A Simplified Model for the Analysis of Thermal Stratification in Cooling Channels

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Abstract

The analysis of the flow in the cooling channels of liquid rocket engine thrust chambers is of paramount importance in their structural thermal design. The complete information can only be obtained by costly experiments or by complex numerical simulations, because of the three-dimensional shape of channels and of the coupling among flow evolution in hot-gas and coolant sides and wall heat transfer. Moreover, the coolant is often a supercritical fluid, which needs its appropriate equation of state. A simplified approach is developed in the present paper which, relying on empirical relationships, is able to study the thermal stratification in both the coolant flow and the cooling channel structures.

1. Introduction

An essential part of the design and realization of liquid rocket engines is the thermal analysis, which is necessary to predict the peak heat flux from the combustion gases to the engine wall and to ensure the structural integrity of the combustion chamber. The need for thermal analysis is especially important in reusable engines, where an effective and efficient cooling system is crucial to extend the engine life, or in expander cycle engines, where coolant warming provides the available power for turbo-machinery. In these cases, usually regenerative cooling is considered, where one of the propellants, typically the fuel (liquid hydrogen, RP1, liquid methane, ...), is forced through passages that are machined inside the thrust chamber wall (flowing in the opposite direction of the rocket main flow), then the heated fuel is injected into the chamber or goes to turbine. Regenerative cooling technique provides: no energy loss (thermal energy is absorbed by the coolant and returned to the injector); no change in wall contour as a function of time; indefinite firing duration; and relatively light-weight construction. However, it has to be kept in mind that it permits only limited throttling with most coolants, has reduced reliability with some coolants (e.g., hydrazine) and requires increased pump power because of the large pressure drop at high heat-flux levels. The trade-off among these aspects makes regenerative cooling interesting for large high-pressure, high heat-flux thrust chambers and for expander (and expander-bleed) cycle engines. Thermal analysis of regeneratively cooled engines is therefore essential to predict not only wall temperature but also coolant temperature and pressure at the channel exit. Moreover, thermal analysis becomes more important if the goal is to search for possible modifications of the cooling channel configuration, providing optimum cooling at high temperature areas rather than under-cooling, that would result in the catastrophic failure of the engine, or over-cooling, that would cause performance-losses because of the need for a bigger coolant pump. An example of such modifications is that of high-aspect-ratio cooling-channels (HARCC).[1,2] In this case the wall temperature on the hot side is reduced increasing the coolant side surface area (relative to the hot gas side surface) by the use of extended surfaces or "fins" (Fig. 1). Increasing the number of passages, and therefore the surface area of the passages that circumferentially line the outer wall of a combustion chamber, necessarily increases their aspect ratio. In turn, the material between them, known as rib, functionally becomes a fin.

To the goal of designing more and more efficient thrust chambers, cooling channel configurations should be studied by optimization methods which will include suitable models for thermal analysis. Unfortunately, a comprehensive thermal model aiming to estimate wall temperature and coolant pressure drop is rather complex, because it must account for different phenomena coupled with each other: convection from hot gases to the wall, conduction within the wall, and convection form the wall to the cold fluid. The coupling of these processes is strongly non-linear because coolant and hot-gas heat transfer depend on the fluid pressure and temperature and on wall temperature. A further complication is that the flow in the cooling channels is strongly three-dimensional.[2–5] Moreover, the coolant is often a supercritical fluid and methods based on the assumption of perfect gas or perfect liquid cannot be used. Finally, it has to be considered that the optimization process consists of many calculation loops which include the thermal and fluid

