 0.3. We compute the stress and the SIFs when an inner pressure 10MPa is applied. The results are shown in Figures 15 and 16. The first one is the equivalent stress on the skin of the half-cylinder when the radius of the two cracks is 6 mm. The second one is the SIF along the two fronts for several radii.

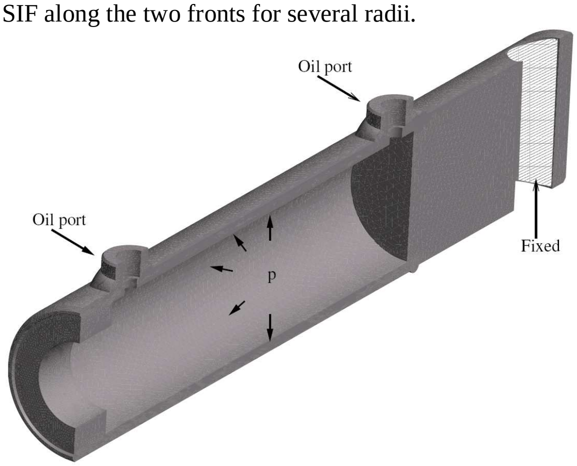

Figure 3: One half of a hydraulic cylinder

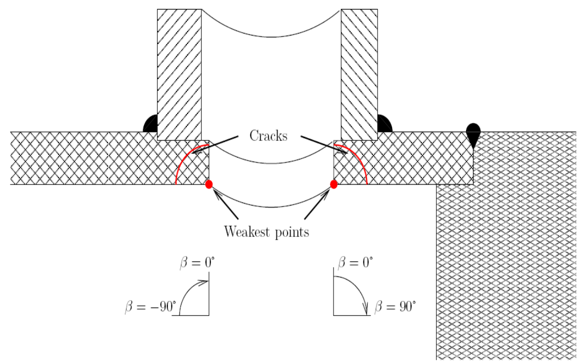

Figure 4: Two quarter-circular corner cracks at the oil port of a hydraulic cylinder

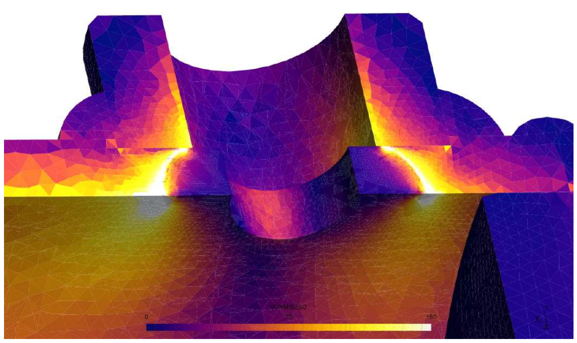

Figure 5: Von Mises stress (MPa) in a hydraulic cylinder with two cracks of radius 6 mm

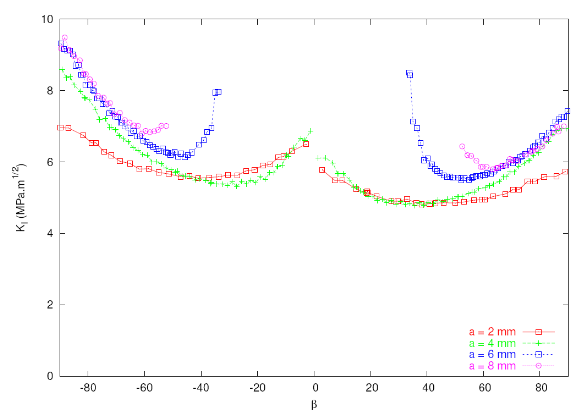

Figure 6: SIF along the front of the pair of cracks in a hydraulic cylinder

The main finding of this study is that the configuration correction factor is a decreasing function of the crack radius. This suggests that a crack may propagate at a constant rate under fatigue loading conditions. The decrease in the configuration factor can be explained by the fact that the crack is propagating towards the welding that connects the oil port to the cylinder. Indeed, when the crack propagates, a fraction of the hoop stress is sustained by the welding and the oil port. The number of load cycles needed for crack propagation until oil leakage was found to be significant compared to the total life of the cylinder, meaning that such a geometry can sustain the presence of defects at the oil port.

