Generalized Reynolds Number for Non-Newtonian Fluids

K. Madlener, B. Frey and H.K. Ciezki*
DLR - German Aerospace Center
Institute of Space Propulsion
Lampoldshausen, Langer Grund, D-74239 Hardthausen, Germany

Abstract

An extended version of the generalized Reynolds number was derived to characterize the pipe flow of non-Newtonian gelled fluids of the Herschel-Bulkley-Extended type (HBE-type). This number allows also to estimate the transition from laminar to turbulent flow conditions. An experimental investigation was conducted with a capillary rheometer for several non-Newtonian gelled fluids to evaluate the introduced HBE-generalized Reynolds number $Re_{gen\,HBE}$. A good correlation between the experimental results and the theory could be found for laminar flow conditions. For one of the examined gelled fuels the necessary high Reynolds numbers could be realised so that the transition from laminar to turbulent flow could be measured. Because of its general description, the HBE-generalized Reynolds number can also be applied to Newtonian liquids as well as to non-Newtonian fluids of the Herschel-Bulkley, Ostwald-de-Waele (power-law) and Bingham type.

1. Introduction

Gelled fuels and propellants are shear thinning non-Newtonian fluids with a significant different viscosity behaviour compared to Newtonian liquids. Due to their safety and performance benefits they are interesting candidates for rocket propulsion systems, see e.g. Ciezki & Natan.[4] During storage and transport, where very low shear forces occur, their viscosity is very high so that they are often described as semi-solid. During the feeding process from the tank through the pipes to the injector unit in a combustion chamber high shear forces are applied to the fluids and their viscosity can be decreased up to its liquefaction. Thus gelled propellants offer the possibility to build enginges with trust variation up to thrust cut-off and re-ignition like in engines with liquid propellants. At the same time they have the similar simple handling and storage characteristics like enginges with solid propellants. Concerning the understanding of the pipe flow characteristic of such non-Newtonian gelled fluids there are still gaps to close. The present paper shall offer a further small step for a better understanding of the flow characteristics by defining a HBE-generalized Reynolds number.

To characterize or to compare the flow characteristics of fluids flowing through pipes, dimensionless numbers are often used. Osborne Reynolds first introduced in 1883 what is today known as the Reynolds number for fully developed pipe flow of Newtonian liquids. The definition of the Newtonian Reynolds number Re_{Newton} is shown in Equation 1, whereas ρ is the density of the fluid, D the pipe diameter, $\bar{u}$ the average flow velocity and η_{Newton} the constant Newtonian viscosity.

$$Re_{Newton} = \frac{\rho \cdot D \cdot \bar{u}}{\eta_{Newton}} \tag{1}$$

The Reynolds number can be interpreted as the ratio of inertial forces to viscous forces. It is used to identify different flow regimes such as laminar or turbulent flow. Furthermore it is used as a criterion for dynamic similitude, that means, if two different flow constellations (different pipe diameter, different flow rate or different fluid properties) have the same dimensionless numbers, they are dynamically similar. As already mentioned, the Reynolds number Re_{Newton} of Equation 1 is only valid for fluids with a constant viscosity. The gelled fluids investigated in this present publication, however, are non-Newtonian fluids with a more complex viscosity characteristic compared to Newtonian liquids. The diagrams in Figure 1 show an example of the shear rate dependent shear stress $\tau(\dot{\gamma})$, indicated with triangles, and shear

*Student from University of Freiburg, Eckerstr.1, 79104 Freiburg, Germany rate dependent viscosity $\eta(\dot{\gamma})$, indicated with circles, for one of the investigated non-Newtonian fluids. The gel based on liquid kerosene has been mixed in a Getzmann dissolver with 7.5 mass-% Thixatrol ST and 7.5 mass-% ketone (5-methyl-2-hexanon). Rheological measurements were conducted with a Haake RS1 rotational rheometer and a Rosand RH2000 capillary rheometer.