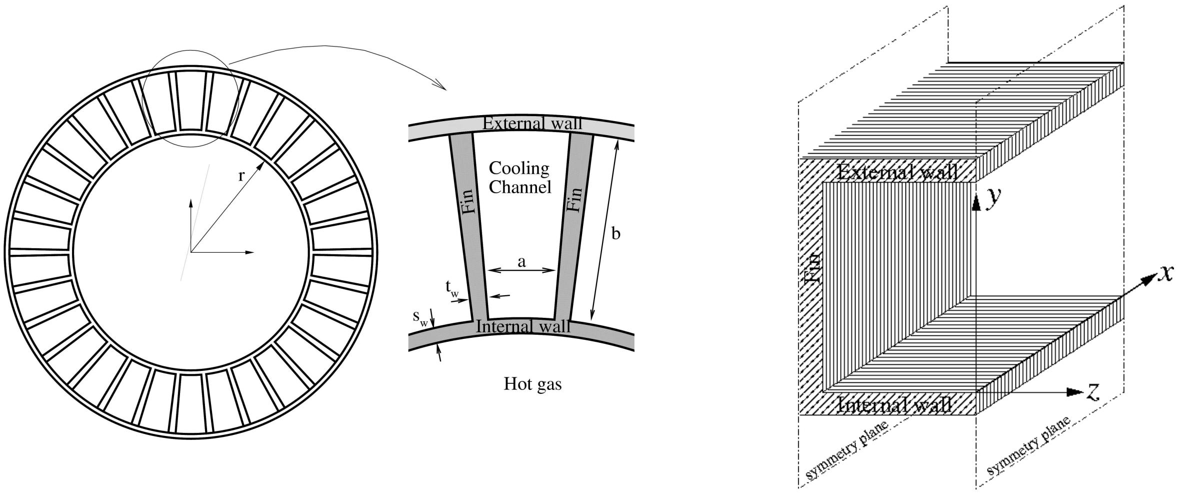

Figure 1: Schematic of cooling channels geometry.

mechanics analysis of the coolant flow in the cooling channels as well as the thermal analysis of the wall structure.[6] Because of all these reasons, the solution of the 3D Navier Stokes equations coupled with the wall thermal analysis, which would be necessary to describe the regenerative cooling system, is not suitable. In fact, its computational cost is too high to be part of an optimization process or to understand the role of the main parameters that affect the cooling system.[2] For that reason one-dimensional models heavily relying on empirical relationships have been widely used.[7] With these methods the complexity of the cooling system can be faced and the main parameters that affect the problem are well described. One of the main drawbacks of conventional one-dimensional calculation methods is that an ideal mixing of the thermal energy into the coolant channel cross section is assumed. This implies that when a significant radial thermal stratification takes place, like in the case of HARCC, a significant error arises.[3] The objective of this study is to overcome the above limitation of simplified approaches by developing a computational tool able to describe the coupled hot-gas/wall/coolant environment that occurs in most liquid rocket engines and to provide a quick and reliable prediction of thermal stratification phenomena in cooling channels. This approach, which is an extension of that presented by Woschnak and Oschwald,[2] is still widely relying on empirical relationships. Nevertheless, it allows to compute the radial stratification of both the wall and the coolant flow temperatures. This result is obtained by considering the one-dimensional steady-state evolution of the hot gas flow, and a “quasi 2-D” flow evolution through the cooling channels. The approach is developed for any fluid evolving through cooling channels, by considering any equation of state, and thus compressible gas, supercritical fluid and liquids can be considered as coolants.

2. Physical and mathematical modeling

Heat transfer in a regeneratively cooled thrust chamber can be described as the heat flux between two moving fluids, separated by a solid wall. In its simplest form regenerative cooling can be modeled as a steady heat flux from a hot gas through a solid wall to a cold fluid. This problem can be divided up into three sub-problems, which are defined as follows:

- The turbulent chemically reacting flow of a mixture of gases in a rocket engine, including combustion chamber and converging-diverging nozzle.
- The heat conduction through the wall of the rocket engine between the hot gases and the liquid coolant.
- The turbulent flow of the coolant in the channels surrounding the rocket engine.

These subproblems are coupled by the two steady-state balances of three heat fluxes: from hot-gases to the wall; through the wall; and from the wall to the coolant.