2.3 Non-through crack in a thin-walled pressure membrane

The third application is a large panel that is a part of an aircraft component separating a pressurised zone and a non-pressurised zone. Under fatigue loading conditions, this type of structure could show multiple arc-circle cracks initiating at the upper fibre of the membrane, close to the flange. The understanding of the initiation and the propagation of the crack is of the utmost importance in order to assess the durability of the structure. In this context, a static crack analysis is performed for an arc-circle crack starting from the upper fibre of the membrane. The position of the crack has been chosen after a stress analysis on the simplified un-cracked geometry. The position of the centre of the crack is defined as the locus of the maximum positive tensile principal stress. The crack zone can be seen on Figure 7.

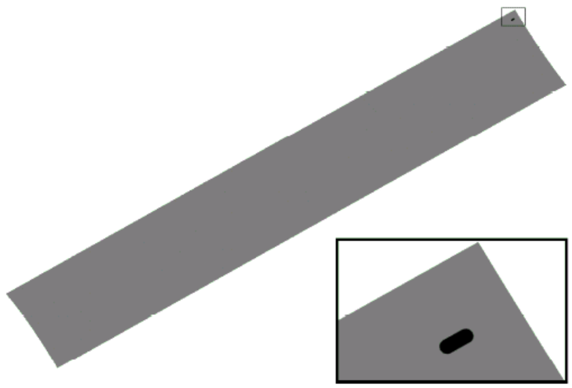

Figure 7: Representation of the FE-domain (light grey) and the XFE-domains (black) on the whole two-dimensional shell domain, the XFE-domain is small in comparison with the FE-domain.

The geometry can be roughly divided into three parts (see Figure 8): the flanges, the contour region, the inner region. Figure 8 shows the variation of the thickness for the different regions. It can be seen that the flanges are thicker than the other parts and the inner region is thinner than the contour region. The “generic panel" is made of aluminium and the whole domain is modelled with shell elements. The boundary conditions are defined as follow: the flange displacement is fixed to zero and a pressure force is applied on the whole structure, see Figure 8. The presence of rivets and contact with other parts of the aircraft structure is not accounted for. Moreover, the model is linear elastic with a small displacement assumption.

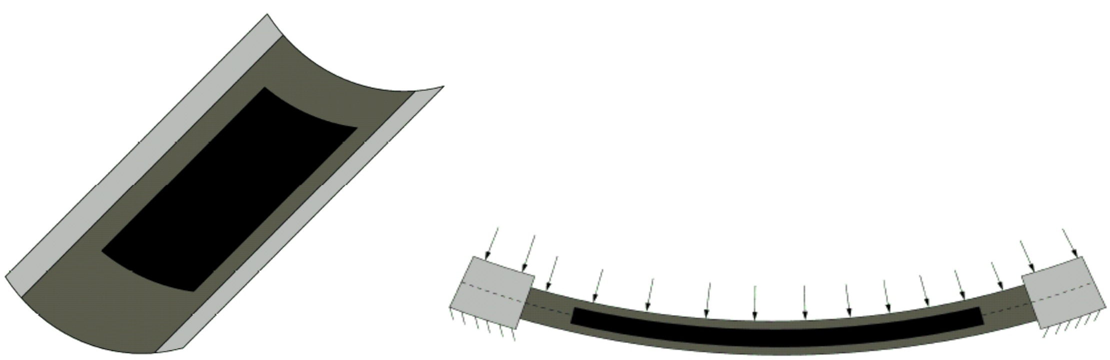

Figure 8: Geometrical description of the “generic panel" decomposed into the flanges (in light grey), the contour region (in dark grey) and the inner region (in black). The panel is subjected to pressure and the flanges are fixed.