The pipe flow of non-Newtonian fluids, especially for fluids with a viscosity characteristic following the Ostwald-de-Waele or power-law equation $\eta_{PL} = K \cdot \dot{\gamma}^{n-1}$, was investigated in the past for example by Böhme,[2] Dodge & Metzner,[5] Malin,[9] Mishra & Triphathi[11] and Ryan.[15] For the identification of different flow regimes or the determination of dynamic similitude, Metzner & Reed[10] introduced a generalized Reynolds number $Re_{gen\,PL}$ valid for pure power-law fluids. This number was derived from its relation to the Darcy friction factor f_{Darcy} and is shown in Equation 2.

$$Re_{gen\,PL} = \frac{\rho \cdot D^n \cdot \bar{u}^{2-n}}{K \cdot \left(\frac{3n+1}{4n}\right)^n \cdot 8^{n-1}} \tag{2}$$

The fluids investigated in the present study, however, have a more complex viscosity characteristic that can not be described by the power-law all over the relevant shear rate range $10^{-2}s^{-1} \leq \dot{\gamma} \leq 10^6 s^{-1}$. The left graph in Figure 1 shows the theoretical approach of the power-law theory (PL) to the experimentally determined viscosity of the kerosene-gel. It can be seen, that especially in the high shear rate range there is not a good agreement between the theory and the experiments.

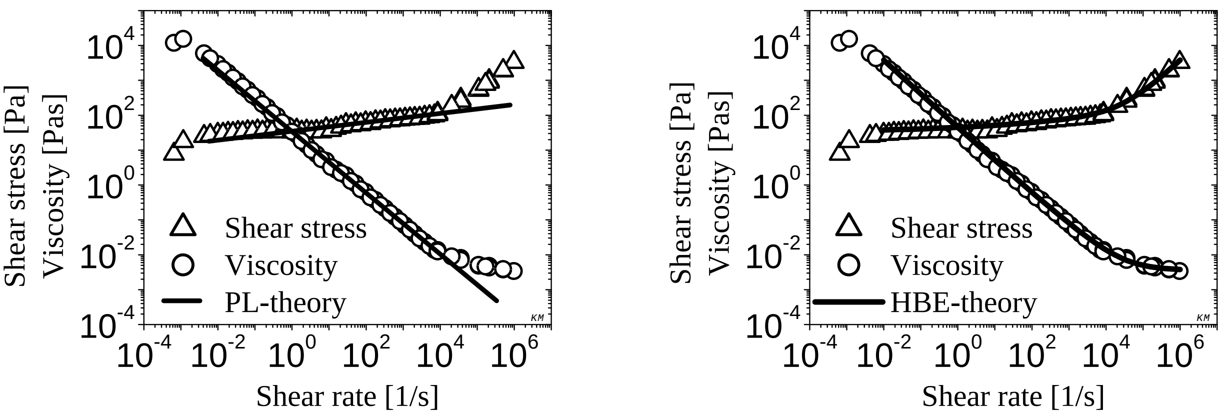

Figure 1: Viscosity measurements for a kerosene-gel and theoretical approach with power-law (left) and Herschel-Bulkley-Extended (right)

For a better description of that viscosity characteristic, Madlener & Ciezki[7] introduced the **H**erschel-**B**ulkley-**E**xtended-equation (HBE-equation) as an extended version of the Herschel-Bukley law. To the term considering the existence of a yield stress τ_0 and to the power-law term $K \cdot \dot{\gamma}^{n-1}$, an additional viscosity term η_∞ was added as the constant viscosity in the very high shear rate range (for more information please see Madlener & Ciezki[7]). The definition of the HBE-theory is shown likewise in the Equations 3 and 4.

$$\eta_{HBE} = \frac{\tau_0}{\dot{\gamma}} + K \cdot \dot{\gamma}^{n-1} + \eta_\infty \tag{3}$$

$$\text{with } \tau = \eta \cdot \dot{\gamma} \text{ in general} \quad \rightarrow \quad \tau_{HBE} = \tau_0 + K \cdot \dot{\gamma}^n + \eta_\infty \cdot \dot{\gamma} \tag{4}$$

In the right diagram of Figure 1 the theoretical approach to the experimental viscosity data is shown for the HBE-law. Compared to the approach with the PL-theory in the left graph, the HBE-theory describes the experimental results over the entire relevant shear rate range. The determined HBE-parameter for the kerosene-gel were $\tau_0 = 33\,Pa$, $n = 0.19$, $K = 11.76\,Pa \cdot s^{0.19}$ and $\eta_\infty = 0.0036\,Pa \cdot s$.