2.1 Hot-gas expansion

The hot-gas flow has been formulated on the basis of a one-dimensional isentropic expansion with chemical reactions. Its thermodynamic and transport properties are evaluated using the software CEA.[8–10] Combustion conditions are obtained with the assumption of chemical equilibrium of the combustion products. The hot-gas expansion is then calculated assuming chemical equilibrium or frozen composition (freezing point at chamber or at throat conditions). The expansion of the hot gases is considered independent of the wall temperature, because the heat transfer from the gases to the wall causes very little change in the gas temperature. The hot-gas heat fluxes are then evaluated using the correlation proposed by Bartz[11] accounting for properties variation across the boundary layer:

$$q_{hg} = h_{w,hg} \cdot \left(T_{aw,hg} - T_{w,hg}\right) \tag{1}$$

where q_{hg} is the wall heat flux from the hot-gases to the wall, $T_{w,hg}$ is the wall temperature (hot-gases side), $T_{aw,hg}$ is the adiabatic wall temperature and $h_{w,hg}$ is the Bartz heat transfer coefficient:

$$h_{w,hg} = \left[\frac{0.026}{D_*^{0.2}}\left(\frac{\mu^{0.2} c_p}{Pr^{0.6}}\right)\left(\frac{p_c}{c^*}\right)^{0.8}\right] \cdot \left(\frac{A_*}{A}\right)^{0.9} \cdot \sigma \tag{2}$$

where μ, c_p and Pr are the viscosity, the specific heat and the Prandtl number of the combustion gases, respectively, evaluated at the chamber conditions, p_c is the chamber pressure, c^* is the characteristic exhaust velocity, D^* is the nozzle diameter at the throat, A_*/A is the nozzle area ratio at the actual axial position and σ is a factor which contains the correction for property variations across the boundary layer. Note that, as the hot-gases parameters are estimated via the one-dimensional isentropic expansion law, at each axial position the wall heat flux q_{hg} is only a function of the unknown temperature $T_{w,hg}$.

2.2 Heat conduction through the wall

Heat is transferred from the hot-gas to the coolant via the solid wall, made of internal wall, fins and external wall (Fig. 1). If steady-state operation is assumed, the heat flux entering the internal wall q_{hg} must be equal to that leaving it, and a simple steady-state wall heat transfer balance can be written:

$$q_{hg} = \frac{k_w}{s_w} \cdot \left(T_{w,hg} - T_{w,co}\right) \tag{3}$$

where k_w and s_w are the wall thermal conductivity and thickness, respectively, and $T_{w,co}$ is the coolant-side wall temperature. Note that a one-dimensional radial heat transfer through the internal wall of thickness s_w has been considered. Then, the heat transfer balance through the fins is computed by assuming again steady-state operation:

$$\frac{\partial}{\partial y}\left(k_w t_w \frac{\partial T_w}{\partial y}\right) = 2 q_w \tag{4}$$

where y is the radial direction, t_w is the fin thickness, T_w is the wall temperature and q_w is the heat flux from the fin to the coolant. This equation assumes one-dimensional heat transfer in the radial direction, a non uniform fluid temperature, a fin thickness that is much smaller than its axial length, and infinitely tall fin. For an actual "fin" in this type of cooling channel the infinite height assumption is approximately valid because the tip is nearly adiabatic in most cases. The boundary conditions at the bottom ($y = 0$) of the fin is, according to (3):

$$q_{hg} = -k_w \left.\frac{\partial T_w}{\partial y}\right|_{y=0} \tag{5}$$

which means that the radial heat flux entering the fin balances with that entering the wall from the hot-gas side (q_{hg}). At the top ($y = b$) of the fin the boundary condition is:

$$0 = -k_w \left.\frac{\partial T_w}{\partial y}\right|_{y=b} \tag{6}$$

which is the adiabatic condition. Finally, the external wall is assumed adiabatic.

2.3 Coolant flow

The cooling channel flow model is developed by using the steady-state conservation laws of mass, momentum, and energy, taking into account the effects of heat transfer and friction. As mentioned above a “quasi 2-D” flow model is assumed for the coolant flow. This model considers a one-dimensional evolution for the velocity $w = w(x)$ (the only component of velocity considered is the axial one) and the pressure $p = p(x)$, whereas temperature is left to vary also in radial direction: $T = T(x, y)$. The other thermodynamic variables are obtained by suitable equations of state (EOS), which are written in the general form:

$$p = F_p(\rho, T) \quad \text{and} \quad h = F_h(\rho, T) \tag{7}$$

The coolant flow governing equations are thus written on the basis of the above model.