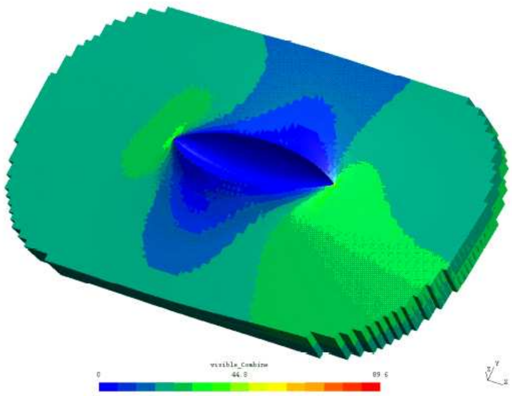

Figure 9: Von Mises stress on the deformed shape for an arc-circle crack in the three-dimensional XFE-domain.

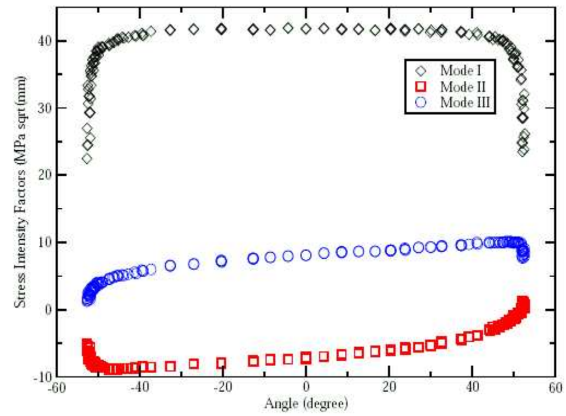

Figure 10: Stress intensity factors along the front for an arc-circle crack in the “generic panel".

Figure 9 shows the Von Mises stress on the three-dimensional XFE-domain and the stress intensity factors for the three modes are shown on Figure 10. The dominant mode is obviously the mode I since the crack is almost perpendicular to the principal stress. The stress intensity factors show that the crack will propagate more rapidly inside the thickness than along the free-surface because the highest stress intensity factors values are obtained in that direction. Furthermore the resistance to fracture along the free-surface will also be larger due to plane stress effects on the fracture mechanism.

2.4 Crack propagation in a blade

A blade with two cracks is considered: one crack is located at the leading edge side and the other at the trailing edge side. The blade is loaded with an external pressure and a temperature gradient. Periodic boundary conditions are enforced. A cyclic pressure is superimposed, under which the propagation of the two cracks occurs.

The cracks start from the point with the highest tensile stress at both edges. The initial length of both cracks is 0.5 mm. An initial iterative step is performed in order to determine the initial direction of the crack as the direction that maximizes the crack opening mode and minimizes the sliding and tearing modes. Then, after each step, the crack grows according to the Paris’ law in the direction given by the maximum principal stress criterion.

For this application, no CAD model is available and the crack analysis is performed starting from a volume mesh (on which the temperature is given) of the component. A fine mesh suitable for the XFEM analysis is obtained by dividing the elements according to the distance to the region where the crack propagation is expected to occur (based on a coarse preliminary analysis). The temperature is interpolated between the initial and the final mesh. Once this mesh is obtained, no remeshing operation is necessary: all the steps are performed on the same mesh with the cracks going through the elements. The fine mesh on the boundary can be seen on Figures 11 and 12.

These figures also show the crack surface – in red inside the blade (only some surfaces are shown in order to see inside the blade). Figures 13 and 14 show the effective stress on the deformed geometry (magnified 10 times) at the last step of the simulation. With the considered cyclic boundary conditions, it is the crack at the trailing edge that propagates faster than the other. It can be seen on Figure 12 that the crack at the trailing edge propagates significantly outside of its initial plane.