Since the power-law theory is not capable to describe the viscosity characteristic of the here examined fluids over the entire relevant shear rate range, the power-law-based Reynolds number $Re_{gen\,PL}$ from Equation 2 can not be used to characterize the pipe flow of such fluids either. For the characterization of flow and spray regimes of fluids following the HBE-viscosity type from Equation 3, a HBE-generalized Reynolds number $Re_{gen\,HBE}$ was derived and is presented below.

2. HBE-generalized Reynolds number

Metzner and Reed[10] derived their generalized Reynolds number $Re_{gen\,PL}$ from its relation to the Darcy friction factor. For laminar and fully developed pipe flow exists a relation between the Reynolds number and the Darcy friction factor, which is generally given for fluids independent of their viscosity characteristic, see herefore Hutter.[6] This relation is given in Equation 5, whereas the definition of the Darcy friction factor is given in Equation 6.

$$Re = \frac{64}{f_{Darcy}} \tag{5}$$

$$f_{Darcy} = \frac{\left(\frac{-\delta p}{\delta l}\right) \cdot D}{\frac{1}{2}\,\rho\,\bar{u}^2} \tag{6}$$

The expression $\frac{-\delta p}{\delta l}$ is the pressure drop per unit length, D is the pipe diameter, ρ the fluid density and $\bar{u}$ the average fluid velocity. The relation between pressure loss Δp and the wall shear stress τ_w is calculated by the equilibrium of forces over the pipe with $\tau_w = \frac{D}{4\,L} \cdot \Delta p$, whereas L is the pipe length and Δp the macroscopic pressure drop. The Darcy friction factor from Equation 6 can then be written as

$$f_{Darcy} = \frac{8\,\tau_w}{\rho\,\bar{u}^2}\ . \tag{7}$$

The wall shear stress τ_w in equation 7 can be calculated by the viscosity type of the examined fluid. For a Newtonian liquid there is a constant relation between the wall shear stress τ_w and the wall shear rate $\dot{\gamma}_w$ with $\tau_w = \eta \cdot \dot{\gamma}_w$, whereas η is the constant Newtonian viscosity. The wall shear rate could then be calculated from the Newtonian velocity profile in pipe flow with $\dot{\gamma}_w = 8\,\bar{u}/D$. With that, the Darcy friction factor would yield to $f_{Darcy} = (64\,\eta)/(\rho\,\bar{u}\,D)$, which equals the relation of Equation 5. For a non-Newtonian fluid of the HBE-type, however, the wall shear stress τ_w has to be calculated by the appropriate viscosity law from Equation 4.

If the shear stress τ_w in Equation 7 is replaced by its definition from Equation 4, the shear rate $\dot{\gamma}$ has to be determined. It is defined as the negative gradient of the velocity profile, so $\dot{\gamma} = -du/dr$. Regarding the velocity gradient at the wall (indicated with w), Equation 4 yields to the following expression.

$$\tau_w = \tau_0 + K \cdot \left(-\frac{du}{dr}\right)_w^n + \eta_\infty \cdot \left(-\frac{du}{dr}\right)_w \tag{8}$$

Rabinowitsch[13] and Mooney[12] developed an expression for the wall shear rate $\dot{\gamma}_w = -(du/dr)_w$ independend of fluid properties and thus valid also for non-Newtonian fluids.

$$\left(-\frac{du}{dr}\right)_w = \frac{3}{4}\left(\frac{8\,\bar{u}}{D}\right) + \frac{1}{4}\left(\frac{8\,\bar{u}}{D}\right)\frac{d\,ln(8\,\bar{u}/D)}{d\,ln(D\,\Delta p/4L)} \tag{9}$$

The expression $8\,\bar{u}/D$ corresponds to the wall shear rate in case of Newtonian fluid flow and is named apparent wall shear rate $\dot{\gamma}_{app\,w}$. The expression $D\Delta p/4L$ corresponds to the wall shear stress τ_w. So the logarithmic expression in Equation 9 can be replaced with the reciprocal value of the local gradient m, which displays the gradient of the shear stress τ_w at a certain apparent wall shear rate $\dot{\gamma}_{app\,w}$ (Note: For a power-law fluid the local gradient m would be identical with the global power-law exponent n, for HBE-fluids this is not the case).