2.3.1 Coolant Mass Equation

The steady-state integral mass conservation equation through the channel cross sections is:

$$\frac{d}{dx} \iint_A \rho w dA = 0 \tag{8}$$

where ρ is the coolant density, A is the cross section area and x is the axial direction. Considering the “quasi 2-D” flow model the mass conservation becomes:

$$\overline{\rho} \cdot w \cdot A = \dot{m} \tag{9}$$

where

$$\overline{\rho} = \frac{1}{A} \iint_A \rho dA \tag{10}$$

is the average coolant density through the channel cross section and $\dot{m}$ is the mass flow rate.

2.3.2 Coolant Momentum Equation

The steady-state integral momentum equation through the channel cross sections is:

$$\frac{d}{dx} \left[\iint_A \left(\rho w^2 + p\right) dA \right] dx - p \frac{dA}{dx} dx = \iint_{S_w} \tau_w dS_w \tag{11}$$

where the left part of the equation represents the momentum flux and the axial component of the pressure force acting on the lateral surface of the channel, while the right part is the integral skin friction force acting on the lateral surface (S_w). Considering the mass equation (9-10) and the “quasi 2-D” flow model (p and w are uniform through the channel cross section), the momentum equation becomes:

$$\left(\dot{m} \frac{dw}{dx} + A \frac{dp}{dx}\right) dx = \iint_{S_w} \tau_w dS \tag{12}$$

where the shear stress τ_w can be related to flow variables by the skin friction factor f_w:

$$\tau_w = \frac{1}{8} \overline{\rho} w^2 f_w \tag{13}$$

The skin factor f_w is estimated using a proper empirical correlation.

2.3.3 Coolant Energy Equation

The steady-state integral energy equation through the channel cross sections is:

$$\frac{d}{dx} \left[\iint_A (\rho w h_0) dA \right] dx = \iint_{S_w} q_w dS_w \tag{14}$$

where h_0 is the total enthalpy ($h_0 = h + w^2/2$), and q_w is the heat flux entering in the coolant through the wall S_w. This is the equation used in the one-dimensional approach. In the present approach, as the temperature is left to vary in the radial direction, some equation suitable to the evaluation of T must be found. As thermal stratification depends on the heat flux through the fluid in radial direction and on the heat flux exchanged with the channel walls, the hypothesis is made of splitting the height of the channel in tiny slices of height dy, all having, at the same abscissa x, the same values of w and p. To solve for $T(y)$, the balance equation (14) has to be written for a slice of height dy rather than for the whole channel height (see Fig. 2):

$$\frac{d}{dx}\Big[\rho w h_0 a(y) dy\Big] dx = 2q_w(y) dy dx + q_c(y) a(y) dx - q_c(y+dy) a(y) dx \qquad (15)$$

where only the dependency of variables on y has been emphasized, because all variables depend on x. The equation

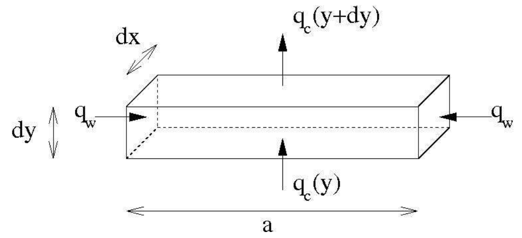

Figure 2: Heat fluxes in a slice of cooling channel of width a and height dy.

(15) becomes a differential equation for T if it is possible to express $q_w(y)$ and $q_c(y)$ as a function of $T(y)$. As regards to $q_c(y)$, this can be made according to Kacynski.[5] If it is assumed that $q_c(y)$ is due to the turbulent mixing:

$$q_c(y) = -k_t \frac{\partial T}{\partial y} \qquad (16)$$

where k_t is the average turbulent conductivity in the radial direction, which can be obtained as a function of the Reynolds number Re and of the fluid thermal conductivity k. For instance in case of hydrogen k_t can be expressed as:

$$\frac{k_t}{k} = 0.008 \cdot Re^{0.9} \qquad (17)$$

With this hypothesis (15) becomes:

$$\frac{\partial}{\partial x}(\rho h_0 w a) = \frac{\partial}{\partial y}\left(k_t a \frac{\partial T}{\partial y}\right) + 2q_w \qquad (18)$$