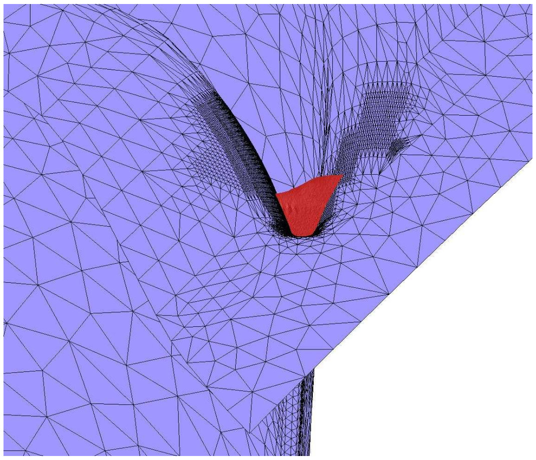

Figure 13: Crack surface at the blade leading edge

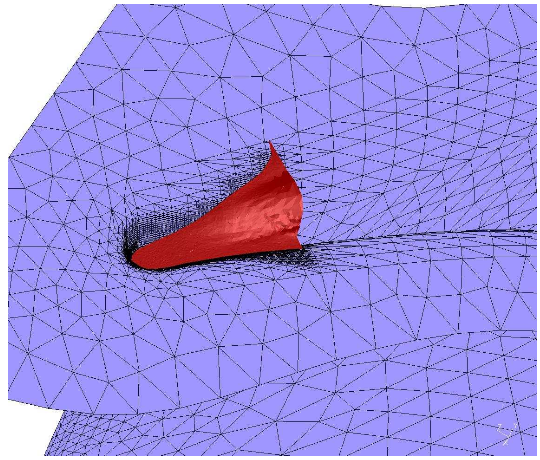

Figure 14: Crack surface at the blade trailing edge

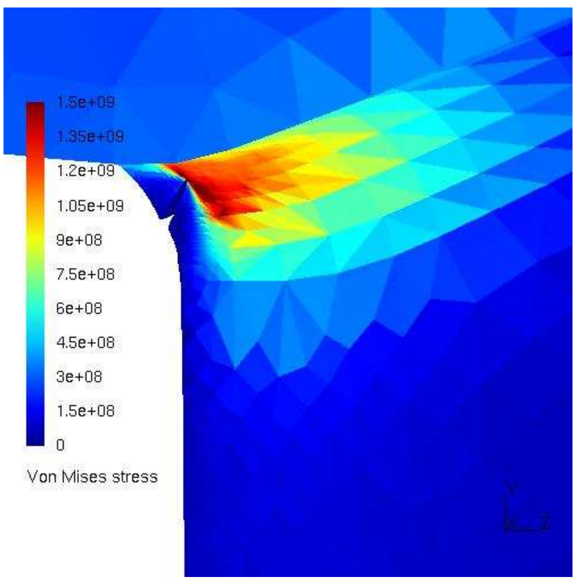

Figure 11: Von Mises stress around the crack at the blade leading edge

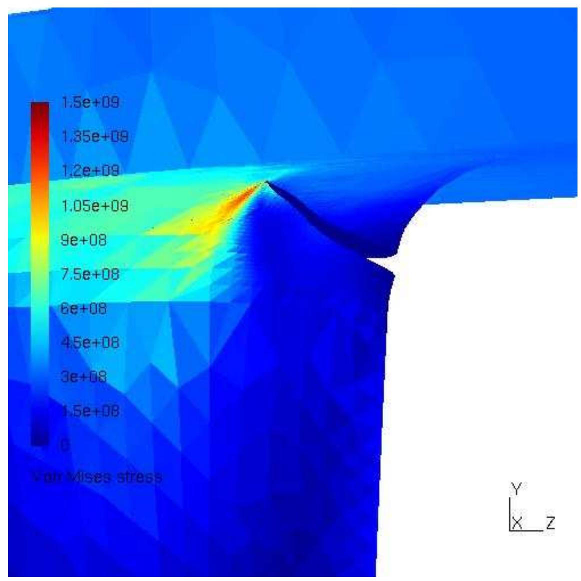

Figure 12: Von Mises stress around the crack at the blade trailing edge

3. Conclusions

Several industrial applications of the XFEM for three-dimensional crack propagation analysis were presented. They show that XFEM has become a very efficient tool for the assessment of the structural integrity of structures and engine components, from the computation of stress intensity factors as a supplement to standard semi-analytical crack analyses, to the study of complex non planar 3D crack propagation and computation of the number of cycles.

Acknowledgements

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