$$\frac{d\,ln(8\bar{u}/D)}{d\,ln(D\Delta p/4L)} = \frac{d\,ln(\dot{\gamma}_{app\,w})}{d\,ln(\tau_w)} = \frac{1}{m} \tag{10}$$

$$\left(\frac{-du}{dr}\right)_w = \frac{3m+1}{4m} \cdot \frac{8\bar{u}}{D} \tag{11}$$

With the expressions from the Equations 8 and 11 the Darcy friction factor from Equation 7 can be written as follows.

$$\begin{aligned} f_{Darcy} &= 8 \cdot \frac{\tau_0 + K\left(\frac{3m+1}{4m}\right)^n \left(\frac{8\bar{u}}{D}\right)^n + \eta_\infty \frac{3m+1}{4m}\frac{8\bar{u}}{D}}{\rho\,\bar{u}^2} \\ &= 64 \cdot \frac{\frac{\tau_0}{8}\left(\frac{D}{\bar{u}}\right)^n + K\left(\frac{3m+1}{4m}\right)^n 8^{n-1} + \eta_\infty \frac{3m+1}{4m}\left(\frac{D}{\bar{u}}\right)^{n-1}}{\rho\,\bar{u}^{2-n} D^n} \end{aligned} \tag{12}$$

The generalized Reynolds number for laminar and fully developed pipe flow valid for non-Newtonian fluids with a viscosity characteristic following the Herschel-Bulkley-Extended equation can then be determined by inserting Equation 12 in Equation 5.

$$Re_{gen\,HBE} = \frac{\rho\,\bar{u}^{2-n}\,D^n}{\frac{\tau_0}{8}\left(\frac{D}{\bar{u}}\right)^n + K\left(\frac{3m+1}{4m}\right)^n 8^{n-1} + \eta_\infty \frac{3m+1}{4m}\left(\frac{D}{\bar{u}}\right)^{n-1}}$$
$$\text{with} \quad m = \frac{n \cdot K\left(\frac{8\bar{u}}{D}\right)^n + \eta_\infty\left(\frac{8\bar{u}}{D}\right)}{\tau_0 + K\left(\frac{8\bar{u}}{D}\right)^n + \eta_\infty\left(\frac{8\bar{u}}{D}\right)} \tag{13}$$

Whereas m is the local gradient of the shear stress versus shear rate graph. It was determined by the differentiation of the logarithmic expression for the HBE-equation according Equation 14. τ_0, K, n and η_∞ are the HBE fluid parameters for the viscosity behaviour from Equation 3, ρ is the fluid density, $\bar{u}$ the average fluid velocity and D the pipe diameter.

$$m = \frac{d\,ln\,(\tau_w)}{d\,ln\left(\dot{\gamma}_{app\,w}\right)} = \frac{d\,ln\left(\tau_0 + K \cdot \dot{\gamma}^n_{app\,w} + \eta_\infty \cdot \dot{\gamma}_{app\,w}\right)}{d\,ln\left(\dot{\gamma}_{app\,w}\right)}$$
$$m = \frac{d\,ln\left(\tau_0 + Ke^{n\,ln\left(\dot{\gamma}_{app\,w}\right)} + \eta_\infty e^{ln\left(\dot{\gamma}_{app\,w}\right)}\right)}{d\,ln\left(\dot{\gamma}_{app\,w}\right)}$$
$$m = \frac{n \cdot K \cdot \dot{\gamma}^n_{app\,w} + \eta_\infty \cdot \dot{\gamma}_{app\,w}}{\tau_0 + K \cdot \dot{\gamma}^n_{app\,w} + \eta_\infty \cdot \dot{\gamma}_{app\,w}} = \frac{n \cdot K\left(\frac{8\bar{u}}{D}\right)^n + \eta_\infty\left(\frac{8\bar{u}}{D}\right)}{\tau_0 + K\left(\frac{8\bar{u}}{D}\right)^n + \eta_\infty\left(\frac{8\bar{u}}{D}\right)} \tag{14}$$

It is claimed, that the introduced HBE-generalized Reynolds number $Re_{gen\,HBE}$ from Equation 13 is valid not only for fluids with a viscosity characteristic of the HBE-type, but also for all viscosity laws included in that equation. Those are the Herschel-Bulkley, the Ostwald-de-Waele (power-law), the Bingham and the Newtonian law. The reduced viscosity laws and their corresponding generalized Reynolds numbers are listed in the following.