Finally the wall heat flux can be related to the coolant and wall temperature using a transfer coefficient form:

$$q_w = h_w \cdot (T_w - T) \qquad (19)$$

where h_w is the heat transfer coefficient and T_w is the wall temperature. The coefficient h_w is estimated using a proper empirical correlation. The boundary conditions at the bottom and at the top of the cooling channel are:

$$q_{hg} = -k_t \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad \text{and} \qquad 0 = -k_t \left.\frac{\partial T}{\partial y}\right|_{y=b} \qquad (20)$$

which are the same as those (5-6) used for the fin and are therefore consistent with the hypothesis of axisymmetric temperature distribution on the internal and external walls. The conditions (20) state that the heat flux q_{hg} enters at the channel bottom and that the channel top is adiabatic.

3. Computational Strategy

The governing equations can be discretized considering a 2D grid: M nodes ($j = 1, ..., M$) for the axial discretization and N nodes ($i = 1, ..., N$) for the radial discretization. The computations proceed starting from the entrance of the coolant and moving along the axial direction. The solution at each axial position is computed from that at the previous one. To simplify the calculations, the empirical coefficients f_w and h_w are evaluated at the previous axial position. This is a minor hypothesis since the variation of the empirical coefficients between contiguous axial positions is negligible. Moreover, the EOS equation has been linearized aroud the actual value of the density ρ and temperature T. Using this hypothesis the governing equations are written for each value of j, assuming known the solution at the previous axial position ($j-1$, or the channel inlet condition). The overall system of equations can be divided into two groups:

- 3N linear equations: coolant energy (18), fin (4), EOS (7) with respect to the variables T_i, $T_{w,i}$ and ρ_i ($i = 1, ..., N$);

- 3 non-linear equations: coolant mass (9) and momentum (12), wall balance (3), with respect to the variables w, p, and $T_{w,hg}$.

To solve the system of equations, the following computation strategy is used at each axial station:

1. A first tentative value for w, p, $T_{w,hg}$ is chosen: these values are taken from the previous axial station;
2. The 3N linear equations system, considering having w, p, $T_{w,hg}$ as parameters, is solved for T_i, $T_i w$, i, ρ_i;
3. The 3 non-linear equations system is solved for a new value of w, p, $T_{w,hg}$, considering T_i, $T_{w,i}$, ρ_i as parameter;
4. The new value of w, p, $T_{w,hg}$ is used for step 2 and the procedure is repeated until these values remain unchanged.

4. Validation and results

The validation of the described numerical tool is made with respect to the test cases presented by Le Bail and Popp,[4] where the coolant flow in the regenerative channels is computed using a numerical solver for the parabolized Navier Stokes equations. This is one of the few papers in the literature in which some data of a regeneratively cooled engine have been published. The test cases address the regenerative cooling of the thrust chamber of Vulcain engine, with two different channel geometries. The main properties of the flow and the main features of the nozzle and cooling channels are reported in Table 1, whereas more details can be found in the reference paper.[4] The coolant flows in the opposite

propellants	LO2-LH2
chamber pressure	100 bar
chamber mixture ratio	$O/F = 5.9$
coolant	H2
inlet coolant temperature	48.7 K
inlet coolant pressure	137.9 bar
maximum channel aspect ratio for case A	AR_{max} =8.5
maximum channel aspect ratio for case B	AR_{max} =7
wall roughness	ϵ =5μm

Table 1: Data for the test case of Le Bail and Popp.[4]

direction with respect to hot gases and cooling channels are divided into three sections of constant height (width varies according to the nozzle radius). Test case B is different from test case A only because channels have 15% lower height.