Herschel-Bulkley-Extended (HBE):

$\eta = \frac{\tau_0}{\dot{\gamma}} + K \cdot \dot{\gamma}^{n-1} + \eta_\infty$ $\quad\rightarrow\quad Re_{gen\,HBE} = ...$ see Equation 13

Herschel-Bulkley (HB):

$\eta_\infty = 0 \quad\rightarrow\quad \eta = \frac{\tau_0}{\dot{\gamma}} + K \cdot \dot{\gamma}^{n-1} \quad\rightarrow\quad Re_{gen\,HB} = \frac{\rho\,\bar{u}^{2-n}\,D^n}{\frac{\tau_0}{8}\left(\frac{D}{\bar{u}}\right)^n + K\left(\frac{3m+1}{4m}\right)^n 8^{n-1}}$ with $m = \frac{n \cdot K\left(\frac{8\bar{u}}{D}\right)^n}{\tau_0 + K\left(\frac{8\bar{u}}{D}\right)^n}$

Power-law (PL):

$\tau_0 = 0,\ \eta_\infty = 0 \quad\rightarrow\quad \eta = K \cdot \dot{\gamma}^{n-1} \quad\rightarrow\quad Re_{gen\,PL} = \frac{\rho\,\bar{u}^{2-n}\,D^n}{K\left(\frac{3m+1}{4m}\right)^n 8^{n-1}}$ with $m = n$ (see Eq. 2)

Bingham:

$K = 0,\ n = 1 \quad\rightarrow\quad \eta = \frac{\tau_0}{\dot{\gamma}} + \eta_\infty \quad\rightarrow\quad Re_{gen\,Bingham} = \frac{\rho\,\bar{u}\,D}{\frac{\tau_0}{8}\left(\frac{D}{\bar{u}}\right) + \eta_\infty\frac{3m+1}{4m}}$ with $m = \frac{\eta_\infty\left(\frac{8\bar{u}}{D}\right)}{\tau_0 + \eta_\infty\left(\frac{8\bar{u}}{D}\right)}$

Newton:

$\tau_0 = 0,\ K = 0,\ n = 1 \quad\rightarrow\quad \eta = \eta_\infty \quad\rightarrow\quad Re_{Newton} = \frac{\rho\,\bar{u}\,D}{\eta_\infty}$ with $m = 1$ (see Eq. 1)

It shall be mentioned, that for a power-law fluid, the HBE-generalized Reynolds number corresponds to the generalized Reynolds number by Metzner and Reed from Equation 2. In case of a Newtonian liquid the HBE-generalized Reynolds number reduces itself to the Newtonian Reynolds number from Equation 1. Hence the HBE-generalized Reynolds number can be applied to non-Newtonian fluids with the mentioned viscosity characteristics above, as well as to Newtonian liquids.

3. Validation and experimental results

For the evaluation of the introduced HBE-generalized Reynolds number the Darcy friction factor f_{Darcy} is plotted versus the HBE-generalized Reynolds number $Re_{gen\,HBE}$. They can be determined with their definitions in the Equation

6 and 13. Both definitions require the information about the fluid properties, the pipe geometry and the measured pressure loss per volumetric flow rate. The examined test fluids (TF1-TF5) were the two Newtonian liquids paraffin and kerosene and three gels based on those two liquids. Their compositions are listed in Table 1.

	Basic fuel	**Gellant**	**Additive**
TF1 (Newtonian)	100% Paraffin	–	–
TF2 (Newtonian)	100% Kerosene	–	–
TF3 (non-Newtonian)	85% Paraffin	7.5% Thixatrol ST	7.5% Miak
TF4 (non-Newtonian)	96% Paraffin	4.0% Aerosil 200	–
TF5 (non-Newtonian)	85% Kerosene	7.5% Thixatrol ST	7.5% Miak

Table 1: Composition of investigated test fuels in wt.-%

The viscosity behaviour of the test fuels could be described by the HBE-equation from Equation 3 over the entire relevant shear rate range $10^{-2}s^{-1} \leq \dot{\gamma} \leq 10^{6}s^{-1}$. The corresponding HBE-parameters needed for the calculation of the HBE-generalized Reynolds number are shown in Table 2.

	TF1	**TF2**	**TF3**	**TF4**	**TF5**
τ_0	–	–	45 Pa	83 Pa	33 Pa
K	–	–	$5,07\,\mathrm{Pas^n}$	$2,54\,\mathrm{Pas^n}$	$11,76\,\mathrm{Pas^n}$
n	1	1	0,38	0,57	0,19
η_∞	$0,026\,\mathrm{Pas}$	$0,0012\,\mathrm{Pas}$	$0,026\,\mathrm{Pas}$	$0,026\,\mathrm{Pas}$	$0,0036\,\mathrm{Pas}$

Table 2: HBE-parameters of the test fuels TF1-TF5

Experiments were conducted herefore with a Rosand RH2000 capillary rheometer. In a capillary rheometer the examined fluid is driven by a piston from the reservoir through a capillary. Pressure losses were messured at several volumetric flow rates with capillaries of diameters $D = 0.2$ mm and $D = 0.3$ mm and lengths $L = 16$ mm and $L = 24$ mm. Necessary corrections were applied to the raw data (Bagley correction, Rabinowitsch correction, wall slip analysis).

The results of the experiments for the liquid kerosene (TF2) and the kerosene-gel (TF5) are plotted in Figure 2, for the liquid paraffin and the two paraffin-gels in Figure 3. The values of the Darcy friction factor were calculated with Equation 6 and the corresponding HBE-generalized Reynolds numbers with the definition from Equation 13. The density of the kerosene-based fuels was $\rho = 800\,\mathrm{kg/m^3}$, the density of the paraffin-based fuels was $\rho = 818\,\mathrm{kg/m^3}$.

For the liquid kerosene and the kerosene-gel the relation $f_{Darcy} = 64/Re$ for laminar flow (indicated with the solid line) is successfully achieved up to Reynolds numbers of $Re_{gen\,HBE} \approx 1000$. Then, compared to the laminar relation, a slight decrease of the friction factor values occure, before in the range between $2000 < Re_{gen\,HBE} < 3000$ the experimental results show a distinct increase with a separation of the obtained data from the line indicating laminar flow conditions. It is assumed, that this increase is indicating the transition from laminar to turbulent flow. It has to be mentioned that this transition was measured for the Newtonian liquid kerosene as well as for the non-Newtonian kerosene-gel. There is still no definition for the critical HBE-generalized Reynolds number valid for non-Newtonian fluids. (Dodge & Metzner[5] and Ryan & Johnson[15] made investigations herefore, which, however, are only valid for purely power-law fluids and cannot be applied to the fluids investigated in the present paper.) Since the parameter $m \approx 0.96$ at those Reynolds numbers indicates that the viscosity characteristic has almost reached the Newtonian plateau, it is assumed, that the transition from laminar to turbulent flow conditions for the kerosene-gel occures near the critical Reynolds numbers of Newtonian liquids ($Re_{crit\,Newton} \approx 2300$).

For the liquid paraffin and the two paraffin-gels the experimental results are plotted in Figure 3. The data show a good agreement with the laminar relation (solid line) up to $Re_{gen\,HBE} \approx 600$. The higher viscosity of the paraffin-based fluids leads to lower Reynolds numbers for the same flow conditions, compared to the kerosene-based fuels. The transition from laminar to turbulent flow was not reached for the conducted experiments. The slight decrease, which was obtained for the kerosene-based fuels before the laminar-turbulent transition, seems to be more pronounced for the paraffin-based fuels. There is not yet an explanation for this decrease, because experimental data are not available for higher Reynolds numbers and further investigations are necessary for its verification.