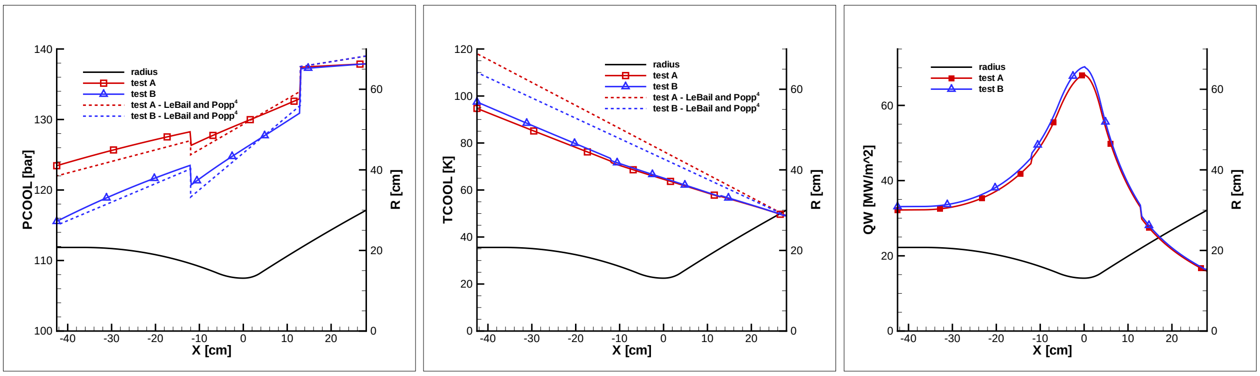

Figure 3: Coolant pressure (left), coolant temperature (center) and wall heat flux (right) for tests A and B.

The computations of test cases A and B carried out with the present model have been obtained by including classical correlations for the skin friction factor (f_w) and heat transfer coefficient (h_w). In particular, Petukhov's correction of Colebrook equation is used for the skin friction factor[12] and Bhatti-Shah expression, again with Petukhov's correction accounting for the variable temperature across the channel section, is used for the heat transfer coefficient.[12,13] The results obtained with the present model for test cases A and B are displayed in Fig. 3 which, from left to right, shows:

coolant pressure, coolant average temperature and wall heat flux, respectively. The behavior of coolant pressure shows a good agreement with the reference results in both test cases. Note that the pressure loss is 14 bar for test A and 22 bar for test B. The behavior of coolant average temperature shows a larger discrepancy. This is due to the different input data: LeBail and Popp[4] used the wall heat flux as an input while in the present computations wall heat flux is an output obtained via the wall energy balance. The wall heat transfer imposed in the reference 3D-computations has a peak of 60 MW/m^2 at the throat while a a value of 70 MW/m^2 has been obtained here. This heat flux mismatch leads to a difference of 20% in coolant exit temperature between present and published data.[4]

Besides to the average evolution of variables along the cooling channels the present models provides the prediction of thermal stratification of coolant and fin. The results obtained for test case A are shown in Fig. 4-5. In particular, Fig. 4 shows the evolution, along the channel of thermal stratification. It can be noticed that a significant stratification takes place in the present test case, especially at the channel exit. An example of cooling channel and fin width and thermal stratification is shown in Fig. 5. The solution are relevant to channel throat and exit.

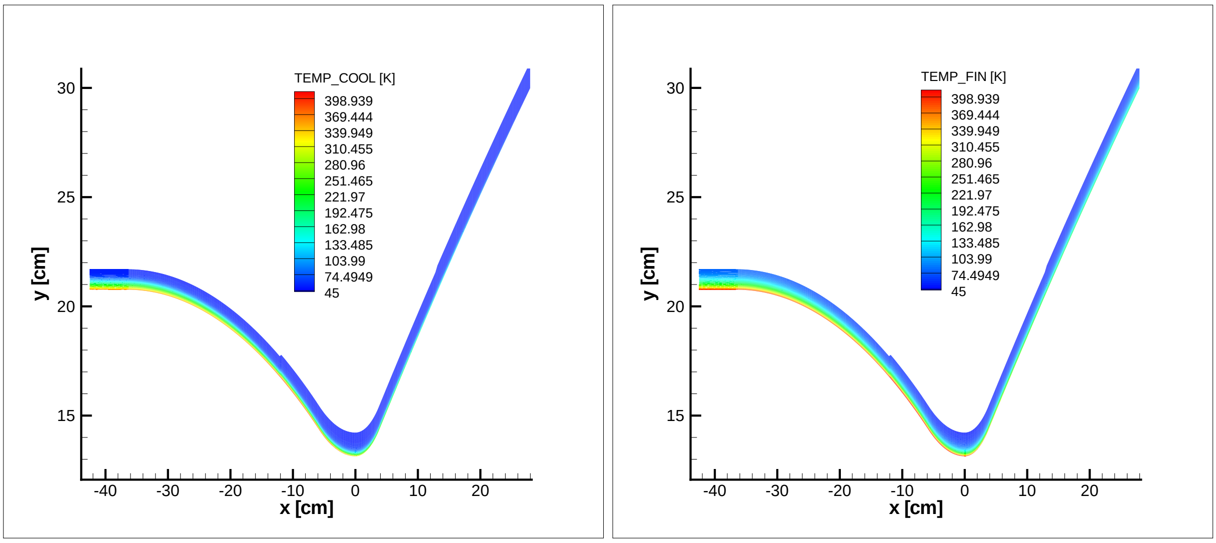

Figure 4: Coolant (left) and fin (right) temperature stratification for test case A.

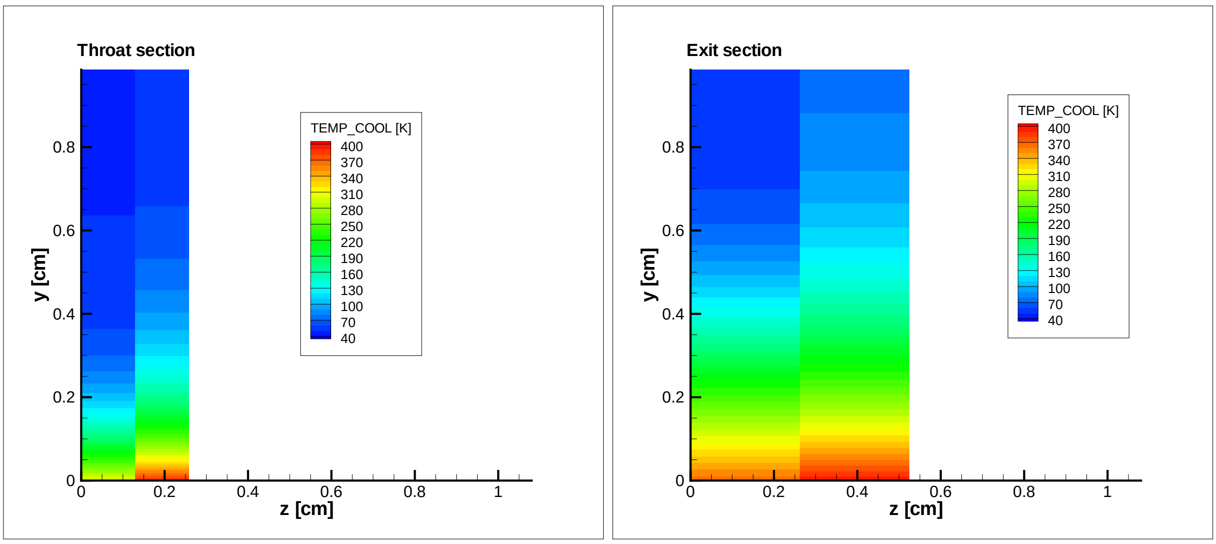

Figure 5: Coolant and fin temperature stratification at throat (left) and exit (right) section for test case A.

5. Conclusions

A simplified model for the analysis of thermal stratification in cooling channels has been developed and, cue to the lack of literature data for regenerative cooling systems, only partially validated. The model allows to evaluate the thermal stratification both of the cooling channel fins and of the coolant flow. The results show reasonable agreement with data published in the literature. In fact, it has to be considered that input data are slightly different and that the model relies on empirical relationships for skin friction factor, heat transfer coefficient, and turbulent conductivity. The accuracy of the predictions is strongly dependent on the accuracy of these relationships. A possible way to improve the knowledge of this relation and to use correctly the model presented in this paper is to validate empirical models on simple channel flows by fully 3D Navier-Stokes simulations and then use the model for the full length of cooling channels and/or in optimization programs.

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