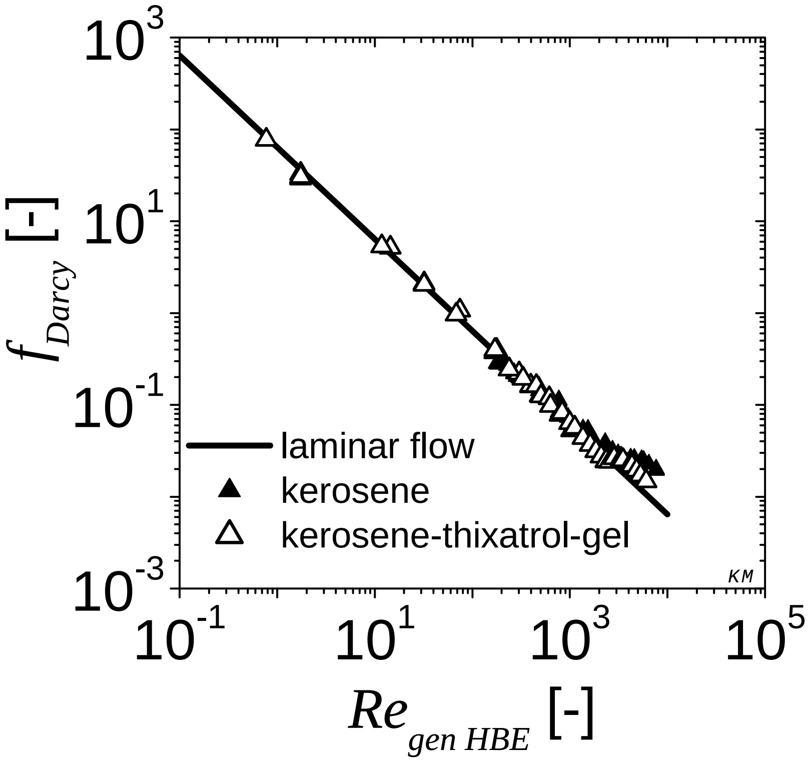

Figure 2: Friction factor versus HBE-generalized Reynolds number for liquid and gelled propellants based on kerosene

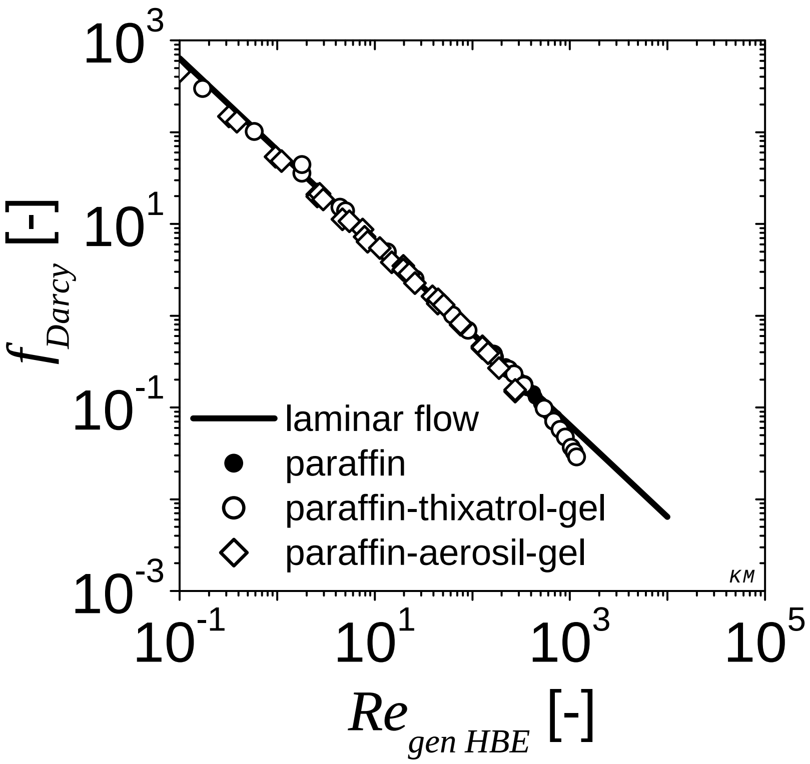

Figure 3: Friction factor versus HBE-generalized Reynolds number for liquid and gelled propellants based on paraffin

4. Summary

Because of their non-Newtonian shear-thinning viscosity behaviour, gelled propellants are interesting candidates for rocket propulsion systems. They offer to build propulsion systems with the similar simple handling and storage characteristics like engines with solid propellants and have at the same time, the advantages of an enginge with liquid propellants like thrust variation. In the present paper a HBE-generalized Reynolds number $Re_{gen\,HBE}$ was suggested to characterize the pipe flow of the here presented non-Newtonian gelled fluids. This number is based on the Herschel-Bulkley-Extended (HBE) viscosity law. Experimental data showed a good correlation in the laminar regime for low Reynolds numbers. For a non-Newtonian kerosene-gel the transition from laminar to turbulent flow conditions could be determined for Reynolds numbers between $2000 < Re_{gen\,HBE} < 3000$. Since the HBE viscosity law also includes several well-known viscosity laws (Herschel-Bulkley, power-law, Bingham and Newtonian), the introduced HBE-generalized Reynolds number $Re_{gen\,HBE}$ can be reduced to the Reynols numbers valid for the corresponding fluid ($Re_{gen\,HB}$, $Re_{gen\,PL}$, $Re_{gen\,Bingham}$ and Re_{Newton}).

Notation

D	pipe diameter
f_{Darcy}	Darcy friction factor
K	pre-factor of power-law
L	pipe length
m	local exponential factor
n	global exponential factor
Re	Reynolds number (in general)
Re_{HBE}	Reynolds number for HBE-fluids
Re_{HB}	Reynolds number for HB-fluids
Re_{PL}	Reynolds number for PL-fluids
$Re_{Bingham}$	Reynolds number for Bingham fluids
Re_{Newton}	Reynolds number for Newtonian fluids
r	radial coordinate
$u_{(r)}$	velocity
$\bar{u}$	average flow velocity

Greek letters

τ	shear stress
τ_0	yield stress
τ_w	wall shear stress
ρ	density
η	viscosity
η_{HBE}	viscosity law for HBE-fluids
η_{PL}	viscosity law for PL-fluids
Δp	pressure loss

Abreviations and subscripts

HBE	Herschel-Bulkley-Extended
HB	Herschel-Bulkley
PL	Power-law
app	apparent
w	wall

References

[1] Bartels N., von Kampen J., Ciezki H.K., Begnini M. Investigation of the spray behaviour of gelled JetA1/Aerosil-mixtures with a like-on-like impinging jet atomizer. *34th Int. Conf. of ICT*, Karlsruhe, Germany, 2003

[2] Böhme G. Strömungsmechanik nichtnewtonscher Fluide. Teubner Verlag, 2. Auflage, 2000

[3] Brinker C.J., Scherer G.W. Sol-Gel Science. Academic Press, Boston, 1990

[4] Ciezki H.K., Natan B. An Overview of Investigations on Gel Fuels for Ramjet Applications. *ISABE 2005-1065, 17th Int. Symposium on Airbreathing Systems*, München, Germany, 2005

[5] Dodge D.W., Metzner A.B. Turbulent flow of non-Newtonian systems. *A.I.Ch.E. Journal*, 5:2:198–204, 1959

[6] Hutter K. Fluid- und Thermodynamik. Springer Verlag, 2. Auflage, 2002

[7] Madlener K., Ciezki H.K. Analytical description of the flow behaviour of Extended-Herschel-Bulkley fluids with regard to gel propellants. *36th Int. Conf. of ICT & 32nd Int. Pyrotecnics Seminar*, Karlsruhe, Germany, 2005

[8] Madlener K., Ciezki H.K. Theoretical investigation of the flow behaviour of gelled fuels of the Extended Herschel Bulkley type. *1st European Conference for Aerospace Science (EUCASS)*, Moscow, Russia, 2005

[9] Malin M.R. Turbulent pipe flow of Herschel-Bulkley fluids. *Int. Comm. Heat Mass Transfer*, 25:321–330, 1998

[10] Metzner A.B., Reed J.C. Flow of non-Newtonian fluids - correlation of the laminar, transition and turbulent-flow regions. *A.I.Ch.E. Journal*, 1:4:434–440, 1955

[11] Mishra P., Triphathi G. Transition from laminar to turbulent flow of purely viscous non-Newtonian fluids in tubes. *Ch.E. Science*, 26:915–921, 1970

[12] Mooney M. Explicit formulas for slip and fluidity. *Journal of Rheology*, 2:210–222, 1931

[13] Rabinowitsch B. Über die Viskosität und Elastizität von Solen. *Z. physik. Chem.*, 145A, 1929

[14] Ryan H.M., Anderson W.E., Pal S., Santoro R.J. Atomization characteristics of impinging liquid jets. *Journal of Propulsion and Power*, 11:1:135–145, 1995

[15] Ryan N.W., Johnson M.M., Transition from laminar to turbulent flow in pipes. *A.I.Ch.E. Journal*, 5:4:433–435, 1959